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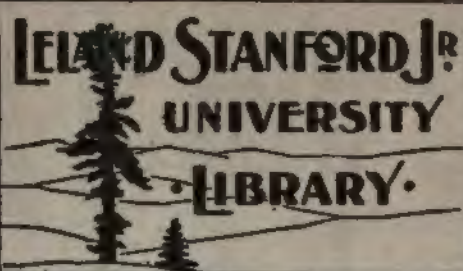
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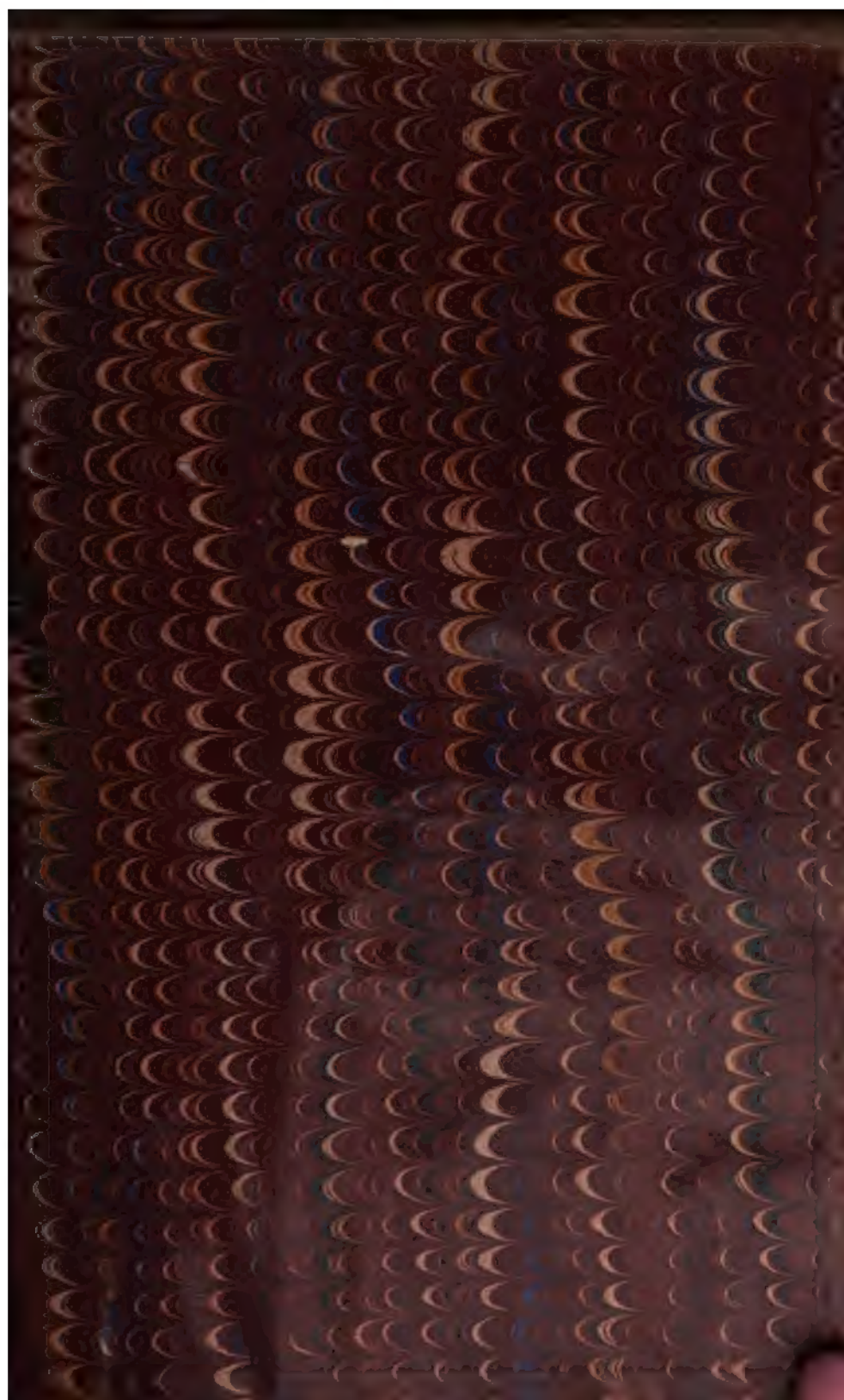
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PREPARED FOR STUDENTS OF
THE INTERNATIONAL CORRESPONDENCE SCHOOLS
/ /
SCRANTON, PA.

Volume I

ARITHMETIC
SPELLING
PENMANSHIP
VERTICAL PENMANSHIP
WITH PRACTICAL QUESTIONS AND EXAMPLES
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First Edition

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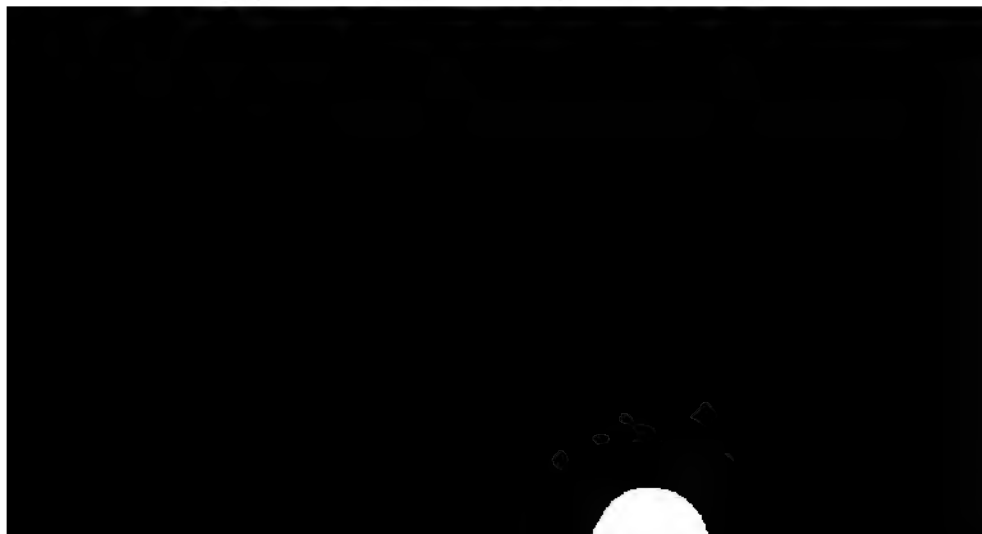
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PREFACE.

In this set of bound volumes are comprised all the Instruction and Question Papers contained in our Complete Commercial Course. These papers have been prepared with great care and are particularly well adapted to the wants of business men. The instruction given is such that a person who completes the Course should have no trouble in satisfactorily performing the duties of a bookkeeper or a stenographer. Each subject taken up is thoroughly and fully explained, and ample opportunity is given for the practical application of the knowledge gained.

The papers on Arithmetic constitute a thorough, complete, and practical treatise on this subject; the arrangement differs greatly from that of any Arithmetic heretofore published, and we believe it to be the most logical and practical of any yet devised.

The papers on Grammar and Punctuation were written by Prof. Wm. B. Ridenour, A.M., a gentleman of established reputation as a successful educator and writer of standard school textbooks. They are original in their treatment of the subject, and form a textbook that will be of great value to all classes.

The papers on Letter Writing have been prepared along somewhat novel lines, and are very complete in their treatment of the subject. To be able to write a good letter is an accomplishment—an art—that cannot be too highly valued, and we have endeavored to give the student such information as will enable him to become a proficient letter writer.

The papers on Bookkeeping form a course of instruction that illustrates and explains the books and business papers in common use, and give considerable actual business practice in connection with the theory of bookkeeping.

The system of Stenography which we have adopted is without doubt the shortest and simplest method of shorthand in use. It is strictly phonetic, and is so arranged that there are practically no exceptions to general rules; more phrasing material is available, and at the same time there are fewer difficult and duplicate forms used than in other systems.

The method of numbering the pages, cuts, articles, etc. is such that each paper and part is complete in itself; hence, in order to make the indexes intelligible, it was necessary to give each paper and part a number. This number is placed at the top of each page, on the headline, opposite the page number; and to distinguish it from the page number, it is preceded by the printer's section mark §. Consequently, a reference such as § 17, page 12, would be readily found as follows: The back stamp on each volume shows the sections (i. e., papers) included in the volume, that for Vol. II reading §§ 14-23; hence, look in Vol. II along the headlines until § 17 is found, and then through § 17 until page 12 is found.

The Question Papers are given the same section numbers as the Instruction Papers to which they belong, and are grouped together at the end of the volumes containing the Instruction Papers to which they refer.

There are three volumes in the set, the subjects being arranged as follows:

Vol. I (§§ 1-13) contains the Instruction and Question Papers on Arithmetic, Spelling, Penmanship, and Vertical Penmanship. The Keys to the Question Papers on Arith-



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ARITHMETIC.

DEFINITIONS.

1. Arithmetic is the science of numbers and the art of computation.

2. A unit is *one*, or a single thing, as *one* boy, *one* horse, *one*, *one* dozen.

3. A number is a unit or a collection of units, as *three* apples, *five* boys, *seven*.

4. The unit of a number is one of the units included in the collection of units forming the number. Thus, the unit of *twelve* is *one*, of *twenty* dollars is *one* dollar.

5. A concrete number is a number applied to some particular kind of object or quantity, as three *horses*, five *dollars*, ten *pounds*.

6. An abstract number is a number not applied to any object or quantity, as *three*, *five*, *ten*.

7. Like numbers are numbers that express units of the *same kind*, as 6 *days* and 10 *days*, 2 *feet* and 5 *feet*.

8. Unlike numbers are numbers that express units of *different kinds*, as ten *months* and eight *miles*, seven *dollars* and five *feet*.

§ 1

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NOTATION AND NUMERATION.

9. Numbers are expressed in three ways: (1) By words.
(2) By figures. (3) By letters.

10. **Notation** is the art of expressing numbers by figures or letters.

11. **Numeration** is the art of reading numbers expressed by figures or letters.

ARABIC NOTATION.

12. The **Arabic notation** is the method of expressing numbers by figures. This method employs ten characters, called **figures**, to represent numbers, viz. :

Figures	0	1	2	3	4	5	6	7	8	9
Names	<i>naught, one cipher, or zero</i>	<i>two</i>	<i>three</i>	<i>four</i>	<i>five</i>	<i>six</i>	<i>seven</i>	<i>eight</i>	<i>nine</i>	

The first figure (0) is called **naught**, **cipher**, or **zero**, and, when standing alone, has no value.

The other nine figures are called **digits**, and each one has a value of its own.

16. The **simple** value of a figure is the value it expresses when standing alone.

17. The **local**, or **place**, value of a figure is its value as determined by its position in a number.

Thus, 6 standing alone means <i>six ones</i>	6
In the second place it denotes <i>six tens</i>	60
In the third place it denotes <i>six hundreds</i>	600
In the fourth place it denotes <i>six thousands</i>	6,000
In the fifth place it denotes <i>six ten-thousands</i>	60,000
In the sixth place it denotes <i>six hundred-thousands</i>	600,000
In the seventh place it denotes <i>six millions</i>	6,000,000

18. The **value** of a figure increases tenfold with each remove to the left.

19. The **cipher** has no value in itself, but it is useful in fixing the place of other figures. To represent the number *four hundred five*, only two significant figures are necessary, one to denote *four hundred*, and the other to denote *five*; but if these two figures are placed together, as 45, the 4, being in the second place, will mean 4 *tens*. To denote 4 *hundreds* it should be in the third place. A cipher, therefore, must be inserted in the *tens* place to show that the number is composed of *hundreds* and *units* only, and that there are no *tens*. *Four hundred five* is, therefore, written 405. If the number were *four thousand five*, two ciphers would be inserted, thus, 4,005. If it were *four hundred fifty*, the cipher would be in units place to show that there are no *units*, but only *hundreds* and *tens*, thus, 450. *Four thousand fifty* is written 4,050, the ciphers indicating that there are no hundreds and no units.

20. In reading numbers, it is usual to divide them by commas into groups of three figures each, called **periods**, beginning at the right. The first figure is said to belong to the *first order*, the second to the *second order*, etc. Each **period** contains three orders, named as shown in the table below.

TABLE.

<i>Name of period.</i>	<i>Trillions.</i>	<i>Billions.</i>	<i>Millions.</i>	<i>Thousands.</i>	<i>Units.</i>
<i>Name of order.</i>	hundred-trillions. ten-trillions. trillions.	hundred-billions. ten-billions. billions.	hundred-millions. ten-millions. millions.	hundred-thousands. ten-thousands. thousands.	hundreds. tens. units.
<i>Number.</i>	9 8 7, 4 3 2, 1 0 8, 7 6 5, 4 3 2				

The first period, beginning at the right, contains *units, tens, hundreds*; the second, *thousands, ten-thousands, hundred-thousands*; the third, *millions, ten-millions, hundred-millions*; etc.

The number in the table is read, *nine hundred eighty-seven trillion, four hundred thirty-two billion, one hundred ninety-eight million, seven hundred sixty-five thousand, four hundred thirty-two.*

21. The writing of numbers is called **notation**, and the reading of numbers is called **numeration**. It will be noticed that in reading and writing numbers the *s* at the end of thousands, millions, etc., is omitted.

ROMAN NOTATION.



V, D, and L are never repeated, and only I, X, C, and M are ever used more than once in any combination.

2. *If a letter precedes one of greater value, their difference is denoted; if it follows, their sum is denoted.*

Thus, IV = 4, VI = 6, IX = 9, XI = 11, XL = 40, LX = 60.

3. *A bar placed over a letter multiplies its value by one thousand.*

Thus, \overline{X} = 10,000, \overline{L} = 50,000, $\overline{XCDXVII}$ = 90,517.

23. The following table illustrates more fully the foregoing principles:

VII	= 7	LX	= 60	CLXIX	= 169	\overline{IXLX}	= 9060
XIII	= 13	LXIX	= 69	CLXXX	= 180	\overline{LCXC}	= 50190
XIV	= 14	LXX	= 70	CCXL	= 240	\overline{XIX}	= 19000
XV	= 15	LXXX	= 80	CCCLIX	= 359	\overline{XLDX}	= 40510
XX	= 20	XC	= 90	CCCCL	= 450	\overline{XCVII}	= 97000
XXV	= 25	XCIX	= 99	CCCCXC	= 490	\overline{DCC}	= 700000
XXVIII	= 28	CIX	= 109	DCCXL	= 740	\overline{XCIX}	= 99000
XXIX	= 29	CXI	= 111	DCCCXI	= 811	\overline{M}	= 1000000
XXX	= 30	CXIX	= 119	DCCCC	= 900	\overline{MM}	= 2000000
XL	= 40	CLVI	= 156	DCCCCXL	= 940	\overline{MDCC}	= 1700000

24. The four fundamental processes of arithmetic are **addition, subtraction, multiplication, and division.** They are called fundamental processes because all operations in arithmetic are based upon them.

ADDITION.

25. Addition is the *process* of *finding* a number that is equal to two or more numbers taken together. The sign of addition is +. It is read *plus*, and means *more*. Thus, 5 + 6 is read 5 *plus* 6, and means that 5 and 6 are to be added.

26. The sign of equality is $=$. It is read *equals*, or *is equal to*. Thus, $5 + 6 = 11$ may be read, *5 plus 6 equals 11*.

27. Only like numbers can be added. Thus, 6 dollars can be added to 7 dollars, and the sum will be 13 dollars, but 6 *dollars* cannot be added to 7 *feet*.

28. The following table gives the sum of any two numbers from 1 to 12:

1 and 1 is 2	2 and 1 is 3	3 and 1 is 4	4 and 1 is 5
1 and 2 is 3	2 and 2 is 4	3 and 2 is 5	4 and 2 is 6
1 and 3 is 4	2 and 3 is 5	3 and 3 is 6	4 and 3 is 7
1 and 4 is 5	2 and 4 is 6	3 and 4 is 7	4 and 4 is 8
1 and 5 is 6	2 and 5 is 7	3 and 5 is 8	4 and 5 is 9
1 and 6 is 7	2 and 6 is 8	3 and 6 is 9	4 and 6 is 10
1 and 7 is 8	2 and 7 is 9	3 and 7 is 10	4 and 7 is 11
1 and 8 is 9	2 and 8 is 10	3 and 8 is 11	4 and 8 is 12
1 and 9 is 10	2 and 9 is 11	3 and 9 is 12	4 and 9 is 13
1 and 10 is 11	2 and 10 is 12	3 and 10 is 13	4 and 10 is 14
1 and 11 is 12	2 and 11 is 13	3 and 11 is 14	4 and 11 is 15
1 and 12 is 13	2 and 12 is 14	3 and 12 is 15	4 and 12 is 16
5 and 1 is 6	6 and 1 is 7	7 and 1 is 8	8 and 1 is 9
5 and 2 is 7	6 and 2 is 8	7 and 2 is 9	8 and 2 is 10
5 and 3 is 8	6 and 3 is 9	7 and 3 is 10	8 and 3 is 11
5 and 4 is 9	6 and 4 is 10	7 and 4 is 11	8 and 4 is 12
5 and 5 is 10	6 and 5 is 11	7 and 5 is 12	8 and 5 is 13
5 and 6 is 11	6 and 6 is 12	7 and 6 is 13	8 and 6 is 14
5 and 7 is 12	6 and 7 is 13	7 and 7 is 14	8 and 7 is 15
5 and 8 is 13	6 and 8 is 14	7 and 8 is 15	8 and 8 is 16
5 and 9 is 14	6 and 9 is 15	7 and 9 is 16	8 and 9 is 17
5 and 10 is 15	6 and 10 is 16	7 and 10 is 17	8 and 10 is 18
5 and 11 is 16	6 and 11 is 17	7 and 11 is 18	8 and 11 is 19
5 and 12 is 17	6 and 12 is 18	7 and 12 is 19	8 and 12 is 20
9 and 1 is 10	10 and 1 is 11	11 and 1 is 12	12 and 1 is 13
9 and 2 is 11	10 and 2 is 12	11 and 2 is 13	12 and 2 is 14

29. For *addition*, place the numbers to be added directly under each other, taking care to place *units* under *units*, *tens* under *tens*, *hundreds* under *hundreds*, and so on.

30. EXAMPLE.—What is the sum of 181, 222, 21, 2, and 413?

SOLUTION.—

$$\begin{array}{r}
 181 \\
 222 \\
 21 \\
 2 \\
 413 \\
 \hline
 \text{sum } 789 \text{ Ans.}
 \end{array}$$

EXPLANATION.—After placing the numbers in proper order, begin at the bottom of the units column and add, mentally repeating the different sums. Thus, three, five, six, eight, nine, the sum of the numbers in the units column. Place the 9 directly beneath, as the units figure in the sum.

The sum of the numbers in the tens column is 8, which is the tens figure in the sum.

The sum of the numbers in the hundreds column is 7, which is the hundreds figure in the sum.

31. EXAMPLE.—What is the sum of 61,803 + 43,429 + 47,712 + 62,188?

SOLUTION.—

$$\begin{array}{r}
 61803 \\
 43429 \\
 47712 \\
 62188 \\
 \hline
 22 \\
 60 \\
 2000 \\
 13000 \\
 200000 \\
 \hline
 \text{sum } 215082 \text{ Ans.}
 \end{array}$$

EXPLANATION.—The sum of the units column is 22; of the tens column it is 6 tens, or 60; of the hundreds column it is 20 hundreds, or 2,000; of the thousands column it is 13 thousands, or 13,000; and of the ten-thousands column the sum is 20 ten-thousands, or 200,000. The sum of these

numbers is 215,082. Ordinarily, the work would be performed as follows, the unnecessary cipher being omitted;

$$\begin{array}{r}
 61803 \\
 43429 \\
 47712 \\
 62138 \\
 \hline
 22 \\
 6 \\
 20 \\
 18 \\
 20 \\
 \hline
 \text{sum } 215082
 \end{array}$$

This method is very convenient to use when adding a long column of figures.

32. In practice, addition is performed as follows:

$$\begin{array}{r}
 425 \\
 36 \\
 9215 \\
 4 \\
 907 \\
 \hline
 \text{sum } 10587 \text{ Ans.}
 \end{array}$$

EXPLANATION.—The sum of the numbers in the units column = 27 units, or 2 tens and 7 units. Write the 7 units as the first, or right-hand, figure in the sum. Reserve the 2 tens and add them to the tens column. The sum of the

33. EXAMPLE.—Add the following numbers:

SOLUTION.—	890
	82
	90
	893
	281
	80
	770
	83
	492
	80
	383
	84
	191
	—
sum	3899
	Ans.

EXPLANATION.—The sum of the first column is 19, or 1 ten and 9 units. Write 9 and carry 1 to the next column. The sum of the second column + 1 = 109 tens, or 10 hundreds and 9 tens. Write 9 and carry 10 to the next column. The sum of this column plus the 10 reserved is 38. Write 38. The total sum is 3,899. Ans.

34. Rule.—I. *Begin at the right, add each column separately, and write the sum, if it be only one figure, under the column added.*

II. *If the sum of any column consists of two or more figures, put the right-hand figure of the sum under that column and add the remaining figure or figures to the next column.*

35. Proof.—*To prove addition, add each column from top to bottom. If the same result is obtained as by adding from bottom to top, the work is probably correct.*

EXAMPLES FOR PRACTICE.

36. Find the sum of:

- (a) 104 + 203 + 613 + 214.
- (b) 1,875 + 3,143 + 5,826 + 10,832.
- (c) 4,865 + 2,145 + 8,173 + 40,084.
- (d) 14,204 + 8,173 + 1,065 + 10,042.
- (e) 10,832 + 4,145 + 3,133 + 5,872.
- (f) 214 + 1,231 + 141 + 5,000.
- (g) 123 + 104 + 425 + 126 + 327.
- (h) 6,854 + 2,145 + 2,042 + 1,111 + 3,333.

- | | | | |
|------|---|-----|---------|
| Ans. | { | (a) | 1,134. |
| | | (b) | 21,676. |
| | | (c) | 55,267. |
| | | (d) | 88,484. |
| | | (e) | 23,982. |
| | | (f) | 6,586. |
| | | (g) | 1,105. |
| | | (h) | 14,985. |

RAPID ADDITION.

37. There is nothing more useful to the bookkeeper and business man than the ability to add rapidly and correctly; but this can be acquired only by persevering practice. Any time employed in practising addition will be well spent. If the student will practise addition ten or fifteen minutes daily for a month or so, he will be greatly benefited.

In order to become expert in adding, it is absolutely essential that when two figures are seen or heard pronounced, the student can instantly give their sum. Thus, 15 should suggest itself as soon as 6 and 9, or 8 and 7, are seen, or heard pronounced. It should not be necessary to say mentally, 6 and 9 are 15, but 6, 15, the 9 not being pronounced either mentally or orally. (In no case should the student contract the habit of adding aloud; it is not necessary, and it is a very difficult habit to break.)

In adding 5, 6, 1, 9, 7, 5, 2, 4, 8, 9, do not say 5 and 6 is 11, and 1 is 12, and 9 is 21, etc., but *think* 5, 11, 12, 21, 28, 33, 35, 39, 47, 56, repeating the sums about as fast as they can be pronounced.

While rapidity is of great importance, accuracy is more important, and accuracy is attained only by practice. Below will be found some examples on which the student can practise. He should construct similar ones for himself.

(1)	(2)	(3)	(4)	(5)
4175	0286	3582	48275	2081753

add the sums of the groups. It is most convenient to choose the groups, as far as possible, so that the sum of each group shall be either 10 or 20. An example will show how this is accomplished:

3	}	Commencing at the bottom of the column, we see at once that 1 and 9 form a group whose sum is 10; hence we mentally say 2, 12 instead of 2, 3, 12. Now the next two figures, 5 and 3, we group together, and instead of adding 5 and 3 separately, we add the sum 8.
9		
2		
5	}	The next two figures, 6 and 4, form a group whose sum is 10, and so do the next two figures, 8 and 2.
8		
2	}	Looking now at the four figures at the top of the column, we readily see that 5, 2, and 3 form a last group whose sum is 10, leaving only the 9 outside of a group. In adding the column, we would repeat mentally 2, 12, 20, 30, 40, 50, 59. Another grouping would readily appear to the skilful accountant; the 2 at the bottom and the 5 and 3 above the 0 form a group whose sum is 10. Recognizing this group, the mental addition would be: 10, 20, 30, 40, 50, 59.
6		
4	}	
3		
5		
0	}	
9		
1		
2	}	
59		

This process of forming groups may be extended to include those whose sums are 15 or 20. By a judicious selection of

5	}	20	the figures composing a group, its sum may usually be made either 10, 15, or 20, and these sums should always be sought in preference to others, since two 15's make 30, and the numbers 10, 20, and 30 are added with little mental labor. Thus, in the following example, the grouping is shown by the braces. The mental addition is 15, 30, 37, 47, 67, 72. After some practice, the student will be able to recognize a group whose sum is 10 or 20 almost instantly; he should persevere in the solution of examples until he is able to form the groups rapidly and with ease. It is not always possible to get consecutive numbers which will form groups whose sums are 10, 15, or 20. Such groups, however, can often be found by skipping one or more figures in the column. For instance, in the following example, the first two figures, 7 and 3, make a 10, and skipping the fourth
9			
3			
8	}	10	
6			
4	}	15	
7			
2			
9	}	15	
4			
8	}	15	
7			
72			

figure 9, the third and fifth figures, 4 and 6, make another 10. Skipping the 8, the 3, 4, and 3 at the top of the column make another 10, thus making three groups of 10 and the figures 9 and 8. In adding, however, we should not leave the figures skipped to be added at the end of the operation, but should add them as they occur. In this example, we would say 10, 20, 29, 39, 47. The order of adding may be shown by the following arrangement:

$$\begin{array}{r} 7 \\ \hline 47 \end{array} \quad \begin{array}{c} 10 \quad 10 \quad 10 \\ \hline 7+3+6+4+9+3+4+3+8. \end{array}$$

The following examples may be used for practice:

(1)	(2)	(3)	(4)	(5)
8	18	817	1127	18108
5	27	226	2632	61706
2	92	282	1946	48285
6	85	516	3217	89184
3	64	896	1184	96286
4	22	435	1936	45173
0	80	707	2003	78135
9	97	992	5114	26282
3	18	811	976	19875
1	83	417	5684	8428

The answers are: (1) 36; (2) 431; (3) 4,551; (4) 25,769; (5) 490,857.

39. It is frequently advantageous to be able to add

by groups of 10, as it is taught; but this is meant the adding of

the amount of grain exported each day, the total amount of each kind of grain exported during the week, and, finally, the total amount of grain exported during the week.

GRAIN EXPORT OF A CITY FOR ONE WEEK (in bushels).

	<i>Mon.</i>	<i>Tues.</i>	<i>Wed.</i>	<i>Thur.</i>	<i>Fri.</i>	<i>Sat.</i>	<i>Totals.</i>
Corn.....	28325	15236	35715	29128	75183	46217	*****
Wheat.....	35719	41719	50108	32546	59275	81126	*****
Oats.....	12136	9237	18265	7268	6950	17290	*****
Barley.....	18230	15738	21375	15928	19263	13637	*****
Rye.....	5275	6829	7201	11325	7825	13261	*****
Totals.....	*****	*****	*****	*****	*****	*****	*****

The student should find the totals, and prove that the results are correct by adding the totals in the right-hand column, and then adding the totals in the bottom row; the two results should be the same, viz., 757,270 bushels. The other results are: corn, 229,804; wheat, 300,493; oats, 71,086; barley, 104,171; rye, 51,716; Mon., 99,685; Tues., 88,759; Wed., 132,664; Thurs., 96,195; Fri., 168,496; Sat., 171,471.

EXAMPLES FOR PRACTICE.

40. Find the sums of the following:

(1)	(2)	(3)	(4)	(5)	(6)
4568	15431	7386	49850	6542	62165
7391	29685	45371	17370	63834	16732
7854	73648	13764	68429	76343	85696
58469	34519	9887	23156	80931	71883
18470	78234	64348	21017	79883	50149
58143	7843	14627	67154	83578	31572
			64353	35647	76844

EXPLANATION.—Having found the sums of the above, add the sums horizontally, and thus obtain the total of all the numbers. Ans.: (1) 144,895; (2) 239,360; (3) 155,383;

(4) 311,329; (5) 426,758; (6) 395,041; total, 1,672,766. Add all of the above groups horizontally, beginning with the numbers in the top row. Ans.: Top row, 145,942; second row, 180,383; third row, 325,734; fourth row, 273,845; fifth row, 307,101; sixth row, 262,917; seventh row, 176,844.

SUBTRACTION.

41. In arithmetic, **subtraction** is the process of finding how much greater one number is than another.

The greater of the two numbers is the **minuend**.

The lesser of the two numbers is the **subtrahend**.

The number left after subtracting the subtrahend from the minuend is the **difference**, or **remainder**.

42. The sign of subtraction is —. It is read **minus**, and means *less*. Thus, $12 - 7$ is read *12 minus 7*, and means that 7 is to be taken from 12.

43. EXAMPLE.—From 7,568 take 3,425.

SOLUTION.—

<i>minuend</i>	7 5 6 8	
<i>subtrahend</i>	3 4 2 5	
<i>remainder</i>	4 1 4 3	Ans.

EXPLANATION.—Begin at the units column and subtract in succession each figure in the subtrahend from the one directly above it in the minuend, and write the remainders below the line. The result is the remainder.

44. When there are more figures in the minuend than in the subtrahend, and when some figures in the minuend are less than the figures directly under them in the subtrahend, proceed as in the following example:

First Method.—EXAMPLE.—From 8,458 take 844.

SOLUTION.—

<i>minuend</i>	8 4 5 8	
<i>subtrahend</i>	8 4 4	
<i>remainder</i>	7 6 0 9	Ans.

EXPLANATION.—Begin at the units column to subtract. We cannot take 4 from 3, and must, therefore, borrow 1 *ten* from 5 in *tens* column, and add, or prefix, it to the 3 units, making 13 units. Then, 4 from 13 equals 9, which is written as the units figure of the remainder.

Since 1 *ten* was borrowed from 5 *tens*, only 4 *tens* remain; 4 from 4 equals 0, the *tens* figure of the remainder.

We cannot take 8 *hundreds* from 4 *hundreds*, and must borrow 1 *thousand* from 8 in the *thousands* column. This equals 10 *hundreds*; and 10 *hundreds* plus 4 *hundreds* equals 14 *hundreds*; 8 from 14 is 6, the *hundreds* figure of the remainder.

Since 1 *thousand* was borrowed from 8 *thousands*, only 7 *thousands* remain. There being nothing to subtract from this, the *thousands* figure of the remainder is 7.

45. Second Method.—**EXAMPLE.**—From 84,532 take 8,447.

SOLUTION.—

<i>minuend</i>	8 4 5 3 2	
<i>subtrahend</i>	8 4 4 7	
	<hr/>	
<i>remainder</i>	7 6 0 8 5	Ans.

EXPLANATION.—As in the preceding example, 7 is taken from 12, leaving 5. Now, instead of subtracting 4 from 12, as explained above, 1 is added to 4, and the sum is subtracted from 13, leaving 8. That is, when 1 is borrowed from the minuend it is added to that figure of the subtrahend under the figure of the minuend from which it was borrowed. Continuing, 5 from 5 is 0, 8 from 14 is 6, and 1 from 8 is 7.

The method here described should be thoroughly learned, since it is easier than the first to apply in practice, and has other advantages that the student will doubtless notice.

46. It often happens that there are ciphers in the minuend, from which, of course, nothing can be borrowed. The following example will explain the difficulty:

EXAMPLE.—From 20,000 take 8,763.

SOLUTION.—

<i>minuend</i>	2 0 0 0 0	
<i>subtrahend</i>	8 7 6 3	
	<hr/>	
<i>remainder</i>	1 1 2 3 7	Ans.

EXPLANATION.—Although 1 cannot be borrowed from tens column, it is customary to regard the ciphers as tens and increase each figure in the subtrahend by 1, explained in Art. 44. Then, in performing the above subtraction, the process would be as follows: 3 from 10 is 7; 7 from 10 is 3; 8 from 10 is 2; 9 from 10 is 1; and 1 from 2 is 1.

47. Rule.—*Write the subtrahend under the minuend in the same manner as for addition, with the units of the same order standing in the same column.*

Commencing at the right, subtract each figure in the lower number from the one above it, and write the difference in the line below.

If any figure in the lower number is greater than the one above it, add 10 to the upper figure, perform the subtraction, and then add 1 to the next figure on the left, in the lower number. So proceed with the remaining figures.

48. Proof.—*Add the remainder to the subtrahend. The sum should equal the minuend. If not, the work is wrong.*

Proof of the last example:

subtrahend	8763
remainder	<u>11287</u>
minuend	20000

49. There is still another method of subtraction better



Since $5 + 0 = 5$, for the next figure in the subtrahend we write 0, as shown. There is nothing to carry; hence, since 8 is greater than 4, we consider 4 to be 14, and find what number added to 8 will make 14; this is 6, which write below the line, as shown. We have 1 to carry, but, as we have no figure in the subtrahend to add it to, we say 1 and 7 is 8, and write the 7 below the line, as shown.

Again, consider the following example:

$$\begin{array}{r}
 \text{minuend} \quad 10000 \\
 \text{subtrahend} \quad 8763 \\
 \hline
 \text{remainder} \quad 1237 \text{ Ans.}
 \end{array}$$

Here we say 3 and 7 is 10, and write the 7; then, 7 and 3 is 10 (carrying the 1 and adding it to the 6), and write the 3; next, 8 and 2 is 10, and write the 2; finally, 9 and 1 is 10, and write the 1. The student may use whichever method he prefers, but we strongly recommend the addition method as being less productive of errors.

EXAMPLES FOR PRACTICE.

50. From:

- (a) 94,278 take 62,574.
- (b) 53,714 take 25,824.
- (c) 71,832 take 58,109.
- (d) 20,804 take 10,408.
- (e) 310,465 take 102,141.
- (f) $(81,043 + 1,041)$ take 14,831.
- (g) $(20,482 + 18,216)$ take 21,214.
- (h) $(2,040 + 1,213 + 542)$ take 3,791.

$$\text{Ans.} \left\{ \begin{array}{l}
 (a) \quad 31,704. \\
 (b) \quad 27,890. \\
 (c) \quad 13,723. \\
 (d) \quad 10,396. \\
 (e) \quad 208,324. \\
 (f) \quad 67,253. \\
 (g) \quad 17,484. \\
 (h) \quad 4.
 \end{array} \right.$$

51. In many cases it is inconvenient to write the subtrahend *under* the minuend. For example, it might be required to find the sum of a series of numbers, and to subtract this sum from some other number. In such cases, both time and space may be saved by writing the minuend under the subtrahend, drawing the line, and then subtracting downwards.

The rule given in Art. 47 may be applied to this case by changing the words *under* to read *above*; *above* to read *below*; *lower* to read *upper*; and *upper* to read *lower*.

EXAMPLE.—Writing the subtrahend above the minuend, subtract 5,267,148 from 10,342,927.

SOLUTION.—

subtrahend	5 2 6 7 1 4 8
minuend	1 0 3 4 2 9 2 7
remainder	5 0 7 5 7 7 9 Ans

EXAMPLES FOR PRACTICE.

52. In the following examples, subtract the upper number from the lower:

(1)	(2)	(3)	(4)
28456789	37529510	9890978	19827
<u>80706040</u>	<u>43184296</u>	<u>100010001</u>	<u>84862</u>
(5)	(6)	(7)	(8)
7090508	9897960	81907	3161746789
<u>8090403</u>	<u>123456789</u>	<u>94871</u>	<u>4218150000</u>

Ans.—(1) 57,249,251; (2) 5,654,786; (3) 90,119,023; (4) 64,535;
(5) 999,895; (6) 113,558,829; (7) 12,464; (8) 1,051,403,211.

GENERAL REMARKS ON SUBTRACTION.



subtrahend above the minuend, without making a mistake, he may then strive for rapidity. A rapid calculator will subtract as fast as he can write the figures of the result.

It will be a great help to the student if he will reverse the addition table. At odd moments, when walking or working, let him say to himself, 6 from 11 is how much? 9 from 17 is what? etc., and in a short time the right answer will present itself without any mental effort whatever.

54. Addition and subtraction form an extremely important part of a bookkeeper's work. The books must *balance*, as it is termed; i. e., the sum of the columns on the debit side must *exactly* equal the sum of the columns on the credit side. If an error of even one cent is made it will manifest itself in the trial balance, and a day or more may be spent in finding the error. Hence, the importance of accuracy. It is also important that no time be wasted in adding the columns, and in subtracting the sums of the two columns to find the balance. To illustrate, we give part of a page of a ledger. The double vertical lines separate the debit side on the left from the credit side on the right.

DR.				PRICE & HOWARD,				CR.			
1888						1888					
June	4	Merchandise,	\$1898	43	June	14	Merchandise,	\$1638	38		
"	20	"	149	37	"	30	Cash,	500			
July	8	Sundries,	115	26	July	17	Bills Receivable,	900			
Aug.	16	Merchandise,	1386	07	Aug.	1	Cash,	875			
"	30	Sundries,	490	63	"	31	Merchandise,	984	68		
Sept.	19	Merchandise,	1065	75	Oct.	1	Balance,	793	14		
			\$5061	40				\$5061	40		
Oct.	1	Balance,	\$ 793	14							

Now, what the bookkeeper has to do is to add the two columns and subtract the less sum from the greater; then he must write the difference in the column that contains the less sum, and also write it under the total of the column containing the greater sum. This difference is indicated by the word *balance*. In order not to mar the appearance of the ledger, no figures except those written above should be used. If the

By the same method perform the following:

	(5)	(6)	(7)
<i>minuend</i>	\$801.28	\$4,214.60	9284621
<i>subtrahend</i>	$\left\{ \begin{array}{l} 24.99 \\ 63.75 \\ 9.87 \\ 48.84 \\ 59.98 \end{array} \right.$	$\left\{ \begin{array}{l} 875.95 \\ 469.98 \\ 93.49 \\ 721.74 \\ 1803.42 \end{array} \right.$	$\left\{ \begin{array}{l} 897749 \\ 3456987 \\ 747678 \\ 498765 \\ 1768496 \end{array} \right.$
<i>remainder</i>			

Ans.— (1) 84,047; (2) 110,790; (3) 81,726; (4) 662,370; (5) \$94.50;
(6) \$250.02; (7) 1,864,953.

MULTIPLICATION.

57. To multiply a number is to *add* the number to itself a certain number of times.

58. Multiplication is the process of multiplying one number by another.

The *number* thus added to itself, or the number to be multiplied, is called the **multiplicand**.

The *number* that shows how many times the *multiplicand* is to be taken, or the *number* by which we *multiply*, is called the **multiplier**.

The result obtained by multiplying is called the **product**.

59. The sign of multiplication is \times . It is read *times*,

1 times 1 is 1	2 times 1 is 2	3 times 1 is 3
1 times 2 is 2	2 times 2 is 4	3 times 2 is 6
1 times 3 is 3	2 times 3 is 6	3 times 3 is 9
1 times 4 is 4	2 times 4 is 8	3 times 4 is 12
1 times 5 is 5	2 times 5 is 10	3 times 5 is 15
1 times 6 is 6	2 times 6 is 12	3 times 6 is 18
1 times 7 is 7	2 times 7 is 14	3 times 7 is 21
1 times 8 is 8	2 times 8 is 16	3 times 8 is 24
1 times 9 is 9	2 times 9 is 18	3 times 9 is 27
1 times 10 is 10	2 times 10 is 20	3 times 10 is 30
1 times 11 is 11	2 times 11 is 22	3 times 11 is 33
1 times 12 is 12	2 times 12 is 24	3 times 12 is 36
4 times 1 is 4	5 times 1 is 5	6 times 1 is 6
4 times 2 is 8	5 times 2 is 10	6 times 2 is 12
4 times 3 is 12	5 times 3 is 15	6 times 3 is 18
4 times 4 is 16	5 times 4 is 20	6 times 4 is 24
4 times 5 is 20	5 times 5 is 25	6 times 5 is 30
4 times 6 is 24	5 times 6 is 30	6 times 6 is 36
4 times 7 is 28	5 times 7 is 35	6 times 7 is 42
4 times 8 is 32	5 times 8 is 40	6 times 8 is 48
4 times 9 is 36	5 times 9 is 45	6 times 9 is 54
4 times 10 is 40	5 times 10 is 50	6 times 10 is 60
4 times 11 is 44	5 times 11 is 55	6 times 11 is 66
4 times 12 is 48	5 times 12 is 60	6 times 12 is 72
7 times 1 is 7	8 times 1 is 8	9 times 1 is 9
7 times 2 is 14	8 times 2 is 16	9 times 2 is 18
7 times 3 is 21	8 times 3 is 24	9 times 3 is 27
7 times 4 is 28	8 times 4 is 32	9 times 4 is 36
7 times 5 is 35	8 times 5 is 40	9 times 5 is 45
7 times 6 is 42	8 times 6 is 48	9 times 6 is 54
7 times 7 is 49	8 times 7 is 56	9 times 7 is 63
7 times 8 is 56	8 times 8 is 64	9 times 8 is 72
7 times 9 is 63	8 times 9 is 72	9 times 9 is 81
7 times 10 is 70	8 times 10 is 80	9 times 10 is 90
7 times 11 is 77	8 times 11 is 88	9 times 11 is 99
7 times 12 is 84	8 times 12 is 96	9 times 12 is 108
10 times 1 is 10	11 times 1 is 11	12 times 1 is 12
10 times 2 is 20	11 times 2 is 22	12 times 2 is 24
10 times 3 is 30	11 times 3 is 33	12 times 3 is 36
10 times 4 is 40	11 times 4 is 44	12 times 4 is 48
10 times 5 is 50	11 times 5 is 55	12 times 5 is 60
10 times 6 is 60	11 times 6 is 66	12 times 6 is 72
10 times 7 is 70	11 times 7 is 77	12 times 7 is 84
10 times 8 is 80	11 times 8 is 88	12 times 8 is 96
10 times 9 is 90	11 times 9 is 99	12 times 9 is 108
10 times 10 is 100	11 times 10 is 110	12 times 10 is 120
10 times 11 is 110	11 times 11 is 121	12 times 11 is 132
10 times 12 is 120	11 times 12 is 132	12 times 12 is 144

This table should be carefully committed to memory.

Since 0 has no value, the product of 0 and any number is 0.

62. Practise the following until you can give them all readily without referring to the table.

3×7	13×9	9×2	4×4	6×7
4×8	7×8	5×9	6×3	13×8
5×12	9×6	7×4	10×11	10×4
6×11	10×10	9×11	8×11	8×10
7×10	11×11	10×7	7×12	4×7
8×9	12×12	8×8	9×10	10×8
9×5	7×7	4×5	10×6	12×2
5×8	8×5	5×10	11×9	4×12
8×10	2×8	6×12	12×11	8×2
4×3	2×12	7×9	2×5	11×3
3×11	9×3	6×10	3×8	11×2
2×10	3×12	6×6	6×2	11×6
5×4	8×3	12×5	10×5	12×10
10×9	3×6	8×2	11×12	10×2
11×8	6×4	9×12	12×4	4×11
8×4	5×11	12×6	7×5	2×7
5×3	12×7	11×5	2×6	3×4
6×8	9×9	4×9	3×5	2×4
7×2	8×12	5×5	4×2	12×3
9×8	3×3	7×6	5×2	9×4
8×7	4×10	9×7	6×9	10×12
3×9	5×7	10×8	7×11	11×4
4×6	6×5	11×10	7×3	2×3
5×6	11×7	2×11	2×9	8×6

63. To multiply any number by a number of one figure.

EXAMPLE.—Multiply 425 by 5.

Solution.—*multiply and* 425



tens, or 1 hundred plus 2 tens. Write the 2 tens in tens place, and reserve the 1 hundred to add to the product of hundreds. Multiplying the third, or last, figure of the multiplicand by the multiplier 5, the result is 20 hundreds, which plus the 1 hundred reserved, is 21 hundreds, or 2 thousands 1 hundred, which we write in the thousands and hundreds places, respectively.

Hence, the product is 2,125.

This result is the same as the sum of five 425's. Thus,

$$\begin{array}{r}
 425 \\
 425 \\
 425 \\
 425 \\
 425 \\
 \hline
 \text{sum } 2125 \text{ Ans.}
 \end{array}$$

EXAMPLES FOR PRACTICE.

64. Find the product of:

- (a) $61,483 \times 6.$
- (b) $12,375 \times 5.$
- (c) $10,426 \times 7.$
- (d) $10,835 \times 3.$
- (e) $98,876 \times 4.$
- (f) $10,873 \times 8.$
- (g) $71,543 \times 9.$
- (h) $218,734 \times 2.$

$$\text{Ans. } \left\{ \begin{array}{l}
 (a) \quad 368,898. \\
 (b) \quad 61,875. \\
 (c) \quad 72,982. \\
 (d) \quad 32,505. \\
 (e) \quad 393,504. \\
 (f) \quad 86,984. \\
 (g) \quad 643,887. \\
 (h) \quad 437,468.
 \end{array} \right.$$

65. To multiply a number by a number of two or more figures.

EXAMPLE.—Multiply 475 by 234.

$$\begin{array}{r}
 \text{SOLUTION.—} \quad \text{multiplicand} \quad 475 \\
 \quad \quad \quad \text{multiplier} \quad 234 \\
 \hline
 \quad \quad \quad 1900 \\
 \quad \quad 1425 \\
 \quad 950 \\
 \hline
 \text{product } 111150 \text{ Ans.}
 \end{array}$$

EXPLANATION.—For convenience, the multiplier is generally written under the multiplicand, placing units under units, tens under tens, etc.

We cannot multiply by 234 at one operation; we must,

therefore, multiply by the parts and then add the **partial products**.

The parts by which we are to multiply are 4 units, 3 tens, and 2 hundreds. 4 times 475 = 1,900, the first partial product; 3 times 475 = 1,425, the second partial product, the right-hand figure of which is written directly under the figure multiplied by, or 3; 2 times 475 = 950, the third partial product, the right-hand figure of which is written directly under the figure multiplied by, or 2.

The sum of these three partial products is 111,150, which is the entire product.

66. Rule. I. Write the multiplier under the multiplicand, so that units are under units, tens under tens, etc.

II. Begin at the right, and multiply each figure of the multiplicand by each successive figure of the multiplier, placing the right-hand figure of each partial product directly under the figure used as a multiplier.

III. The sum of the partial products will be the required product.

67. Proof.—Review the work carefully, or multiply the multiplier by the multiplicand; if the results agree, the work is correct.

68. The student will find the following test useful in determining whether the answer in multiplication is correct:

Find the sum of the digits in the multiplicand. If the sum consists of more than one figure, add the digits of the sum, and so continue until the sum is one figure. Do the same with the multiplier. Multiply together the final sums thus obtained, and if the result consists of more than one figure, add its digits until one figure is obtained. If this result is the same as is obtained by adding the digits of the product until one figure is obtained, the work is probably correct.

To illustrate, multiply 837,295 by 4,631.

SOLUTION.—	<i>multiplicand</i>	837295
	<i>multiplier</i>	4631
	<i>product</i>	3877513145

PROOF.— $8+8+7+2+9+5 = 34; 3+4 = 7$
 $4+6+3+1 = 14, 1+4 = 5$
 $35; 3+5 = 8.$
 $8+8+7+7+5+1+8+1+4+5 = 44, 4+4 = 8.$

The proof given in this article is not absolute, because two or more errors might cause the product to fulfil the conditions of the test. But if, upon trial, the final sum of the digits of the product does not agree with that of the product of the final sums of the multiplicand and the multiplier, it is certain that the work is wrong.

69. There are many short methods of multiplication, and some of these will be given under the heading "Aliquot Parts." It is important that the student should notice the abbreviation that is possible when a cipher occurs in the multiplier, and when the multiplicand or the multiplier ends with one or more ciphers.

70. EXAMPLE.—Multiply 49,076 by 40,807.

SOLUTION.—	<i>multiplicand</i>	49076
	<i>multiplier</i>	40807
		<hr style="width: 100px; border: 0.5px solid black;"/>
		343532
	partial products {	392608
		196304
		<hr style="width: 100px; border: 0.5px solid black;"/>
	product	2002644332 Ans.

EXPLANATION.—The process is exactly the same as the preceding, except that when a cipher occurs in the multiplier it is not used to multiply by, the next digit of the multiplier being used instead. The first figure of the partial product is always written directly under the figure by which we multiply, as stated in the rule for multiplication, Art. 66.

EXAMPLE.—Multiply 49,076 by 48,700.

SOLUTION.—	<i>multiplicand</i>	49076
	<i>multiplier</i>	48700
		<hr style="width: 100px; border: 0.5px solid black;"/>
		343532
	partial products {	392608
		196304
		<hr style="width: 100px; border: 0.5px solid black;"/>
	product	2390001200 Ans.

EXPLANATION.—Here the multiplier consists of three digits

and two ciphers, as in the preceding examples, but in this case the two ciphers occupy the units and tens places in the multiplier. Write the multiplier as shown, so that the ciphers lie to the right of the right-hand figure of the multiplicand, or, in other words, so that the right-hand *digit* of the multiplier lies under the right-hand digit of the multiplicand. Then, without paying attention to the ciphers on the right of the multiplier, multiply in the usual manner, annexing the two ciphers (the number of ciphers to the right of the right-hand digit of the multiplier) to the product, as shown. If the multiplicand ends in ciphers, the process is exactly the same; thus, multiplying 4,907,600 by 487:

$$\begin{array}{r}
 \text{multiplicand} \quad 4907600 \\
 \text{multiplier} \quad \underline{487} \\
 \text{partial products } \left\{ \begin{array}{r} 343532 \\ 392608 \\ 196304 \end{array} \right. \\
 \text{product} \quad \underline{2390001200} \text{ Ans.}
 \end{array}$$

If both multiplicand and multiplier end in ciphers, place the right-hand digits under each other, as above, and add to the product as many ciphers as are contained in both multiplicand and multiplier on the right of their right-hand digits.

EXAMPLE.—Multiply 590,000 by 420.

$$\begin{array}{r}
 \text{SOLUTION.—} \quad \text{multiplicand} \quad 590000 \\
 \text{multiplier} \quad \underline{420} \\
 \quad \quad \quad 118 \\
 \quad \quad \quad 236 \\
 \text{product} \quad \underline{24780000} \text{ Ans.}
 \end{array}$$

71. It would be well to apply the principle given in Art. 68 to all cases of multiplication, until the student has attained confidence. Thus, in the first example of Art. 70, the number obtained by adding the digits of the multiplicand is $4+7+9+6=26$, and $2+6=8$; by adding the digits of the multiplier, $4+8+7=19$, $1+9=10$, and $1+0=1$. Whence, $8 \times 1 = 8$. Adding the digits of the product, $2+2+6+4+4+3+3+2=26$, or $2+6=8$; hence, the work is very probably correct.

72. To be able to multiply rapidly is almost as valuable to a bookkeeper or business man as to be able to add rapidly. The only way that any one can become expert in multiplying is by practice. To give the student a good idea of how a rapid multiplier would proceed, we will now give the entire process pursued, choosing two numbers whose digits consist of 7's, 8's, and 9's only, as these are the hardest to use.

EXAMPLE. — $987789 \times 897 = ?$

SOLUTION.—	987789	sum of digits = 48 = 12, or 3
	897	sum of digits = 24, or 6
	6014523	18, or 9
	8890101	
	7902312	
	886046733	sum of digits = 45, or 9.

EXPLANATION.—Say 7 times 9 is 63, write the 3 and carry 6. Say 7 times 8 is 56 and 6 is 62; write the 2 and carry 6. Say 7 times 7 is 49 and 6 is 55; write the 5 and carry 5. Say 7 times 7 is 49 and 5 is 54; write the 4 and carry 5. Say 7 times 8 is 56 and 5 is 61; write the 1 and carry 6. Say 7 times 9 is 63 and 6 is 69, write the 9. We multiply by 9 and 8 in the same way, and then prove the work by the test given in Art. 68.

73. After the student has attained considerable proficiency in multiplying as described in Art. 72, he may shorten his work considerably by merely repeating the digit by which he multiplies, the product of that digit and the desired digit in the multiplicand, and the sum of this product and the number carried. Thus, instead of saying 7 times 8 is 56 and 6 is 62, think 7, 56, 62. In other words, in multiplying 987,789 by 9, think 9, 81 (write the 1); 9, 72, 80 (write the 0); 9, 63, 71; 9, 54, 70; 9, 45, 79; and, finally, 9, 81, 88. By practising this method for some time, he should be able to multiply nearly as fast as he can write the results.

74. Besides working the examples for practice which follow, the student should make up many others and work them out. He should continue to do this until he can

multiply rapidly, and with ease and certainty. This remark applies, also, to addition and subtraction, and to division, which follows. He should study the multiplication table until he can name the product of any two numbers between 1 and 12 instantly, without any hesitation whatever. He can learn to do this only by repeating the table over and over again. The result attained will be well worth the time and labor spent.

EXAMPLES FOR PRACTICE.

75. Find the product of:

(a) $3,842 \times 26.$	(a) 99,892.
(b) $3,716 \times 45.$	(b) 167,220.
(c) $1,817 \times 124.$	(c) 225,308.
(d) $675 \times 88.$	(d) 25,650.
(e) $1,875 \times 83.$	(e) 61,875.
(f) $4,836 \times 47.$	(f) 227,292.
(g) $5,682 \times 543.$	(g) 3,085,826.
(h) $3,257 \times 246.$	(h) 801,222.
(i) $2,875 \times 302.$	(i) 868,250.
(j) $17,819 \times 1,004.$	(j) 17,890,276.
(k) $38,674 \times 205.$	(k) 7,928,170.
(l) $18,304 \times 100.$	(l) 1,830,400.
(m) $7,834 \times 10.$	(m) 78,340.
(n) $87,543 \times 1,000.$	(n) 87,543,000.
(o) $48,763 \times 100.$	(o) 4,876,300.

78. The **divisor** is the number by which the dividend is divided.

79. The **quotient** is the number showing how many times the dividend contains the divisor. In *partition* the quotient shows one of the equal parts of the dividend.

80. The **sign of division** is \div . It is read *divided by*. Thus, $54 \div 9$ denotes that 54 is to be divided by 9. In this case 54 is the *dividend* and 9 is the *divisor*.

81. To divide when the divisor consists of but one figure.

EXAMPLE.—What is the quotient of $875 \div 7$?

divisor	dividend	quotient
7	875	125 Ans.
	7	
	17	
	14	
	35	
	35	
remainder	0	

EXPLANATION.— 7 is contained in 8 hundreds 1 hundred times. Place the 1 as left-hand figure of the quotient. Multiply the divisor 7 by the 1 hundred of the quotient, and place the product, 7 hundreds, under the 8 hundreds in the dividend, and subtract. On the right of the remainder 1, bring down the next, or *tens* figure of the dividend, in this case 7, making 17 tens; 7 is contained in 17, 2 times. Write the 2 as the second figure of the quotient. Multiply the divisor 7 by the 2 in the quotient, and subtract the product from 17. To the remainder, 3, annex the next, or *units* figure of the dividend, in this case 5, making 35 units. 7 is contained in 35, 5 times, which is placed in the quotient. Multiplying the divisor by the last figure of the quotient, 5 times 7 = 35, which subtracted from 35, under which it is placed, leaves 0. Therefore, the quotient is 125. This method is called **long division**.

82. In **short division**, only the divisor, dividend, and quotient are written.

$$\begin{array}{r} \text{divisor } 7 \overline{) 8175} \\ \text{quotient } 125 \text{ Ans.} \end{array}$$

The operation is as follows: 7 is contained in 8 once and 1 remainder; 1 placed before 7 makes 17; 7 is contained in 17 2 times and 3 over; the 3 placed before 5 makes 35, 7 is contained in 35, 5 times. These partial quotients placed in order as they are found, make the entire quotient, 125.

83. If the divisor consists of 2 or more figures, proceed as in the following example:

EXAMPLE.—Divide 2,702,826 by 63.

$$\begin{array}{r} \text{divisor } 63 \overline{) 2702826} \\ \text{SOLUTION. —} \quad 252 \\ \quad 182 \\ \quad 126 \\ \quad \hline \quad 568 \\ \quad 567 \\ \quad \hline \quad 126 \\ \quad 126 \\ \quad \hline \text{remainder } 0 \end{array} \begin{array}{l} \text{dividend} \\ \text{quotient} \\ \text{Ans.} \end{array}$$

EXPLANATION.—As 63 is not contained in the first two figures, 27, we must use the first three figures, 270. Now, by trial we must find how many times 63 is contained in 270; 6 is contained in the first two figures of 270, 4 times. Place the 4 as the first figure in the quotient. Multiply the divisor, 63, by 4, and subtract the product 252 from 270. The remainder is 18, to which we annex the next figure of the dividend, 2, making 182. Now, 6 is contained in the first two figures of 182, 3 times, but on multiplying 63 by 3, we see that the product 189 is too great, so we try 2 as the second figure of the quotient. Multiplying the divisor 63 by 2, and subtracting the product 126 from 182, the remainder is 56, to which we annex the next figure of the dividend, making 568; 6 is contained in 56 about 9 times. Multiply the divisor 63 by 9, and subtract the product 567 from 568.

The remainder is 1, and bringing down the next figure of the dividend, 2, gives 12. As 12 is less than 63, we write 0 in the quotient and bring down the next figure, 6, making 126; 63 is contained in 126, 2 times, without a remainder. Therefore, 42,902 is the quotient.

84. Rule.—I. *Write the divisor at the left of the dividend with a curved line between them.*

II. *Find how many times the divisor is contained in the least number of the left-hand figures of the dividend that will contain it, and write the result at the right of the dividend, with a line between, as the first figure of the quotient.*

III. *Multiply the divisor by this quotient; write the product under the partial dividend used, and subtract, annexing to the remainder the next figure of the dividend. Divide as before, and continue thus until all the figures of the dividend have been used.*

IV. *If any partial dividend will not contain the divisor, write a cipher in the quotient, annex the next figure of the dividend and proceed as before.*

V. *If there is at last a remainder, write it after the quotient, with the divisor underneath.*

85. Proof.—*Multiply the quotient by the divisor, and add the remainder, if any, to the product. The result will be the dividend.*

	<i>divisor</i>	<i>dividend</i>	<i>quotient</i>	
Thus,	63) 4235	(67 $\frac{1}{3}$	Ans.
		378		
		455		
		441		
	<i>remainder</i>	14		
PROOF.—	<i>quotient</i>	67		
	<i>divisor</i>	63		
		201		
		402		
		4221		
	<i>remainder</i>	14		
	<i>dividend</i>	4235		

SHORT METHOD OF DIVISION.

86. The following method saves about half the figures required by the method just given, and we think that there will be fewer mistakes made when using it.

87. EXAMPLE.—Divide 39,913,910 by 5,494.

SOLUTION.—	$\begin{array}{r} \text{dividend} \quad \text{divisor} \\ 39913910 \,) 5494 \\ \underline{14559} \\ 35711 \\ \underline{27470} \\ 0000 \end{array}$	7265 quotient
------------	--	-------------------------

EXPLANATION.—The addition method of subtraction (see Art. 49) is used in this case; the divisor is written on the right of the dividend, and the quotient underneath the divisor. The different figures of the quotient are obtained in exactly the same manner as by the preceding method. Thus, the divisor is contained in the first five figures of the dividend 7 times, and 7 is written for the first figure of the quotient. Now, instead of multiplying the divisor by 7, writing the product under the first five figures of the dividend, and then subtracting, we multiply each figure of the divisor by 7, and by the addition method, subtract from the dividend, writing only the remainder. Thus, 7 times 4 is 28, and 5 is 33; write the 5 under the 3 in the dividend, and carry 3. Then, 7 times 9 is 63 and 3 is 66, and 66 and 5 is 71; write the 5 and carry 7. 7 times 4 is 28 and 7 is 35, 35 and 4 is 39; write the 4 and carry 3. 7 times 5 is 35 and 3 is 38, and 38 and 1 is 39; write the 1. Now bring down the next figure of the dividend, 9, and annex it to the remainder. $14,559 \div 5,494 = 2$; write 2 as the second figure of the quotient. Then, as above, $2 \times 4 = 8$, and $8 + 1 = 9$; write the 1 under the 9, as shown. $2 \times 9 = 18$, and $18 + 7 = 25$; write the 7 and carry the 2. $2 \times 4 = 8$, $8 + 2 = 10$, and $10 + 5 = 15$; write the 5 and carry the 1. $2 \times 5 = 10$, $10 + 1 = 11$, and $11 + 3 = 14$; write the 3. Bringing down 1, the next figure of the dividend, $35,711 \div 5,494 = 6$, the third figure of the quotient. Proceed as above with the remaining figures.

A fast computer would work as follows: In multiplying by 6, he would repeat to himself 6, 24, and 7 is 31 (writing the 7 and carrying the 3). 6, 54, 57, and 4 is 61. 6, 24, 30, and 7 is 37. 6, 30, 33, and 2 is 35.

The object of writing the divisor on the right is to make it easier to multiply by the figures of the quotient; it also saves space, as may readily be seen. The student is strongly advised to learn this method thoroughly, and always to use it. The best way to attain facility in division, is first to practise dividing by small numbers, from 2 to 12, and using the method of short division. After he has become proficient in this, he should practise long division by the method just described. Some special methods which may be used when dividing by certain numbers will be mentioned farther on.

88. The principle given in Art. **68** may be used to test the work of division, when the principle has been slightly modified. Add the digits of the divisor, the dividend, the quotient, and the remainder, if any, as described in Art. **68**, obtaining a single figure for the sum of each. Multiply the number thus obtained for the divisor by the number obtained for the quotient, and add to the product the number obtained for the remainder, if any. If the work has been done correctly, the result must equal the number obtained for the dividend. Thus, in the last example, the sum of the digits in the divisor (reduced to a single figure) is 4, of those in the quotient, 2, and in the remainder, 0. Hence, $4 \times 2 = 8$; $8 + 0 = 8$. Adding the digits in the dividend, the result (reduced to a single figure) is also 8; hence, the work is very probably correct.

Applying this method to the example in Art. **85**, we have, for the divisor 9, for the quotient 4, for the remainder 5. Hence, $9 \times 4 + 5 = 41$, and $4 + 1 = 5$. For the dividend, $4 + 2 + 3 + 5 = 14$, and $1 + 4 = 5$, also. The student will find this principle very useful.

Addition, subtraction, multiplication, and division, are the four corner stones of arithmetic; everything else in arithmetic depends upon them.

EXAMPLES FOR PRACTICE.

89. Divide the following

(a)	126 498 by 58.	Ans.	(a)	2,181.
(b)	3 207,594 by 767		(b)	4,182.
(c)	11 408,202 by 234.		(c)	48,753.
(d)	2,100,315 by 581.		(d)	3,615.
(e)	969 046 by 4,008.		(e)	242.
(f)	7 484 888 by 1,021.		(f)	7,328.
(g)	1,525 915 by 5 003.		(g)	305.
(h)	1,646 301 by 381.		(h)	4,321.

CANCELATION.

90. Cancellation is the process of shortening operations in division by casting out equal factors from both dividend and divisor.

91. The factors of a number are those numbers which, when multiplied together, will equal that number. Thus, 5 and 3 are the factors of 15, since $5 \times 3 = 15$. Likewise, 8 and 7 are the factors of 56, since $8 \times 7 = 56$.

92. A prime number is a number that cannot be divided by any number except itself and 1; 1 is not considered a factor. Thus, 2, 3, 11, 29, etc. are prime numbers.

93. A prime factor of a number is any factor that is a prime number.

Any number that is not a prime is called a composite number, and may be produced by multiplying together its prime factors. Thus, 60 is a composite number, and is equal to the product of its prime factors, $2 \times 2 \times 3 \times 5$.

Two numbers are said to be prime to each other when they have no common factor, as, for example, 15 and 28; there is no number, except 1, that will divide both 15 and 28 without a remainder.

94. Canceling equal factors from both dividend and divisor does *not* change the quotient.

The canceling of a factor in both dividend and divisor is the same as dividing them both by the same number, which, by a principle of division, does not change the quotient.

Write the numbers forming the dividend above the line, and those forming the divisor below it.

95. EXAMPLE.—Divide $4 \times 45 \times 60$ by 9×24 .

SOLUTION.—Placing the dividend over the divisor, and canceling

$$\frac{\overset{5}{4} \times \overset{10}{\cancel{45}} \times \cancel{60}}{\underset{6}{9} \times \underset{6}{\cancel{24}}} = 50. \text{ Ans.}$$

EXPLANATION.—The 4 in the dividend and 24 in the divisor are both divisible by 4, since 4 divided by 4 equals 1, and 24 divided by 4 equals 6. Cancel the 4 and the 24, and write the 6 under 24. Thus,

$$\frac{\cancel{4} \times 45 \times 60}{9 \times \underset{6}{\cancel{24}}} =$$

60 in the dividend and 6 in the divisor, are divisible by 6, since 60 divided by 6 equals 10, and 6 divided by 6 equals 1. Cancel the 60 and write 10 over it; also, cancel the 6. Thus,

$$\frac{\cancel{4} \times 45 \times \overset{10}{\cancel{60}}}{9 \times \underset{6}{\cancel{24}}} =$$

Again, 45 in the dividend and 9 in the divisor are each divisible by 9, since 45 divided by 9 equals 5, and 9 divided by 9 equals 1. Cancel the 45 and write the 5 over it; also, cancel the 9. Thus,

$$\frac{\overset{5}{\cancel{4}} \times \cancel{45} \times \overset{10}{\cancel{60}}}{\underset{6}{\cancel{9}} \times \underset{6}{\cancel{24}}} =$$

Since there are no two remaining numbers (one in the dividend and one in the divisor) divisible by any number greater than 1 without a remainder, it is impossible to cancel further.

Multiply together all the uncanceled numbers in the dividend and divide their product by the product of all the uncanceled numbers in the divisor. The result will be the quotient. The product of all the uncanceled numbers in

the dividend is $5 \times 10 = 50$, and there are no uncanceled numbers in the divisor.

$$\text{Hence, } \frac{5 \times 10}{\cancel{4} \times \cancel{4} \times \cancel{9} \times \cancel{9}} = 5 \times 10 = 50. \text{ Ans.}$$

96. Rule.—I. *Cancel the common factors from both the dividend and divisor.*

II. *Then divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor, and the result will be the quotient.*

EXAMPLES FOR PRACTICE.

97. Divide:

- | | |
|--|---|
| (a) $14 \times 18 \times 16 \times 40$ by $7 \times 8 \times 6 \times 5 \times 3$. | Ans. { <div style="display: inline-block; vertical-align: middle;"> (a) ■
 (b) 250.
 (c) 1.
 (d) 48.
 (e) ■
 (f) 100.
 (g) 43.
 (h) 5. </div> |
| (b) $8 \times 65 \times 50 \times 100 \times 60$ by $30 \times 60 \times 18 \times 10$. | |
| (c) $8 \times 4 \times 8 \times 9 \times 11$ by $11 \times 9 \times 4 \times 8 \times 8$. | |
| (d) $164 \times 321 \times 6 \times 7 \times 4$ by $82 \times 321 \times 7$. | |
| (e) $50 \times 100 \times 200 \times 72$ by $1,000 \times 144 \times 100$. | |
| (f) $48 \times 63 \times 55 \times 49$ by $7 \times 21 \times 11 \times 48$. | |
| (g) $110 \times 150 \times 84 \times 32$ by $11 \times 15 \times 100 \times 64$. | |
| (h) $115 \times 120 \times 400 \times 1,000$ by $28 \times 1,000 \times 80 \times 800$. | |

ARITHMETIC.

FRACTIONS.

1. A **fraction** is one or more of the equal parts of a unit.
2. Two numbers are required to express a fraction, one called the **numerator**, and the other, the **denominator**.
3. The numerator is placed above the denominator, with a *line* between them, as $\frac{3}{4}$. Here 3 is the *denominator*, and shows into how many equal parts the unit is divided. The *numerator* 2 shows how many of these equal parts are taken or considered. The denominator also indicates the name of the parts.

$\frac{1}{2}$ is read one-half.

$\frac{3}{4}$ is read three-fourths.

$\frac{3}{8}$ is read three-eighths.

$\frac{5}{16}$ is read five-sixteenths.

$\frac{29}{47}$ is read twenty-nine forty-sevenths.

4. In the expression " $\frac{3}{4}$ of an apple," the denominator 4 shows that the apple is divided into four *equal* parts, and the numerator 3 shows that three of these parts, or fourths, are taken or considered.

If each of the parts, or fourths, of the apple were cut into two equal pieces, there would then be twice as many pieces as before, or $4 \times 2 = 8$ pieces in all; one of these pieces would be called one-eighth, and would be expressed in figures as $\frac{1}{8}$. Three of these pieces would be called three-eighths, written $\frac{3}{8}$. The words three-fourths, three eighths, five-sixteenths, etc. are abbreviations of three one-fourths, three

§ 2

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one-eighths, five one-sixteenths, etc. It is evident that, the greater the denominator, the greater is the number of parts into which the unit is divided; consequently, the parts themselves are smaller, and the value of the fraction is less for the same number of parts taken. In other words, $\frac{7}{9}$, for example, is less than $\frac{7}{8}$, because, if a unit is divided into 9 parts, the parts are less than if the same unit had been divided into 8 parts; and, since $\frac{1}{9}$ is less than $\frac{1}{8}$, it is clear that 7 one-ninths is less than 7 one-eighths. Hence, also, $\frac{7}{9}$ is less than $\frac{7}{8}$.

5. The value of a fraction is the *numerator* divided by the *denominator*; as, $\frac{4}{2} = 2$, $\frac{6}{2} = 3$.

6. The line between the *numerator* and the *denominator* means *divided by*, or \div .

$\frac{3}{4}$ is equivalent to $3 \div 4$.

$\frac{5}{8}$ is equivalent to $5 \div 8$.

7. The *numerator* and *denominator* of a fraction are called the *terms* of a fraction.

8. The *value* of a fraction when its terms are equal is 1.

$\frac{4}{4}$, or four-fourths = 1.

$\frac{8}{8}$, or eight-eighths = 1.

$\frac{64}{64}$, or sixty-four sixty-fourths = 1.

9. A *proper fraction* is a fraction whose *numerator* is *less* than its *denominator*. Its value is *less* than 1, as $\frac{1}{2}$, $\frac{5}{8}$, $\frac{1}{10}$.

10. An *improper fraction* is a fraction whose *numerator* *equals* or is *greater* than the *denominator*. Its value is *not less* than 1, as $\frac{4}{4}$, $\frac{5}{4}$, $\frac{11}{10}$.

$\frac{3}{4}$ is reduced to $\frac{6}{8}$ by multiplying both terms of the fraction by 2.

$$\frac{3 \times 2}{4 \times 2} = \frac{6}{8}$$

The *value* is not changed. For, suppose that a unit, say an apple, is divided into 8 equal parts. If these parts be arranged in 4 piles, each containing 2 parts, it is evident that each pile will be composed of the same part of the apple as would have been the case had the apple been originally cut into 4 equal parts. Now, if one of these piles (containing 2 parts) be removed, there will be 3 piles left, each containing 2 equal parts, or 6 equal parts in all, i. e., six-eighths. But, since one pile, or one-quarter, was removed, there are three-quarters left. Hence, $\frac{3}{4} = \frac{6}{8}$. The same reasoning may be applied to any similar case. Therefore, multiplying both terms of a fraction by the same number does not alter its value.

14. To reduce a fraction to an equivalent fraction having a given denominator.

EXAMPLE.—Reduce $\frac{7}{8}$ to an equivalent fraction having 96 for a denominator.

SOLUTION.—Both the numerator and the denominator must be multiplied by the same number in order not to change the value of the fraction. The denominator must be multiplied by some number which will make the product 96; this number is evidently $96 \div 8 = 12$, since $8 \times 12 = 96$. Hence, $\frac{7 \times 12}{8 \times 12} = \frac{84}{96}$. Ans.

15. Rule.—*Divide the given denominator by the denominator of the given fraction, and multiply both terms of the fraction by the quotient.*

EXAMPLE.—Reduce $\frac{3}{4}$ to 100ths.

SOLUTION.— $100 \div 4 = 25$; hence, $\frac{3 \times 25}{4 \times 25} = \frac{75}{100}$. Ans.

16. *A fraction is reduced to lower terms by dividing both terms by the same number.* Thus, $\frac{8}{10}$ is reduced to $\frac{4}{5}$ by dividing both terms by 2.

$$\frac{8 \div 2}{10 \div 2} = \frac{4}{5}$$

That $\frac{8}{10} = \frac{4}{5}$ is readily seen from the explanation given in

Art. 13; for, multiplying both terms of the fraction $\frac{4}{5}$ by 2, $\frac{4 \times 2}{5 \times 2} = \frac{8}{10}$, and, if $\frac{4}{5} = \frac{8}{10}$, $\frac{8}{10}$ must equal $\frac{4}{5}$. Hence, dividing both terms of a fraction by the same number does not alter its value.

17. *A fraction is reduced to lowest terms, or simplest form, when its numerator and denominator cannot both be divided by the same number without a remainder. As, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{11}{13}$, $\frac{1}{17}$.*

EXAMPLES FOR PRACTICE.

18. Reduce the following:

- | | | |
|--|--------|-------------------------|
| (a) $\frac{7}{8}$ to 128ths. | Ans. { | (a) $\frac{11}{13}$. |
| (b) $\frac{14}{15}$ to its lowest terms. | | (b) $\frac{1}{17}$. |
| (c) $\frac{14}{15}$ to its lowest terms. | | (c) $\frac{1}{18}$. |
| (d) $\frac{4}{5}$ to 49ths. | | (d) $\frac{4}{15}$. |
| (e) $\frac{1}{8}$ to 10,000ths. | | (e) $\frac{1}{10000}$. |

19. To reduce a whole number or a mixed number to an improper fraction.

EXAMPLE 1—How many *fourths* in 5?

SOLUTION.—Since there are 4 *fourths* in 1, in 5 there will be 5×4 fourths, or 20 fourths; i.e., $5 \times \frac{1}{4} = \frac{20}{4}$. Ans.

EXAMPLE 2.—Reduce $8\frac{1}{4}$ to an improper fraction.

SOLUTION.— $8 \times \frac{1}{4} = 2$. $2 + \frac{1}{4} = \frac{9}{4}$. Ans.

20. Rule.—Multiply the whole number by the denominator of the fraction, add the numerator to the product and place the denominator under the result. If it is desired to reduce a whole number to a fraction, multiply the whole

22. To reduce an improper fraction to a whole or a mixed number.

EXAMPLE.—Reduce $2\frac{1}{4}$ to a mixed number.

SOLUTION.—4 is contained in 21, 5 times and 1 remaining (see Art. 5); as this remainder is also divided by 4, its value is $\frac{1}{4}$. Therefore, $5 + \frac{1}{4}$, or $5\frac{1}{4}$, is the number.

23. Rule.—*Divide the numerator by the denominator, and write the result as in ordinary division.* (See part V of Rule, Art. 84, § 1.)

EXAMPLES FOR PRACTICE.

24. Reduce to whole or mixed numbers:

- (a) $1\frac{1}{2}$.
 (b) $1\frac{1}{3}$.
 (c) $1\frac{1}{4}$.
 (d) $1\frac{1}{5}$.
 (e) $1\frac{1}{6}$.
 (f) $1\frac{1}{7}$.

- Ans. $\left\{ \begin{array}{l} (a) \ 24\frac{1}{2}. \\ (b) \ 61\frac{1}{3}. \\ (c) \ 116\frac{1}{4}. \\ (d) \ 49\frac{1}{5}. \\ (e) \ 4. \\ (f) \ 5. \end{array} \right.$

25. A common denominator of two or more fractions is a number that will contain (i. e., which may be divided by) all of the *denominators* of the fractions without a remainder. The least common denominator is the least number that will contain all the denominators of the fractions without a remainder.

26. To find the least common denominator.

EXAMPLE.—Find the least common denominator of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{8}$.

SOLUTION.—We first place the denominators in a row, separated by commas.

$$\begin{array}{r} 2 \) \ 4, \ 8, \ 9, \ 16 \\ \hline 2 \) \ 2, \ 8, \ 9, \ 8 \\ \hline 3 \) \ \quad 8, \ 9, \ 4 \\ \hline \quad \quad 8, \ 4 \end{array}$$

$2 \times 2 \times 3 \times 3 \times 4 = 144$, the least common denominator. Ans.

EXPLANATION.—Divide each of the denominators by some prime number that will divide at least two of them without a remainder if (possible), bringing down to the row below those

denominators which will not contain the divisor without a remainder. Dividing each of the numbers by 2, the second row becomes 2, 3, 9, 8, since 2 will not divide 3 and 9 without a remainder. Dividing again by 2, the result is 3, 9, 4. Dividing the third row by 3, the result is 3, 4. The numbers in the fourth row are now prime to each other (see Art. 93, § 1), and the product of these numbers multiplied by the divisors will be the least common denominator. Thus, $2 \times 2 \times 3 \times 3 \times 4 = 144$, the least common denominator.

27. EXAMPLE.—Find the least common denominator of $\frac{1}{4}$, $\frac{1}{3}$, and $\frac{1}{12}$.

SOLUTION.—

$$\begin{array}{r} 3 \overline{) 9, 12, 18} \\ 3 \overline{) 3, 4, 6} \\ 2 \overline{) 4, 2} \\ \hline 2 \end{array}$$

$$3 \times 3 \times 2 \times 2 = 36. \text{ Ans.}$$

28. If one (or more) of the denominators is a factor of some other denominator, it need not be considered in the process of finding the least common denominator, for if the least common denominator will contain the larger denominator it will also contain any factor of it. Thus, in the last example, since 9 is a factor of 18, it need not be considered, and all that is necessary is to find the least common denominator of 12 and 18. Also, if *all* of the denominators have a common factor, whether prime or composite, that factor may be used as a divisor. For example, since 12 and 18 have the common factor 6, 6 may be used as a divisor instead of its

exactly contain two or more numbers is called the **least common multiple** of those numbers. Hence, the least common denominator of two or more fractions is the least common multiple of the denominators of the fractions.

30. The least common multiple of several numbers may often be determined by inspection. For example, if it is desired to find the least common multiple of 3, 5, and 10, simple inspection will show that the least common multiple is 30, the mental process being as follows: Since 5 is a factor of 10, 5 need not be considered; and since 3 and 10 are prime to each other, their least common multiple is their product, i. e., 3×10 , or 30. Therefore, 30 is the least common multiple of 3, 5, and 10.

Again, consider the example of Art. 27. Here, it is required to find the least common multiple of 9, 12, and 18. Since 9 is a factor of 18, it need not be considered. The least common multiple of 12 and 18 must, of course, contain 18 a certain number of times. To ascertain how many times 18 the least common multiple must be, divide 12 (the other number) by the greatest factor common to both 12 and 18; the quotient multiplied by 18 will be the required least common multiple. In the present case, the greatest factor is 6, and $12 \div 6 = 2$; $2 \times 18 = 36$, the least common multiple.

31. To reduce two or more fractions to equivalent fractions having the least common denominator.

EXAMPLE.—Reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{7}{8}$ to equivalent fractions having the least common denominator.

SOLUTION.—The least common denominator is the least number that can be exactly divided by 3, 4, and 8. This number is readily seen by inspection to be 24. Now, reducing each fraction to a fraction having a denominator of 24 (Art. 14), we obtain

$$\frac{2 \times 8}{3 \times 8} = \frac{16}{24}, \quad \frac{3 \times 6}{4 \times 6} = \frac{18}{24}, \quad \frac{7 \times 3}{8 \times 3} = \frac{21}{24} \quad \text{Ans.}$$

32. Rule.—*Divide the least common denominator by the denominator of the given fraction, and multiply both terms of the fraction by the quotient.*

EXAMPLES FOR PRACTICE.

33. Reduce to fractions having a common denominator:

(a) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$	Ans	(a) $\frac{6}{12}, \frac{4}{12}, \frac{3}{12}$
(b) $\frac{1}{6}, \frac{1}{4}, \frac{1}{12}$		(b) $\frac{2}{12}, \frac{3}{12}, \frac{1}{12}$
(c) $\frac{1}{8}, \frac{1}{12}, \frac{1}{16}$		(c) $\frac{2}{24}, \frac{2}{24}, \frac{1}{24}$
(d) $\frac{1}{5}, \frac{1}{10}, \frac{1}{15}$		(d) $\frac{2}{30}, \frac{2}{30}, \frac{2}{30}$
(e) $\frac{1}{10}, \frac{1}{20}, \frac{1}{30}$		(e) $\frac{3}{60}, \frac{3}{60}, \frac{2}{60}$
(f) $\frac{1}{12}, \frac{1}{18}, \frac{1}{24}$		(f) $\frac{2}{36}, \frac{2}{36}, \frac{1}{36}$

ADDITION OF FRACTIONS.

34. *Fractions cannot be added unless they have a common denominator* We cannot add $\frac{1}{4}$ to $\frac{1}{8}$ as they now stand, since the denominators represent different parts of a unit. Fourths can be added to fourths, but not to eighths.

Suppose we divide an apple into 4 equal parts, and then divide 2 of these parts into 2 equal parts. It is evident that we shall have 2 one-fourths and 4 one-eighths. Now if we add these parts the result is $2 + 4 = 6$ something. But what is this something? It is not fourths, for six fourths are $1\frac{1}{2}$, and we had only one apple to begin with; neither is it eighths, for six eighths are $\frac{3}{4}$, which is less than 1 apple. By reducing the fourths to eighths, we have $\frac{1}{4} = \frac{2}{8}$; and, adding the other 4 eighths, $4 + 2 = 6$ eighths. The result is correct, since $\frac{6}{8} = 1$. Or we can, in this case, reduce the eighths to fourths. Thus, $\frac{1}{8} = \frac{1}{4}$; whence, adding $2 + 2 = 4$ quarters or fourths, a correct result, since $\frac{4}{4} = 1$.

Before adding, fractions should be reduced to a common denominator, preferably the *least* common denominator.

35. EXAMPLE Find the sum of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{6}$

SOLUTION The *least common denominator*, or the *least number* that will exactly contain all the *denominators*, is 6.

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{6} = \frac{3}{6} + \frac{2}{6} + \frac{1}{6} = \frac{4+2+1}{6} = \frac{7}{6} = 1\frac{1}{6}. \text{ Ans.}$$

EXPLANATION —As the *denominator* indicates the names of the *parts*, only the *numerators* are added to obtain the total number of *parts* indicated by the *denominator*. Thus, 4 one-eighths plus 6 one-eighths plus 5 one-eighths = 15 one-eighths.

36. EXAMPLE 1.—What is the sum of $12\frac{3}{4}$, $14\frac{1}{2}$, and $7\frac{5}{8}$?

SOLUTION.—The least common denominator in this case is 16.

$$\begin{array}{r} 12\frac{3}{4} = 12\frac{12}{16} \\ 14\frac{1}{2} = 14\frac{8}{16} \\ 7\frac{5}{8} = 7\frac{10}{16} \\ \hline \text{sum } 33 + \frac{30}{16} = 33 + 1\frac{15}{8} = 34\frac{15}{8}. \text{ Ans.} \end{array}$$

The sum of the fractions = $\frac{30}{16}$, or $1\frac{15}{8}$, which added to the sum of the whole numbers = $34\frac{15}{8}$.

EXAMPLE 2.—What is the sum of 17, $13\frac{3}{8}$, $\frac{9}{2}$, and $3\frac{1}{4}$?

SOLUTION.—The least common denominator is 32. $13\frac{3}{8} = 13\frac{12}{32}$, $3\frac{1}{4} = 3\frac{8}{32}$.

$$\begin{array}{r} 17 \\ 13\frac{12}{32} \\ \frac{9}{2} \\ 3\frac{8}{32} \\ \hline \text{sum } 33\frac{20}{32} \text{ Ans.} \end{array}$$

37. Rule.—I. *Make the fractions similar; write the sum of the numerators over the least common denominator.*

II. *When there are integers or mixed numbers, add them separately and then add the results.*

EXAMPLES FOR PRACTICE.

38. Find the sum of:

- (a) $\frac{1}{2}$, $\frac{7}{8}$, $\frac{5}{8}$.
- (b) $\frac{2}{3}$, $\frac{5}{12}$, $2\frac{1}{6}$.
- (c) $\frac{1}{2}$, $\frac{3}{8}$, $\frac{5}{16}$.
- (d) $\frac{5}{8}$, $1\frac{1}{2}$, $1\frac{3}{8}$.
- (e) $1\frac{1}{11}$, $\frac{5}{33}$, $2\frac{2}{3}$.
- (f) $2\frac{2}{3}$, $1\frac{1}{6}$, $1\frac{1}{6}$.
- (g) $\frac{4}{11}$, $\frac{7}{22}$, $1\frac{1}{2}$.
- (h) $\frac{3}{7}$, $1\frac{1}{3}$, $\frac{2}{7}$.

- Ans. {
- (a) $1\frac{7}{8}$.
 - (b) $1\frac{11}{12}$.
 - (c) $1\frac{9}{16}$.
 - (d) $1\frac{17}{8}$.
 - (e) $1\frac{11}{33}$.
 - (f) $1\frac{2}{3}$.
 - (g) $1\frac{7}{11}$.
 - (h) 1.

SUBTRACTION OF FRACTIONS.

39. *Fractions cannot be subtracted without first reducing them to a common denominator. This can be shown in the same manner as in the case of addition of fractions. (Art. 34.)*

EXAMPLE.—Subtract $\frac{3}{4}$ from $1\frac{1}{2}$.

SOLUTION.—The least common denominator is 16.

$$\frac{1}{2} = \frac{8}{16}, \quad 1\frac{1}{2} - \frac{3}{4} = \frac{18-6}{16} = \frac{12}{16} = \frac{3}{4}. \quad \text{Ans.}$$

40. EXAMPLE.—From 7 take $\frac{3}{4}$.

SOLUTION.— $1 = \frac{4}{4}$; therefore, since $7 = 6 + 1$, $7 = 6 + \frac{4}{4} = 6\frac{4}{4}$, and $6\frac{4}{4} - \frac{3}{4} = 6\frac{1}{4}$. Ans.

41. EXAMPLE.—What is the difference between $17\frac{1}{2}$ and $9\frac{1}{4}$?

SOLUTION.—The least common denominator of the fraction is 83.
 $17\frac{1}{2} = 17\frac{1}{2}$.

$$\begin{array}{r} \text{minuend} \quad 17\frac{1}{2} \\ \text{subtrahend} \quad 9\frac{1}{4} \\ \hline \text{difference} \quad 8\frac{1}{4} \quad \text{Ans.} \end{array}$$

42. EXAMPLE.—From $9\frac{1}{2}$ take $4\frac{3}{4}$.

SOLUTION.—The least common denominator of the fractions is 16.
 $9\frac{1}{2} = 9\frac{1}{2}$.

$$\begin{array}{r} \text{minuend} \quad 9\frac{1}{2} \text{ or } 8\frac{2}{2} \\ \text{subtrahend} \quad 4\frac{3}{4} \quad 4\frac{3}{4} \\ \hline \text{remainder} \quad 4\frac{1}{4} \quad 4\frac{1}{4} \quad \text{Ans.} \end{array}$$

EXPLANATION.—As the fraction in the subtrahend is greater than the fraction in the minuend, it cannot be subtracted; therefore, borrow 1, or $\frac{4}{4}$, from the 9 in the minuend and add it to the $\frac{1}{2}$; $\frac{1}{2} + \frac{4}{4} = \frac{5}{4}$. $\frac{3}{4}$ from $\frac{5}{4} = \frac{2}{4}$. Since 1 was borrowed from 9, 8 remains; 4 from 8 = 4; $4 + \frac{2}{4} = 4\frac{1}{2}$.

43. EXAMPLE.—From 9 take $8\frac{3}{4}$.

SOLUTION.—

$$\begin{array}{r} \text{minuend} \quad 9 \text{ or } 8\frac{4}{4} \\ \text{subtrahend} \quad 8\frac{3}{4} \quad 8\frac{3}{4} \end{array}$$

II. *When there are mixed numbers, subtract the fractions and whole numbers separately.*

III. *When the fraction in the subtrahend is greater than the fraction in the minuend, borrow 1 from the whole number in the minuend, and add it to the fraction in the minuend, from which subtract the fraction in the subtrahend.*

IV. *When the minuend is a whole number, borrow 1 from it; reduce the 1 to a fraction whose denominator is the same as the denominator of the fraction in the subtrahend, and then subtract.*

EXAMPLES FOR PRACTICE.

45. Subtract:

- (a) $\frac{19}{4}$ from $1\frac{1}{2}$.
- (b) $\frac{7}{4}$ from $\frac{1}{2}$.
- (c) $\frac{4}{8}$ from $\frac{5}{10}$.
- (d) $\frac{1}{8}$ from $\frac{5}{8}$.
- (e) $\frac{1}{8}$ from $\frac{5}{4}$.
- (f) $13\frac{1}{4}$ from $30\frac{1}{2}$.
- (g) $12\frac{1}{2}$ from 27.
- (h) $5\frac{1}{4}$ from 30.

- Ans. {
- (a) $\frac{1}{2}$.
 - (b) $\frac{3}{8}$.
 - (c) $\frac{1}{10}$.
 - (d) $\frac{3}{8}$.
 - (e) $\frac{1}{4}$.
 - (f) $17\frac{1}{4}$.
 - (g) $14\frac{1}{2}$.
 - (h) $24\frac{3}{4}$.

MULTIPLICATION OF FRACTIONS.

46. In multiplication of fractions it is not necessary to reduce the given fractions to fractions having a common denominator.

47. *Multiplying the numerator or dividing the denominator multiplies the fraction.*

EXAMPLE.—Multiply $\frac{3}{4}$ by 4.

SOLUTION.— $\frac{3}{4} \times 4 = \frac{3 \times 4}{4} = \frac{12}{4} = 3$. Ans.

Or, $\frac{3}{4} \times 4 = \frac{3}{4 \div 4} = \frac{3}{1} = 3$. Ans.

The word “of” in multiplication of fractions means the same as \times , or times. Thus,

$$\frac{3}{4} \text{ of } \frac{1}{2} = \frac{3}{4} \times \frac{1}{2} = \frac{3}{8}.$$

$$\frac{1}{8} \text{ of } \frac{5}{16} = \frac{1}{8} \times \frac{5}{16} = \frac{5}{128}.$$

EXAMPLE.—Multiply 2 by $\frac{1}{3}$.

SOLUTION.— $2 \times \frac{1}{3} = \frac{2 \times 1}{3} = \frac{2}{3}$. Ans.

Or $2 \times \frac{1}{3} = \frac{2}{3 \div 2} = \frac{2}{\frac{3}{2}} = \frac{2}{3}$. Ans.

48. EXAMPLE.—What is the product of $\frac{4}{16}$ and $\frac{7}{8}$?

SOLUTION.— $\frac{4}{16} \times \frac{7}{8} = \frac{4 \times 7}{16 \times 8} = \frac{28}{128} = \frac{7}{32}$. Ans.

Or, by cancelation, $\frac{4 \times 7}{\frac{16}{4} \times 8} = \frac{7}{4 \times 8} = \frac{7}{32}$. Ans.

49. EXAMPLE.—What is $\frac{1}{3}$ of $\frac{2}{3}$ of $\frac{1}{2}$?

SOLUTION.— $\frac{1}{3} \times \frac{2}{3} \times \frac{1}{2} = \frac{1 \times 2 \times 1}{3 \times 3 \times 2} = \frac{2}{18} = \frac{1}{9}$. Ans.

50. EXAMPLE.—What is the product of $9\frac{1}{2}$ and $5\frac{1}{2}$?

SOLUTION.— $9\frac{1}{2} = \frac{19}{2}$; $5\frac{1}{2} = \frac{11}{2}$.

$\frac{19}{2} \times \frac{11}{2} = \frac{19 \times 11}{2 \times 2} = \frac{209}{4} = 52\frac{1}{4}$. Ans.

51. EXAMPLE.—Multiply $15\frac{1}{2}$ by 3.

SOLUTION.—

$15\frac{1}{2}$	$15\frac{1}{2}$
$\frac{1}{2}$	$\frac{1}{2}$
<hr style="width: 50px; border: 0.5px solid black;"/>	<hr style="width: 50px; border: 0.5px solid black;"/>
$47\frac{1}{2}$	$45 + 2\frac{1}{2} = 47\frac{1}{2}$

Ans.

52. Rule.—I. *Divide the product of the numerators by*

EXAMPLES FOR PRACTICE.

53. Find the product of :

$$(a) \quad 7 \times \frac{3}{15}.$$

$$(b) \quad 14 \times \frac{5}{18}.$$

$$(c) \quad \frac{3}{4} \times \frac{5}{12}.$$

$$(d) \quad \frac{1}{4} \times 4.$$

$$(e) \quad \frac{1}{2} \times 7.$$

$$(f) \quad 17\frac{1}{2} \times 7.$$

$$(g) \quad \frac{1}{2} \times 32.$$

$$(h) \quad \frac{1}{2} \times 14.$$

$$\text{Ans. } \left\{ \begin{array}{ll} (a) & 1\frac{1}{5}. \\ (b) & 4\frac{5}{9}. \\ (c) & \frac{5}{8}. \\ (d) & 2\frac{1}{2}. \\ (e) & 7\frac{1}{2}. \\ (f) & 125. \\ (g) & 15. \\ (h) & 7\frac{1}{2}. \end{array} \right.$$

54. Short Methods of Multiplying by a Mixed Number.—In all business transactions, the multiplication of a mixed number by an integer, an integer by a mixed number, or a mixed number by a mixed number, is of very frequent occurrence. Unless the numbers are quite small, which is not usually the case, it is very inconvenient to reduce the mixed numbers to improper fractions, multiply, and then reduce the product to a mixed number. A better way is to use one of the methods given below:

55. EXAMPLE.—Multiply $126\frac{7}{8}$ by 27.

SOLUTION.—

$$\begin{array}{r} 126\frac{7}{8} \\ 27 \\ \hline 3\frac{3}{8} \\ 181\frac{1}{8} \\ 882 \\ 252 \\ \hline 3425\frac{1}{8} \text{ Ans.} \end{array}$$

EXPLANATION.—First multiply the $\frac{7}{8}$ by 27; this may be conveniently done as follows: Multiply $\frac{1}{8}$ by 27; to do this all that is necessary is to divide 27 by 8 (this is evidently correct, since $\frac{1}{8} \times 27 = \frac{27}{8} = 27 \div 8$), obtaining $3\frac{3}{8}$. Now multiply the result just obtained ($3\frac{3}{8}$) by the numerator of the fraction less 1, or in this case, by $7 - 1 = 6$, getting $181\frac{1}{8}$ for the product, which write under $3\frac{3}{8}$, as shown. Then multiply 126 by 27 in the usual manner, placing the unit figure under the unit figures of the two mixed numbers, which may be

regarded as partial products. That this method is correct is readily seen. For $3\frac{1}{3}$ is $\frac{1}{3}$ of 27, and $18\frac{1}{3}$ is $\frac{2}{3}$ of 27; therefore, $3\frac{1}{3} + 18\frac{1}{3} = \frac{1}{3}$ of 27 + $\frac{2}{3}$ of 27 = $\frac{3}{3}$ of 27.

56. EXAMPLE.—Multiply 825 by $29\frac{1}{3}$.

SOLUTION.—*First Method.*

$$\begin{array}{r} 825 \\ 29\frac{1}{3} \\ \hline 4)2475 \\ \hline 618\frac{1}{3} \\ 7425 \\ \hline 1650 \\ \hline 24543\frac{1}{3} \text{ Ans.} \end{array}$$

Second Method.

$$\begin{array}{r} 825 \\ 29\frac{1}{3} \\ \hline 206\frac{1}{3} \\ 412\frac{1}{3} \\ 7425 \\ \hline 1650 \\ \hline 24543\frac{1}{3} \text{ Ans.} \end{array}$$

EXPLANATION.—*First Method:* Since $\frac{1}{3}$ of 825 is the same as $\frac{1}{3}$ of 3 times 825, we first find 3 times 825 and take $\frac{1}{3}$ of the product. The remainder of the operation needs no explanation. The *second method* is similar to that used in the preceding example.

57. EXAMPLE.—Multiply $89\frac{1}{3}$ by $75\frac{1}{3}$.

SOLUTION.—

First Method.

$$\begin{array}{r} 89\frac{1}{3} \\ 75\frac{1}{3} \\ \hline 8)267 \\ \hline 3)150 \\ \hline 33\frac{1}{3} \\ 50\frac{1}{3} \\ \hline 445 \end{array}$$

$$\frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$$

Second Method.

$$\begin{array}{r} 89\frac{1}{3} \\ 75\frac{1}{3} \\ \hline 11\frac{1}{3} \\ 22\frac{2}{3} \\ 25 \\ 25 \\ \hline 445\frac{1}{3} \end{array}$$

75 by 2 and indicating the division by 3. 267 (i. e., 3×89) is now divided by 8, obtaining $33\frac{3}{8}$, which is written as shown. 150 (i. e., 2×75) is divided by 3, obtaining 50, which is written under the $33\frac{3}{8}$. The two fractions are multiplied and the product, $\frac{1}{4}$, placed alongside of the 50. The two integers are now multiplied and all the separate products added together, as shown.

Second Method: This is a combination of the method of Art. 55, and the second method of Art. 56, and should be understood without further explanation. The product of the two fractions, $\frac{1}{4}$, is written alongside of the 445 and added to the other two fractions, $\frac{1}{8}$ and $\frac{3}{8}$, as shown.

58. EXAMPLE.—At $3\frac{1}{2}$ cents per pound, what will $66\frac{1}{2}$ pounds of sugar cost?

SOLUTION.—*First Method.*

$$\begin{array}{r}
 66\frac{1}{2} \\
 3\frac{1}{2} \\
 \hline
 8 \overline{) 330} \\
 2 \overline{) 3} \\
 \hline
 41\frac{1}{4} \\
 1\frac{1}{2} \\
 \hline
 198\frac{5}{16} \\
 \hline
 241\frac{1}{16} \text{ cents. Ans.}
 \end{array}$$

Second Method.

$$\begin{array}{r}
 66\frac{1}{2} \\
 3\frac{1}{2} \\
 \hline
 8\frac{1}{2} \\
 33 \\
 1\frac{1}{2} \\
 \hline
 198\frac{5}{16} \\
 \hline
 241\frac{1}{16}
 \end{array}$$

$$\frac{1}{2} \times \frac{5}{8} = \frac{5}{16}$$

EXPLANATION.—First multiply 66 by $\frac{5}{8}$, then 3 by $\frac{1}{2}$, then 66 by 3, and finally $\frac{1}{2}$ by $\frac{5}{8}$. The $\frac{5}{16}$ is written under and added to the other fractions.

EXAMPLES FOR PRACTICE.

59. Solve the following examples:

1. Find the cost of $89\frac{3}{4}$ yards of silk velvet at $\$4\frac{3}{4}$ per yard.

Ans. $\$425\frac{1}{4}$.

2. How much must be paid for $83\frac{1}{2}$ tons of hay at $\$16\frac{1}{2}$ a ton?

Ans. $\$1,388\frac{3}{4}$.

3. How far can a man ride on a bicycle in $14\frac{1}{2}$ hours at the rate of $9\frac{1}{2}$ miles per hour?

Ans. $144\frac{1}{2}$ miles.

4. Find the following products: (a) $28\frac{7}{8} \times 17\frac{3}{4}$. (b) $44\frac{7}{8} \times 16\frac{1}{2}$. (c) $127\frac{1}{2} \times 69\frac{3}{4}$. (d) $86\frac{5}{12} \times 78\frac{1}{2}$. (e) $53\frac{3}{4} \times 27\frac{1}{2}$.

Ans. (a) $510\frac{1}{2}$. (b) $744\frac{3}{4}$. (c) $8,854\frac{5}{8}$. (d) $6,809\frac{1}{2}$. (e) $1,478\frac{1}{2}$.

DIVISION OF FRACTIONS.

60. In division of fractions it is not necessary to reduce the given fractions to fractions having a common denominator.

61. *Dividing the numerator or multiplying the denominator of a fraction, divides the fraction.*

EXAMPLE.—Divide $\frac{6}{8}$ by 3.

SOLUTION.—When *dividing the numerator*, we have

$$\frac{6}{8} \div 3 = \frac{6 \div 3}{8} = \frac{2}{8} = \frac{1}{4}. \text{ Ans.}$$

When *multiplying the denominator*, we have

$$\frac{6}{8} \div 3 = \frac{6}{8 \times 3} = \frac{6}{24} = \frac{1}{4}. \text{ Ans.}$$

EXAMPLE.—Divide $\frac{3}{16}$ by 2.

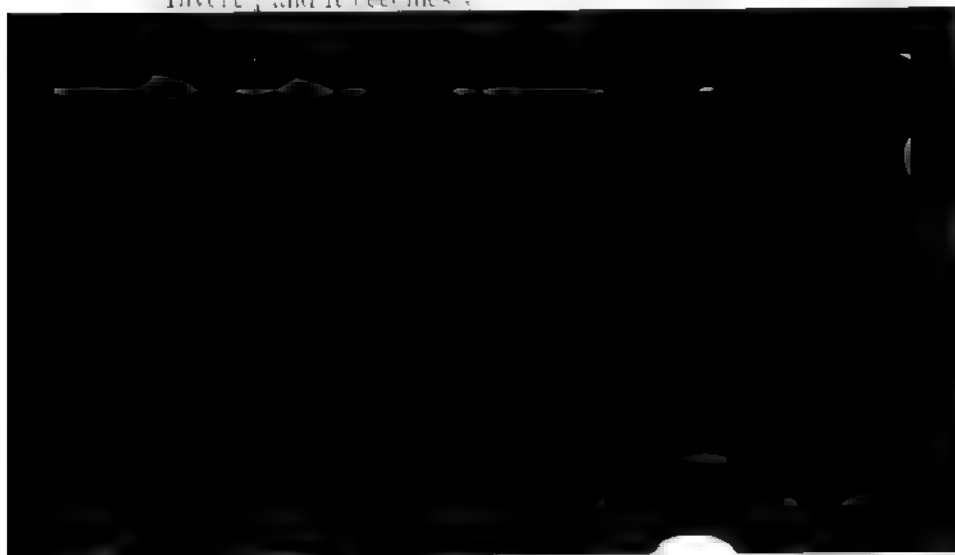
SOLUTION.— $\frac{3}{16} \div 2 = \frac{3}{16 \times 2} = \frac{3}{32}. \text{ Ans.}$

EXAMPLE.—Divide $1\frac{1}{2}$ by 7.

SOLUTION.— $1\frac{1}{2} \div 7 = \frac{14 \div 7}{22} = \frac{2}{11} = \frac{1}{5.5}. \text{ Ans.}$

62. To invert a fraction is to *turn it upside down*, that is, make the numerator and denominator change places.

Invert $\frac{3}{4}$ and it becomes $\frac{4}{3}$.



64. EXAMPLE.—Divide $\frac{3}{2}$ by $\frac{1}{2}$.

SOLUTION.—We cannot divide $\frac{3}{2}$ by $\frac{1}{2}$, as in the first case above, for the denominators are not the same; therefore, we must solve as in the second case.

$$\frac{3}{2} \div \frac{1}{2} = \frac{3}{2} \times \frac{2}{1} = \frac{3 \times 2}{2 \times 1} = \frac{6}{2} = 3 \text{ or } 1\frac{1}{2}. \text{ Ans.}$$

65. EXAMPLE.—Divide 5 by $\frac{5}{8}$.

SOLUTION.— $\frac{5}{8}$ inverted becomes $\frac{8}{5}$.

$$5 \times \frac{16}{10} = \frac{5 \times 16}{10} = 8. \text{ Ans.}$$

66. EXAMPLE.—How many times is $3\frac{1}{2}$ contained in $7\frac{1}{8}$?

SOLUTION.— $3\frac{1}{2} = \frac{7}{2}$; $7\frac{1}{8} = \frac{57}{8}$.

$\frac{7}{2}$ inverted equals $\frac{2}{7}$.

$$\frac{119}{16} \times \frac{4}{15} = \frac{119 \times 4}{16 \times 15} = \frac{119}{60} = 1\frac{1}{60}. \text{ Ans.}$$

67. Rule.—*Invert the divisor and proceed as in multiplication.*

68. We have learned that a line placed between two numbers indicates that the number above the line is to be divided by the number below it. Thus, $\frac{18}{3}$ denotes that 18 is to be divided by 3. This is also true if a fraction or a fractional expression be placed above or below a line.

$\frac{9}{\frac{3}{8}}$ means that 9 is to be divided by $\frac{3}{8}$; $\frac{3 \times 7}{\frac{8+4}{16}}$ means that

3×7 is to be divided by the value of $\frac{8+4}{16}$.

$\frac{1}{\frac{3}{8}}$ is the same as $\frac{1}{3} \div \frac{3}{8}$.

69. It will be noticed that there is a heavy line between the 9 and the $\frac{3}{8}$. This is necessary, since otherwise there would be nothing to show whether 9 is to be divided by $\frac{3}{8}$, or

$\frac{3}{4}$ is to be divided by 8. Whenever a heavy line is used, as shown here, it indicates that *all above the line* is to be divided by *all below it*.

EXAMPLES FOR PRACTICE.

70. Divide:

- (a) 15 by $6\frac{3}{4}$.
 (b) 30 by $\frac{5}{8}$.
 (c) 172 by $\frac{1}{4}$.
 (d) $\frac{1}{2}$ by $1\frac{3}{4}$.
 (e) $1\frac{3}{4}$ by $14\frac{1}{2}$.
 (f) $1\frac{3}{4}$ by $17\frac{1}{2}$.
 (g) $\frac{1}{2}$ by $1\frac{1}{2}$.
 (h) $1\frac{3}{4}$ by $72\frac{1}{2}$.

- Ans. $\left\{ \begin{array}{l} (a) 2\frac{1}{2}. \\ (b) 40. \\ (c) 215. \\ (d) 1\frac{1}{2}. \\ (e) 1\frac{1}{2}. \\ (f) 1\frac{1}{2}. \\ (g) 1\frac{1}{2}. \\ (h) 1\frac{1}{2}. \end{array} \right.$

COMPLEX FRACTIONS.

71. Whenever an expression like one of the three following is obtained, it may always be simplified by transposing the denominator from *above* to *below* the line, or from *below* to *above*, as the case may be, taking care, however, to indicate that the denominator, when so transferred, is a multiplier. These expressions are called **complex fractions**.

1. $\frac{\frac{3}{4}}{9} = \frac{3}{9 \times 4} = \frac{1}{36} = \frac{1}{36}$; for, regarding the fraction above the heavy line as the numerator of a fraction whose denominator is 9, $\frac{\frac{3}{4} \times 4}{1 \times 4} = \frac{3}{4}$, as before

This principle may be used to great advantage in cases like $\frac{1}{4} \times 310 \times \frac{27}{12} \times 72$. Reducing the mixed numbers to fractions, the expression becomes $\frac{1}{4} \times 310 \times \frac{27}{12} \times 72$. Now, transferring the denominators of the fractions and canceling,

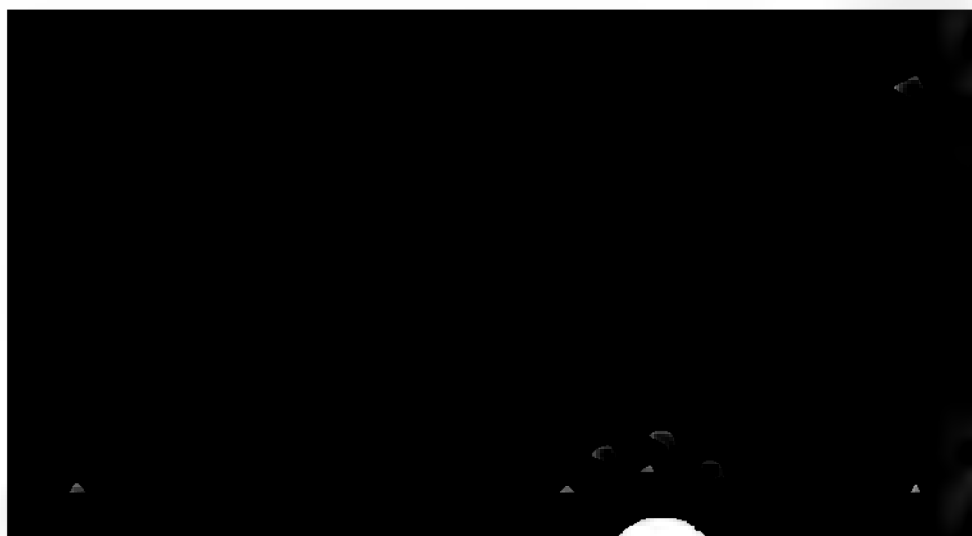
$$\frac{1 \times 310 \times 27 \times 72 \times 2 \times 6}{40 \times 9 \times 31 \times 4 \times 12} = \frac{1 \times \overset{10}{\cancel{310}} \times \overset{3}{\cancel{27}} \times \overset{3}{\cancel{72}} \times \overset{3}{\cancel{2}} \times \overset{3}{\cancel{6}}}{\underset{\substack{4 \\ 2}}{\cancel{40}} \times \underset{\substack{3 \\ 2}}{\cancel{9}} \times \underset{\substack{3 \\ 2}}{\cancel{31}} \times \underset{\substack{2 \\ 2}}{\cancel{4}} \times \underset{\substack{2 \\ 2}}{\cancel{12}}} = \frac{27}{2} = 13\frac{1}{2}.$$

Greater exactness in results can usually be obtained by using this principle than can be obtained by reducing the fractions to decimals. The principle, however, should not be employed *if a sign of addition or subtraction occurs either above or below the dividing line.*

EXAMPLES FOR PRACTICE.

72. Simplify the following:

$$\begin{array}{ll} (a) \frac{4\frac{1}{2} \times 10\frac{1}{2} \times 26\frac{1}{2}}{8\frac{1}{2} \times 4\frac{1}{2} \times 12\frac{1}{2}} & (b) \frac{10\frac{1}{2} \times 12\frac{1}{2} \times 15\frac{1}{2}}{4\frac{1}{2} \times 4\frac{1}{2} \times 8\frac{1}{2}} \\ (c) \frac{20\frac{1}{2} \times 34\frac{1}{2} \times 8\frac{1}{2}}{14\frac{1}{2} \times 3\frac{1}{2} \times 18\frac{1}{2}} & (d) \frac{15\frac{1}{2} \times 32\frac{1}{2} \times 25\frac{1}{2}}{28\frac{1}{2} \times 88\frac{1}{2} \times 17\frac{1}{2}} \end{array} \quad \text{Ans. } \left\{ \begin{array}{l} (a) \ 2\frac{1}{2}. \\ (b) \ 80. \\ (c) \ 6. \\ (d) \ 1\frac{1}{2}. \end{array} \right.$$



ARITHMETIC.

DECIMALS.

NOTATION AND NUMERATION.

1. A **decimal**, or a **decimal fraction**, is a fraction whose denominator is 10, 100, 1,000, etc.

2. The denominator is always 10 or a multiple of 10, and is not expressed as in common fractions, by writing it under the numerator, with a line between them; as $\frac{3}{10}$, $\frac{8}{100}$, $\frac{3}{1000}$. The denominator is always understood, the numerator consisting of the figures on the right of the *unit* figure of the number. In order to distinguish the unit figure, a period (.), called the **decimal point**, is placed between the unit figure and the next figure on the right. The decimal point may be regarded in two ways: first, as indicating that the number on the right is the numerator of a fraction whose denominator is 10, 100, 1,000, etc.; and, second, as a part of the Arabic system of notation, each figure on the right being 10 times as large as the next succeeding figure, and 10 times as small as the next preceding figure, serving merely to point out the unit figure.

3. The reading of a *decimal* depends upon the number of decimal places in it; i. e., upon the number of figures to the *right* of the unit figure.

The first figure to the right of the unit figure expresses *tenths*.

The second figure to the right of the unit figure expresses *hundredths*.

The third figure to the right of the unit figure expresses *thousandths*.

§ 3

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The fourth figure to the right of the unit figure expresses *ten-thousandths*.

The fifth figure to the right of the unit figure expresses *hundred-thousandths*.

The sixth figure to the right of the unit figure expresses *millionths*.

Thus:

.3	=	$\frac{3}{10}$	= 3 tenths.
.03	=	$\frac{3}{100}$	= 3 hundredths.
.003	=	$\frac{3}{1000}$	= 3 thousandths.
.0003	=	$\frac{3}{10000}$	= 3 ten-thousandths.
.00003	=	$\frac{3}{100000}$	= 3 hundred-thousandths.
.000003	=	$\frac{3}{1000000}$	= 3 millionths.

The first figure to the right of the unit figure is called the *first decimal place*; the second figure, the *second decimal place*, etc. We see in the above that the number of decimal places in a decimal equals the *number of ciphers to the right of the figure 1 in the denominator of its equivalent fraction*. This fact kept in mind will be of much assistance in reading and writing decimals.

Whatever may be written to the *left* of a decimal point is a whole number. The decimal point affects only the figures to its *right*.

When a whole number and decimal are written together, the expression is a *mixed number*. Thus, 8.12 and 17.25 are mixed numbers.

The relation of decimals and whole numbers to each other is clearly shown by the following table:

The figures to the left of the decimal point represent whole numbers; those to the right are decimals.

In both the decimals and whole numbers, the *units* place is made the starting point of notation and numeration. The *decimals decrease* on the scale of ten to the right, and the *whole numbers increase* on the scale of ten to the left. The first figure to the left of units is *tens*, and the first figure to the right of units is *tenths*. The second figure to the left of units is *hundreds*, and the second figure to the right is *hundredths*. The third figure to the left is *thousands*, and the third to the right is *thousandths*, and so on. The figures equally distant from units place correspond in name, but the decimals have the ending *ths*, which distinguishes them from whole numbers. The following is the numeration of the number in the above table: nine hundred eighty-seven million, six hundred fifty-four thousand, three hundred twenty-one, and twenty-three million, four hundred fifty-six thousand, seven hundred eighty-nine hundred-millionths.

The decimals increase to the left, on the scale of ten, the same as whole numbers; for, beginning at, say, 4 thousandths, in the table, the next figure to the left is hundredths, which is ten times as great, and the next tenths, or ten times the hundredths, and so on through both decimals and whole numbers.

PRINCIPLES OF DECIMALS.

4. I. *The value of a decimal is not changed by annexing or rejecting a cipher at the right.*

Thus, $.5 = .50$. For $.5 = \frac{5}{10} = \frac{1}{2}$, and $.50 = \frac{50}{100} = \frac{1}{2}$.

II. *A decimal is divided by 10 by inserting a cipher after the decimal point.*

Thus, $.5 = \frac{5}{10}$; $\frac{5}{10} \div 10 = \frac{5}{100} = .05$.

III. *A decimal is multiplied by 10 by rejecting a cipher from its left.*

Thus, $.05 = \frac{5}{100}$; $\frac{5}{100} \times 10 = \frac{5}{10} = .5$.

EXAMPLES FOR PRACTICE.

5. Read the following numbers:

- | | |
|------------------|----------------|
| 1. .00707. | 5. .004001008. |
| 2. .100707. | 6. 75040.0742. |
| 3. 114.75206. | 7. 234.542000. |
| 4. 40032.890001. | 8. 961724.009. |

Write the following numbers:

9. Five hundred forty-six ten-millionths.
10. Three thousand, four, and three thousand four hundred seven-
teen hundred-thousandths.
11. Five hundred four, and three-tenths.
12. Nineteen thousand thirteen, and one hundred four thousand
five hundred one ten-billionths.
13. Six hundred thirty-eight million, four hundred twenty-five
thousand, six hundred seventy-two, and thirty-two million, six hundred
seventy-two thousand, five hundred forty-five hundred-millionths.

 ADDITION OF DECIMALS.

6. To add decimals, they must be written so that the units of the same order are in the same column. That this may be, it is necessary only to see that the decimal points are in the same vertical column.

<i>whole numbers</i>	<i>decimals</i>	<i>mixed decimals</i>
342 .	.342	342.032
4234	.4234	4234 5
20	.26	26.6782
3	.03	3.01

8. Rule.—*Place the numbers to be added so that the decimal points shall be directly under each other. Add as in whole numbers, and place the decimal point in the sum directly under the decimal points above.*

EXAMPLES FOR PRACTICE.

9. Find the sum of:

(a)	.2143, .105, 2.3042, and 1.1417.	Ans.	(a)	3.7652.
(b)	783.5, 21.473, .2101, and .7816.		(b)	805.9647.
(c)	21.781, 138.72, 41.8738, .72, and 1.413.		(c)	204.5078.
(d)	.3724, 104.15, 21.417, and 100.042.		(d)	225.9814.
(e)	200.172, 14.105, 12.1465, .705, and 7.2.		(e)	234.3285.
(f)	1,427.16, .244, .32, .032, and 10.0041.		(f)	1,437.7601.
(g)	2,473.1, 41.65, .7243, 104.067, and 21.073.		(g)	2,640.6143.
(h)	4,107.2, .00375, 21.716, 410.072, and .0345.		(h)	4,539.02625.

SUBTRACTION OF DECIMALS.

10. For the same reason as in addition of decimals, the numbers are placed so that the decimal points shall be in the same vertical column.

EXAMPLE.—Subtract .132 from .3063.

SOLUTION.—

<i>minuend</i>	.3 0 6 3
<i>subtrahend</i>	.1 3 2
<i>difference</i>	.1 7 4 3 Ans.

11. EXAMPLE.—What is the difference between 7.895 and .725?

SOLUTION.—

<i>minuend</i>	7.8 9 5
<i>subtrahend</i>	.7 2 5
<i>difference</i>	7.1 7 0 or 7.17 Ans.

12. EXAMPLE.—Subtract .625 from 11.

SOLUTION.—

<i>minuend</i>	1 1.0 0 0
<i>subtrahend</i>	.6 2 5
<i>difference</i>	1 0.3 7 5 Ans.

13. Rule.—*Place the subtrahend under the minuend, so that the decimal points shall be in the same vertical column. Subtract as in whole numbers, and place the decimal point in the remainder directly under the decimal points above.*

When there are more decimal places in the subtrahend than in the minuend, place ciphers in the minuend above them, and subtract as before.

EXAMPLES FOR PRACTICE.

14. From:

(a) 407.885 take 235.0004.	Ans. {	(a) 172.8846.
(b) 22.718 take 1.7042.		(b) 21.0138.
(c) 1,868.17 take 13.6817.		(c) 1,854.4883.
(d) 70.00017 take 7.000017.		(d) 63.000153.
(e) 630.630 take .6304.		(e) 629.9996.
(f) 421.73 take 217.162.		(f) 204.568.
(g) 1.000014 take .00001.		(g) 1.000004.
(h) .789652 take .542314.		(h) .241338.

MULTIPLICATION OF DECIMALS.

15. In multiplication of decimals, no attention is paid for the time being to the decimal points. Write the multiplier under the multiplicand, so that the right-hand figure of the one is under the right-hand figure of the other, and proceed *exactly as in multiplication of whole numbers*. After multiplying, *count the number of decimal places in both multiplicand and multiplier, and point off the same number in the product*.

EXAMPLR.—Multiply .825 by 13.

SOLUTION.—	<i>multiplicand</i>	.825
	<i>multiplier</i>	13
		2475

In this example there are 3 decimal places in the multiplier and none in the multiplicand; therefore, 3 decimal places are pointed off in the product.

17. It is not necessary to multiply by the ciphers on the *left* of a decimal; they merely *determine the number of decimal places*. Ciphers to the *right* of a decimal should be removed, as they only make more figures to deal with, and do not change the value.

18. EXAMPLE.—Multiply 1.205 by 1.15.

SOLUTION.—	<i>multiplicand</i>	1.2 0 5	
	<i>multiplier</i>	1.1 5	
		<hr/>	
		6 0 2 5	
		1 2 0 5	
		<hr/>	
	<i>product</i>	1.3 8 5 7 5	Ans.

In this example there are 3 decimal places in the multiplicand, and 2 in the multiplier; therefore, $3 + 2$, or 5 decimal places must be pointed off in the product.

19. EXAMPLE.—Multiply .232 by .001.

SOLUTION.—	<i>multiplicand</i>	.2 3 2	
	<i>multiplier</i>	.0 0 1	
		<hr/>	
	<i>product</i>	.0 0 0 2 3 2	Ans.

In this example we multiply the multiplicand by the digit in the multiplier, which makes 232 in the product, but since there are 3 decimal places in the multiplier and 3 in the multiplicand, we must prefix 3 ciphers to the 232, to make $3 + 3$, or 6 decimal places in the product.

20. Rule.—*Place the multiplier under the multiplicand, disregarding the position of the decimal points. Multiply as in whole numbers, and in the product point off as many decimal places as there are decimal places in both multiplier and multiplicand, prefixing ciphers if necessary.*

EXAMPLES FOR PRACTICE.

21. Find the product of:

(a) $.000492 \times 4.1418$.	Ans. {	(a) $.0020877656$.
(b) $4,008.2 \times 1.2$.		(b) $4,809.84$.
(c) 78.6591×1.06 .		(c) 81.012698 .
(d) $.8685 \times .042$.		(d) $.015477$.
(e) $178,352 \times .01$.		(e) $1,783.52$.
(f) $.00045 \times .0045$.		(f) $.000002025$.
(g) $.714 \times .00002$.		(g) $.00001428$.
(h) $.00004 \times .008$.		(h) $.0000032$.

DIVISION OF DECIMALS.

22. In division of decimals we pay no attention to the decimal point until after the division is performed. Divide exactly as in whole numbers. *If the divisor contains more decimal places than the dividend, annex ciphers to the dividend until the number of decimal places in the dividend equals the number of decimal places in the divisor, before dividing. Subtract the number of decimal places in the divisor from the number of decimal places in the dividend, and point off as many decimal places in the quotient as there are units in the remainder thus found.*

23. EXAMPLE.—Divide .625 by 25.

	divisor	dividend	quotient	
SOLUTION.—	25	.625	.025	Ans.
		50		
		125		
		125		
	remainder	0		

than in the divisor; therefore, one decimal place is pointed off in the quotient.

25. EXAMPLE.—Divide .125 by .005.

SOLUTION.—

<i>divisor</i>	<i>dividend</i>	
.005).125	
	<u> </u>	
		<i>quotient</i> 25 Ans.

In this example there are the same number of decimal places in the dividend as in the divisor; therefore, the quotient has no decimal places, and is a whole number.

26. EXAMPLE.—Divide 326 by .25.

SOLUTION.—

<i>divisor</i>	<i>dividend</i>	<i>quotient</i>	
.25) 326.00	(1304	Ans.
	<u>25</u>		
	76		
	<u>75</u>		
	100		
	<u>100</u>		
	<i>remainder</i> 0		

In this problem two ciphers were annexed to the dividend to make the number of decimal places equal to the number in the divisor. The quotient is a whole number.

27. EXAMPLE.—Divide .0025 by 1.25.

SOLUTION.—

1.25).00250	(.002	Ans.
	<u>250</u>		
	<i>remainder</i> 0		

EXPLANATION.—In this example we are to divide .0025 by 1.25. Consider the dividend as a whole number or 25 (disregarding the two ciphers at its left, for the present); also, consider the divisor as a whole number, or 125. It is evident that the dividend 25 will not contain the divisor, 125; we must, therefore, annex one cipher to the 25, thus making the dividend 250. 125 is contained twice in 250, so we place the figure 2 in the quotient. In pointing off the decimal places in the quotient, it must be remembered that there were only four decimal places in the dividend; but one cipher was annexed, making 4 + 1, or 5, decimal places. Since there are 5 decimal places in the dividend and 2 decimal places in the

divisor, we must point off 5—2, or 3, decimal places in the quotient. In order to point off 3 decimal places, two ciphers must be prefixed to the figure 2, thereby making .002 the quotient. It is not necessary to consider the ciphers at the left of a decimal when dividing, except when determining the position of the decimal point in the quotient.

28. Rule.—I. *Place the divisor to the left of the dividend, and proceed as in division of whole numbers; in the quotient, point off as many decimal places as the number of decimal places in the dividend exceeds those in the divisor, prefixing ciphers to the quotient, if necessary.*

II. *If in dividing one number by another there is a remainder, the remainder can be placed over the divisor, as a fractional part of the quotient, but it is generally better to annex ciphers to the remainder, and continue dividing until there are 3 or 4 decimal places in the quotient, and then if there is still a remainder, terminate the quotient by the plus sign (+), to show that the division can be carried farther.*

29. EXAMPLE.—What is the quotient of 199 divided by 15?

SOLUTION.—

$$15 \overline{) 199} (13 + \frac{4}{15} \text{ Ans.}$$

$$\begin{array}{r} 15 \\ \underline{45} \end{array}$$

$$\begin{array}{r} 45 \\ \underline{45} \end{array}$$

$$\begin{array}{r} 45 \\ \underline{45} \end{array}$$

$$\text{remainder } 4$$

$$\text{Or, } 15 \overline{) 199.000} (13.266 + \text{ Ans.}$$

$$\begin{array}{r} 15 \\ \underline{45} \end{array}$$

30. It frequently happens, as in the above example, that the division will never terminate. In such cases, decide how many decimal places are desired in the quotient and then carry the work one place farther. If the last figure of the quotient thus obtained is 5 or a greater number, increase the preceding figure by 1, and write after it the minus sign ($-$), thus indicating that the quotient is not quite so great as indicated; if the figure thus obtained is less than 5, write the plus sign ($+$) after the quotient, thus indicating that the number is slightly greater than indicated. In the last example, had it been desired to obtain the answer correct to four decimal places, the work would have been carried to five places, obtaining 13.26666, and the answer would have been given as 13.2667 $-$. This remark applies to any other calculation involving decimals, when it is desired to omit some of the figures in the decimal. Thus, if it is desired to retain three decimal places in the number .2471253, it would be expressed as .247 $+$; if it were desired to retain five decimal places, it would be expressed as .24713 $-$. Both the $+$ and $-$ signs are frequently omitted; they are seldom used in this connection outside of arithmetic, except in exact calculations, when it is desired to call particular attention to the fact that the result obtained is not *quite* exact.

EXAMPLES FOR PRACTICE.

31. Divide:

(a) 101.6888 by 2.38.	Ans	(a) 43.08.
(b) 187.12264 by 123.107.		(b) 1.52.
(c) .08 by .008.		(c) 10.
(d) .0003 by 3.75.		(d) .00008.
(e) .0144 by .024.		(e) .6.
(f) .00375 by 1.25.		(f) .003.
(g) .004 by 400.		(g) .00001.
(h) .4 by .008.		(h) 50.

TO REDUCE A FRACTION TO A DECIMAL.

32. EXAMPLE.— $\frac{3}{4}$ equals what decimal?

SOLUTION.—

$$4 \overline{) 3.00} \quad \text{or } \frac{3}{4} = .75 \quad \text{Ans.}$$

EXAMPLE.—What decimal is equivalent to $\frac{3}{4}$?

SOLUTION.—
$$8 \overline{) 7.000} \text{ or } \frac{3}{4} = .875. \text{ Ans.}$$

33. Rule.—*Annex ciphers to the numerator and divide by the denominator. Point off as many decimal places in the quotient as there are ciphers annexed.*

EXAMPLES FOR PRACTICE.

34. Reduce the following common fractions to decimals:

(a) $\frac{11}{16}$.	Ans. {	(a) .6875.
(b) $\frac{7}{8}$.		(b) .875.
(c) $\frac{11}{16}$.		(c) .65625.
(d) $\frac{7}{8}$.		(d) .796875.
(e) $\frac{1}{16}$.		(e) .16.
(f) $\frac{1}{8}$.		(f) .625.
(g) $\frac{1}{20}$.		(g) .05.
(h) $\frac{1}{1000}$.		(h) .001.

35. To reduce inches to decimal parts of a foot.

EXAMPLE.—What decimal part of a foot is 9 inches?

SOLUTION.—Since there are 12 inches in one foot, 1 inch is $\frac{1}{12}$ of a foot, and 9 inches is $9 \times \frac{1}{12}$ or $\frac{3}{4}$ of a foot. This reduced to a decimal by the above rule, shows what decimal part of a foot 9 inches is.

$$12 \overline{) 9.00} \text{ (.75 of a foot. Ans.}$$

$$\underline{84}$$

EXAMPLES FOR PRACTICE.

37. Reduce to the decimal part of a foot:

(a) 3 in.	Ans. {	(a) .25.
(b) $4\frac{1}{2}$ in.		(b) .375.
(c) 5 in.		(c) .4167.
(d) $6\frac{1}{8}$ in.		(d) .5521.
(e) 11 in.		(e) .9167.

TO REDUCE A DECIMAL TO A FRACTION.

38. EXAMPLE.—Reduce .125 to a fraction.

SOLUTION.— $.125 = \frac{125}{1000} = \frac{5}{8} = \frac{1}{8}$.

EXAMPLE.—Reduce .875 to a fraction.

SOLUTION.— $.875 = \frac{875}{1000} = \frac{7}{8}$. Ans.

39. Rule.—*Under the figures of the decimal, place 1 followed by as many ciphers at its right as there are decimal places in the decimal, and reduce the resulting fraction to its lowest terms.*

EXAMPLES FOR PRACTICE.

40. Reduce the following to common fractions:

(a) .125.	Ans. {	(a) $\frac{1}{8}$.
(b) .625.		(b) $\frac{5}{8}$.
(c) .3125.		(c) $\frac{5}{16}$.
(d) .04.		(d) $\frac{1}{25}$.
(e) .06.		(e) $\frac{3}{50}$.
(f) .75.		(f) $\frac{3}{4}$.
(g) .15625.		(g) $\frac{5}{32}$.
(h) .875.		(h) $\frac{7}{8}$.

41. To express a decimal approximately as a fraction having a given denominator.

EXAMPLE.—Express .5827 in 64ths.

SOLUTION.— $.5827 \times \frac{64}{64} = \frac{37.2928}{64}$, say, $\frac{37}{64}$.

Hence, $.5827 = \frac{37}{64}$, nearly. Ans.

EXAMPLE.—Express .3917 in 12ths.

SOLUTION.— $.3917 \times \frac{12}{12} = \frac{4.7004}{12}$, say, $\frac{5}{12}$.

Hence, $.3917 = \frac{5}{12}$, nearly. Ans.

42. Rule.—Reduce 1 to a fraction having the given denominator. Multiply the given decimal by the fraction so obtained, and the result will be the fraction required.

EXAMPLES FOR PRACTICE.

43. Express:

(a)	.625 in 8ths.	Ans.	(a)	$\frac{5}{8}$.
(b)	.3125 in 16ths.		(b)	$\frac{1}{16}$.
(c)	.15625 in 32ds.		(c)	$\frac{1}{32}$.
(d)	.77 in 64ths.		(d)	$\frac{11}{16}$.
(e)	.81 in 48ths.		(e)	$\frac{3}{16}$.
(f)	.923 in 96ths.		(f)	$\frac{11}{16}$.

UNITED STATES MONEY.

44. The sign for dollars is \$. It is read dollars. \$25 is read 25 dollars.

Since there are 100 cents in a dollar, one cent is 1 one-hundredth of a dollar; the first two figures of a decimal part of a dollar represent *cents*. Since a mill is $\frac{1}{10}$ of a cent, or $\frac{1}{1000}$ of a dollar, the third figure represents *mills*. Thus, \$25.16 is read twenty-five dollars and sixteen cents; \$25.168 is read twenty-five dollars, sixteen cents, and eight mills.

45. To change dollars to cents, move the decimal point *two* places to the *right*; to change dollars to mills, move the decimal point *three* places to the right. Thus, to change \$143.75 to cents, we have \$143.75 = 14,375 cents, or 143,750 mills. The decimal point is always understood as following the unit figure, whether written or not; hence, to change \$100 to cents, write it thus, \$100.; to move the decimal point two places to the right, it is necessary to annex two ciphers, thus, 10000; in other words, \$100 = 10,000 cents.

Moving the decimal point two places to the right is evidently the same thing as multiplying by 100, since it changes the unit figure from the first order to the third order. Thus, in 143.75, 3 is a figure of the first order; but, in 14375., 3 is a figure of the third order; and, since all the other figures have also been advanced two orders, the number has been multiplied by 100.

46. To change cents to dollars, move the decimal point *two* places to the *left*; or, to change mills to dollars, move the decimal point *three* places to the left. Thus, to change 143,750 mills to dollars, move the decimal point (understood to follow the 0) three places to the left, obtaining $\$143.750 = \143.75 . Similarly, 14,375 cents = $\$143.75$, and 10,000 cents = $\$100.00$.

REPEATING DECIMALS.

47. Consider the fraction $\frac{2}{3}$. Reducing this to a decimal, it becomes $.66666+$, and it is evident that, no matter how far the operation is carried, the process will never terminate, and that all succeeding figures will be 6's. Now, instead of writing the result as above, it may be written $.\dot{6}$, the dot over the 6 indicating that the 6 *repeats*; that is, that $.\dot{6}$ is equivalent to $.66666+$. Again, consider the fraction $\frac{10}{11}$; this, reduced to a decimal, becomes $.909090+$, in which two figures, 9 and 0, repeat. This fact may be indicated by placing a dot over the 9 and over the 0, thus, $.\dot{9}\dot{0}$, whence, $\frac{10}{11} = .909090+ = .\dot{9}\dot{0}$. Had the fraction been $\frac{1}{11}$ instead of $\frac{10}{11}$, the result would have been $\frac{1}{11} = .090909+ = .\dot{0}\dot{9}$. Again, $\frac{1}{7}$ equals $.142857142857+$. Here it will be noticed that the figures 142857 repeat; this is indicated by placing a dot over the 1 and another dot over the 7, thus, $.\dot{1}4285\dot{7}$; that is, $\frac{1}{7} = .\dot{1}4285\dot{7}$.

48. Any decimal containing figures that repeat at regular intervals is called a **repeating decimal**, a **circulating decimal**, or a **circulate**.

The repeating part of the decimal is called the **repetend**. When the repetend begins with the first figure to the right of the decimal point, it is called a **pure repetend**. All of the above repetends are pure repetends.

49. Any fraction whose denominator contains only the prime factors, 2 and 5, will not repeat when converted into a decimal; that is, the division will be exact if carried far enough. But if the denominator contains any other prime factors than 2 and 5, as 3, 7, 11, etc., the fraction will produce a repeating decimal.

50. If the denominator contains the prime factor 2 or 5, or both, and other prime factors also, and the fraction itself is reduced to its lowest terms, the first figure to the right of the decimal point will not be a part of the repetend; the resulting decimal is then called a **mixed repetend**. Thus, $5 \times 3 = 15$; whence, $\frac{1}{15} = .5333\overline{3} = .53$, a mixed repetend. Similarly, $2 \times 11 = 22$, and $\frac{5}{22} = .2272\overline{7} = .22\overline{7}$.

51. Since most fractions will not reduce to exact decimals, it is frequently convenient to express the resulting decimal as a common fraction, or as a decimal and common fraction combined. This may always be done; for, any repetend may be converted into a common fraction.

Since $\frac{1}{9} = .1111\overline{1} = .\dot{1}$, $\frac{1}{99} = .010101\overline{1} = .0\dot{1}$, $\frac{1}{999} = .001001001\overline{1} = .00\dot{1}$, etc. $.1 = \frac{1}{9}$, $.0\dot{1} = \frac{1}{99}$, $.00\dot{1} = \frac{1}{999}$, etc.; hence, we may derive the denominator of any circulate from its relation to these given circulates. To illustrate, reduce $.6$, $.4\dot{5}$, and $.32\dot{4}$ to common fractions. Since $.1 = \frac{1}{9}$, $.5$ which is 5 times $.1 = 5$ times $\frac{1}{9} = \frac{5}{9}$. $.0\dot{1} = \frac{1}{99}$ and $.4\dot{5}$ which is 45 times $.0\dot{1} = \frac{45}{99}$, and, reduced to its lowest terms, $= \frac{5}{11}$. In the same way $.32\dot{4}$ may be reduced to the fraction $\frac{324}{999}$. Hence, the following rule:

52. Rule.—Write the repetend as the numerator of a fraction, and for the denominator, write as many nines as there are different figures that repeat.

53. EXAMPLE.—Express (a) $.6$; (b) $.00$; (c) $.09$; (d) $.14285\overline{7}$ as common fractions.

SOLUTION.—(a) Applying the rule, $.6 = \frac{6}{10} = \frac{3}{5}$. Ans.

(b) $.00 = \frac{00}{99} = \frac{0}{11}$. Ans.

(c) $.09 = \frac{09}{99} = \frac{1}{11}$. Ans.

(d) $.14285\overline{7} = \frac{142857}{999999} = \frac{1}{7}$. Ans.

In the case of a mixed repetend, apply the rule to the part that repeats.

EXAMPLE.—Reduce (a) $.5\dot{3}$ and (b) $.2\dot{2}\overline{7}$ to a common fraction.

SOLUTION.—(a) $.5\dot{3} = .5\frac{3}{10} = .5\frac{3}{10}$, and $\frac{5\frac{3}{10}}{10} = \frac{5\frac{3}{10} \times 3}{10 \times 3} = \frac{16}{30} = \frac{8}{15}$. Ans.

(b) $.2\dot{2}\overline{7} = .2\frac{2}{11} = .2\frac{2}{11} = \frac{2\frac{2}{11}}{10} = \frac{2\frac{2}{11} \times 11}{10 \times 11} = \frac{25}{110} = \frac{5}{22}$. Ans.

54. When a decimal and a common fraction are used in combination, *the fraction belongs to the order of the figure next preceding it.* Thus, in $.37\frac{1}{2}$, the fraction belongs to the second order, and not to the third. In other words,

$$.37\frac{1}{2} = \frac{37\frac{1}{2}}{100} = \frac{37.5}{100} = \frac{375}{1,000} = .375.$$

So, also, $.0\frac{1}{2} = \frac{\frac{1}{2}}{10} = \frac{1}{20}$, and $0.\frac{1}{2} = .\frac{1}{2} = \frac{1}{2}$. The decimal point serves only to separate the order of units from that of tenths. In all cases in which a decimal and a common fraction are used in combination, as, for example, $.67\frac{5}{8}$, if it is desired to express the common fraction as a decimal, proceed as if there were no decimal fraction before it, and then annex the figures of the result to the original decimal. Thus, in $.67\frac{5}{8}$, $\frac{5}{8} = .625$; hence, $.67\frac{5}{8} = .67625$. Proof: $.67\frac{5}{8} = \frac{67\frac{5}{8}}{100} = \frac{541}{800} = .67625$. For the want of a better name, we shall call such fractions as $.67\frac{5}{8}$ **combined fractions**.

Combined fractions frequently occur in business. They possess nearly all of the advantages of decimals, and at the same time give the absolutely exact results obtainable by common fractions. If a repeating decimal has a mixed repetend, it may be converted into a combined fraction by means of the rule given in Art. 52.

55. Repeating decimals frequently arise in ordinary division. Thus, $1,188 \div 56 = 21.2142857142857+$. Here it is seen that the six figures 142857 form a repetend. We can place the first dot over any figure of the repetend we choose; then look to the right *five* figures further, and place the second dot over the fifth figure. Thus, in the above expression, suppose that we desired a combined fraction having three figures in the decimal part. Place the first dot over the fourth figure from the decimal point, and the second dot over the ninth figure. Thus, $21.214\dot{2}857\dot{1}4$. Reducing this to a combined fraction it becomes $21.214\frac{2}{7}$.

SYMBOLS OF AGGREGATION.

56. The vinculum —, parenthesis (), brackets [], and brace { } are called **symbols of aggregation**, and are used to include numbers that are to be considered together; thus, $13 \times \overline{8-3}$, or $13 \times (8-3)$ shows that 13 is to be multiplied by the difference between 8 and 3.

$$13 \times (8-3) = 13 \times 5 = 65. \text{ Ans.}$$

$$13 \times 8 - 3 = 13 \times 5 = 65. \text{ Ans.}$$

When the vinculum or parenthesis is not used, we have

$$13 \times 8 - 3 = 104 - 3 = 101. \text{ Ans.}$$

57. In any series of numbers connected by the signs +, —, \times , and \div , the operations indicated by the signs must be performed in order from left to right, *except* that no addition or subtraction may be performed if a sign of multiplication or division *follows* the number on the *right* of a sign of addition or subtraction, until the indicated multiplication or division has been performed. In all cases the sign of multiplication takes the precedence, the reason being that when two or more numbers or expressions are connected by the sign of multiplication, the numbers thus connected are regarded as factors of the product indicated, and not as separate numbers.

EXAMPLE.—What is the value of $4 \times 24 - 8 + 17$?

SOLUTION.—Performing the operations in order from left to right, $4 \times 24 = 96$, $96 - 8 = 88$; $88 + 17 = 105$. Ans.

58. EXAMPLE.—What is the value of the following expression: $1,296 \div 12 + 160 - 22 \times 3\frac{1}{2}$?

SOLUTION.— $1,296 \div 12 = 108$; $108 + 160 = 268$, here we cannot subtract 22 from 268 because the sign of multiplication *follows* 22; hence multiplying 22 by $3\frac{1}{2}$, we get 77, and $268 - 77 = 191$. Ans.

Had the above expression been written $1,296 \div 12 + 160 - 22 \times 3\frac{1}{2} \div 7 + 25$, it would have been necessary to divide $22 \times 3\frac{1}{2}$ by 7 before subtracting, and the final result would have been $22 \times 3\frac{1}{2} = 77$; $77 \div 7 = 11$; $268 - 11 = 257$; $257 + 25 = 282$. Ans. In other words, it is necessary to

perform *all* of the multiplication or division included between the signs $+$ and $-$, or $-$ and $+$, before adding or subtracting. Also, had the expression been written $1,296 \div 12 + 160 - 24\frac{1}{2} \div 7 \times 3\frac{1}{2} + 25$, it would have been necessary to multiply $3\frac{1}{2}$ by 7 before dividing $24\frac{1}{2}$, since the sign of multiplication takes the precedence, and the final result would have been $3\frac{1}{2} \times 7 = 24\frac{1}{2}$; $24\frac{1}{2} \div 24\frac{1}{2} = 1$; $268 - 1 = 267$; $267 + 25 = 292$.

Ans.

It likewise follows that if a succession of multiplication and division signs occur, the indicated operations must not be performed in order, from left to right—the multiplication must be performed first. Thus, $24 \times 3 \div 4 \times 2 \div 9 \times 5 = \frac{1}{3}$. Ans. In order to obtain the same result that would be obtained by performing the indicated operations in order, from left to right, symbols of aggregation must be used. Thus, by using two vinculum, the last expression becomes $24 \times \overline{3 \div 4} \times \overline{2 \div 9} \times 5 = 20$, the same result that would be obtained by performing the indicated operations in order, from left to right.

EXAMPLES FOR PRACTICE.

59. Find the values of the following expressions:

(a) $(8 + 5 - 1) + 4$.	Ans. {	(a) 3.
(b) $5 \times 24 - 82$.		(b) 88.
(c) $5 \times 24 + 15$.		(c) 8.
(d) $144 - 5 \times 24$.		(d) 24.
(e) $(1,691 - 540 + 559) + 3 \times 57$.		(e) 10.
(f) $2,080 + 120 - 80 \times 4 - 1,670$.		(f) 210.
(g) $(\overline{90 + 60} + 25) \times 5 - 29$.		(g) 1.
(h) $\overline{90 + 60 + 25} \times 5$.		(h) 1.2.

ALIQUOT PARTS.

60. An aliquot part of a number is a number that will divide it without a remainder. For example, 15 is an aliquot part of 60, because 15 is an exact divisor of 60. For the same reason, 5, 6, 10, 12, etc. are also aliquot parts of 60.

61. About the only case of any importance to the business man is that concerning the aliquot parts of 100, which are given in the following table:

$2\frac{1}{2} = \frac{1}{40}$	$12\frac{1}{2} = \frac{1}{8}$	$50 = \frac{1}{2}$
$3\frac{1}{3} = \frac{1}{30}$	$16\frac{2}{3} = \frac{1}{6}$	$62\frac{1}{2} = \frac{5}{8}$
$4 = \frac{1}{25}$	$20 = \frac{1}{5}$	$66\frac{2}{3} = \frac{2}{3}$
$5 = \frac{1}{20}$	$25 = \frac{1}{4}$	$75 = \frac{3}{4}$
$6\frac{1}{2} = \frac{1}{15}$	$33\frac{1}{3} = \frac{1}{3}$	$87\frac{1}{2} = \frac{7}{8}$
$10 = \frac{1}{10}$	$87\frac{1}{2} = \frac{7}{8}$	

The aliquot parts given in the above table should be carefully memorized; in many cases calculations may be shortened by using them. Thus, in order to multiply any number by one of the numbers in the above table:

62. Rule.—*Move the decimal point of the multiplicand two places to the right, and divide the multiplicand by the denominator of the fraction opposite the multiplier in the table. Then multiply the result by the numerator of the fraction.*

63. EXAMPLE.—Multiply 478.4 by 25.

SOLUTION.—Applying the rule in Art. 62, the fraction opposite 25 is $\frac{1}{4}$. Hence, $478.4 \times 25 = 47,840 \div 4 \times 1 = 11,960$. Ans.

The result may be proved to be true by actual multiplication.

64. The reasoning on which the rule is based is this: The last example required the multiplication of 478.4 by 25. But 25 is $\frac{1}{4}$ of 100; that is, $25 = \frac{100}{4}$. Hence, in multiplying by 25, $\frac{100}{4}$ can be used as a multiplier, and the operation

65. It matters not where the decimal point is placed in the number denoting the aliquot part—the principle can still be applied with a slight modification. Thus, suppose it is required to multiply 72 by 625. Now, $62\frac{1}{2}$, or 62.5, is $\frac{5}{8}$ of 100; that is, $62.5 = 100 \times \frac{5}{8} = \frac{500}{8}$. It is an axiom in mathematics that if equals be multiplied or divided by equals, the result will be equal; that is, if $4 = 4$, and both 4's be multiplied or divided by the same number, the results will be equal. Thus, multiplying by 10, $40 = 40$; dividing by 10, $.4 = .4$. Hence, if in the expression $62.5 = \frac{500}{8}$, both numbers are multiplied by 10, the result is $625 = \frac{5,000}{8}$, or $625 = 1,000 \times \frac{5}{8}$. Therefore, to multiply 72 by 625, move the decimal point three places to the right, divide by 8 and multiply by 5. (It makes no difference whether we divide by the denominator or multiply by the numerator first.) Then, $72 \times 625 = \frac{72,000}{8} \times 5 = 45,000$. Had it been required to multiply 72 by .0625, we note that $62.5 = 100 \times \frac{5}{8}$, and that moving the decimal point in 62.5 three places to the left will give .0625. Hence, moving the decimal point in the other number (the 100) three places to the left, we have $.0625 = .1 \times \frac{5}{8} = \frac{.5}{8}$; whence, $72 \times .0625 = \frac{72}{8} \times .5 = 4.5$.

To multiply any number by 5, multiply by 10 and divide by 2.

66. To divide by one of the aliquot parts of 100, simply reverse the rule. Thus:

Rule.—*Move the decimal point two places to the left, multiply by the denominator of the equivalent fraction in the table, and divide by the numerator.*

67. EXAMPLE.—Divide 1,844 by $16\frac{2}{3}$.

SOLUTION.—Since $16\frac{2}{3} = 100 \times \frac{1}{6}$, $1,844 \div 16\frac{2}{3} = 1,844 \times \frac{6}{100} = 110.64$.
Ans.

Or, applying the rule, $1,844 \div 16\frac{2}{3} = 18.44 \times 6 \div 1 = 110.64$. Ans.

EXAMPLE.—Divide 71,025 by 37.5.

SOLUTION.—Applying the rule, $71,025 \div 37.5 = 710.25 \times 8 \div 8$
 $= 5,682 \div 8 = 1,894$. Ans.

68. To divide by a number having all the figures of the aliquot part, but with the decimal point in a different place, proceed in the same manner as in Art. 65. Thus, to divide 45,000 by 625, we have $62.5 = 100 \times \frac{5}{8}$; hence,
 $625 = 1,000 \times \frac{5}{8} = \frac{5,000}{8}$, and $45,000 \div 625 = 45,000 \div \frac{5,000}{8}$
 $= 45,000 \times \frac{8}{5,000} = 72$. Or, $45,000 \times \frac{8}{5,000} = 45,000 \times \frac{1}{1,000}$
 $\times \frac{8}{5} = 45 \times \frac{8}{5} = 72$. Ans.

69. The student will find the principle of aliquot parts extremely convenient for accurate and rapid work. Such numbers as $12\frac{1}{2}$, $16\frac{2}{3}$, 25, $33\frac{1}{3}$, $37\frac{1}{2}$, $62\frac{1}{2}$, 75, and $87\frac{1}{2}$ are of very frequent occurrence in business accounts, and the method can be readily employed in such cases. If these numbers are given as $.12\frac{1}{2}$, $.16\frac{2}{3}$, etc., as is usually the case, the example becomes even easier, as they are then equivalent to $\frac{1}{8}$, $\frac{1}{6}$, etc.

EXAMPLE.—What will be the cost of 48 yards of carpeting at \$1.83 $\frac{1}{2}$ per yard?

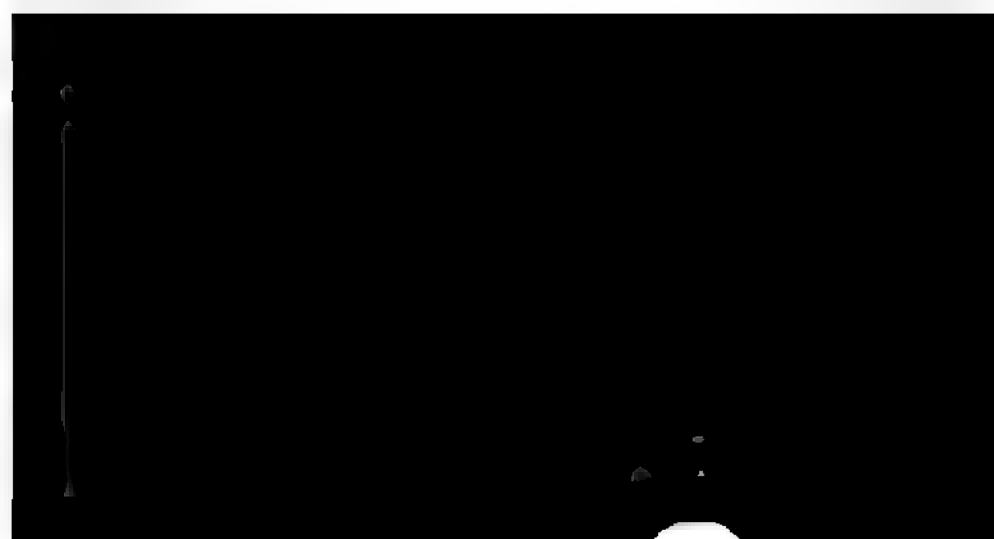
SOLUTION.—Since $.33\frac{1}{3} = \frac{1}{3}$, $\$1.83\frac{1}{2} = 1\frac{1}{3}$, and $48 \times 1\frac{1}{3} = \64 . Ans.

$$\begin{array}{r} 48 \\ 1\frac{1}{3} \\ \hline 16 \\ 48 \end{array}$$

Divide:

(f) 542.7 by 75.	Ans. {	(f) 7.236.
(g) 90,309 by 125.		(g) 722.472.
(h) 3.1416 by $66\frac{2}{3}$.		(h) .047124.
(i) 20,412 by 50.		(i) 408.24.
(j) 1,729 by 875.		(j) 1.976.

1. Find the cost of 12 dozen hats at $\$4.12\frac{1}{2}$ per dozen. Ans. $\$49.50$.
2. Find the cost of 24 boxes of note paper at $16\frac{1}{2}$ cents per box.
Ans. $\$4$.
3. Find the cost of 75 books at 25 cents each. Ans. $\$18.75$.
4. Find the cost of 30.19 hundredweight of bran at $62\frac{1}{2}$ cents per hundredweight.
Ans. $\$18.86\frac{1}{2}$.
5. Find the cost of 36 pairs of shoes at $\$2.25$ per pair. Ans. $\$81$.
6. Find the cost of 87 pounds of sugar at 5 cents per pound.
Ans. $\$4.35$.



ARITHMETIC.

COMPOUND NUMBERS.

1. Numbers may be expressed according to a *uniform* or a *varying scale*. By **scale** is meant the relation of a unit of one order to the unit of the next higher or lower order. When the relation is the same for any two consecutive orders, the scale is said to be **uniform**; otherwise, it is **varying**. For example, the scale by which numbers are expressed in the Arabic notation is a uniform scale, since a unit of any order is 10 times as great as the unit of the next lower order; for 100 is 10 times 10, and 10 is 10 times 1, etc. The Arabic notation, the metric system, and United States money are the leading examples of the application of a uniform scale. All other numbers in commercial use in English-speaking countries require the use of a varying scale. Thus, to express 4 yards 2 feet 7 inches, it is necessary to write the words *yards*, *feet*, and *inches*, or their abbreviations, since 12 inches equal 1 foot, and 3 feet equal 1 yard.

2. A **simple number** is one which expresses one or more units of the same name or denomination; as, 5, 6 yards, etc.

3. A **compound number** is one which expresses units of two or more denominations of the *same kind*, the denominations increasing or decreasing according to a varying scale; as, 4 yards 2 feet 7 inches. But 4 yards and 5 ounces is not a compound number, since there is no relation between yards and ounces; that is, no number of ounces can equal a yard. Compound numbers are also frequently called **denominate numbers**.

§ 4

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4. Compound numbers form a very important section of arithmetic. It is necessary for the student to obtain a clear idea of their use, and to be able to add, subtract, multiply, and divide them. The only real difficulty that arises is the memorizing of the tables. The best way to do this is to read them over very carefully several times, and then, after carefully studying the sections on Reduction, Addition, etc., work the examples (all of them), constantly referring to the tables for help. But, before leaving the subject to take up the next paper, he should thoroughly memorize *all* of the tables, so that when any one asks him how many square rods there are in an acre, or a similar question, he can answer instantly, without being obliged to stop and think.

5. A measure is a *standard unit* established by law or custom, by which the length, surface, capacity, and weight of things are estimated.

6. Measures are of six kinds, as follows:

- | | |
|----------------|---------------------|
| (1) Extension. | (4) Time. |
| (2) Weight. | (5) Angles or Arcs. |
| (3) Capacity. | (6) Money or Value. |

We shall consider them in the above order.

MEASURES OF EXTENSION.

7. Measures of extension are used in measuring lengths (distances), surfaces (areas), and solids (volumes), and are divided, accordingly, into *linear measure*, *square measure*, and *cubic measure*.

LINEAR MEASURE.

8. The standard to which all measures of extension are referred is the *yard*, which is the distance between two points on a brass bar kept at Washington. The yard is subdivided into feet and inches; and multiples of the yard are termed rods and miles. The relations between the different units are shown in the following table; the letters in *Italics* are the abbreviations of the names of the units.

LINEAR MEASURE.

TABLE I.

12 inches (<i>in.</i>)	=	1 foot.....	<i>ft.</i>
3 feet	=	1 yard.....	<i>yd.</i>
5½ yards	=	1 rod.....	<i>rd.</i>
320 rods	=	1 mile.....	<i>mi.</i>

in.	ft.	yd.	rd.	mi.
12 =	1			
36 =	3 =	1		
198 =	16½ =	5½ =	1	
63,360 =	5,280 =	1,760 =	320 =	1

9. The inch is usually divided into halves, quarters, eighths, and sixteenths; by civil engineers and scientists, into tenths, hundredths, thousandths, etc., and in other ways. In measuring cloth, ribbons, and other goods that are sold by the yard, the yard is divided into halves, quarters, eighths, and sixteenths.

A furlong is one-eighth of a mile, or 40 rods. The mile of 5,280 feet is a **statute mile**, so called to distinguish it from the geographical or sea mile, which equals 6,081 feet.

10. Another abbreviation frequently used for inches and feet is (") and ('). Thus, instead of writing 4 feet 6 inches as 4 ft. 6 in., it may be written 4' 6"; but when so written, it is customary to place a dash between the feet and inches; thus, 4'-6". Still another way of writing the above is 4 ft. 6".

SURVEYOR'S LINEAR MEASURE.

TABLE II.

7.92 inches (<i>in.</i>)	=	1 link.....	<i>li.</i>
25 links.....	=	1 rod.....	<i>rd.</i>
4 rods } 100 links }	=	1 chain.....	<i>ch.</i>
80 chains.....	=	1 mile.....	<i>mi.</i>

in.	li.	ft.	rd.	ch.	mi.
7.92 =	1				
198 =	25 =	16½ =	1		
792 =	100 =	66 =	4 =	1	
63,360 =	8,000 =	5,280 =	320 =	80 =	1

11. Surveyor's linear measure is used in measuring land, roads, etc. The unit is a steel chain 66 feet long, and made of 100 links, all of equal length; therefore, the length of a link is $66 \times 12 \div 100 = 7\frac{1}{2}$ inches. For railroad surveying and other purposes, civil engineers use a steel tape 100 feet long, the feet being divided into tenths and hundredths. In computations, the links are written as so many hundredths of a chain.

SQUARE MEASURE.

TABLE III.

144 square inches (<i>sq. in.</i>) . . .	= 1 square foot. <i>sq. ft.</i>
9 square feet.	= 1 square yard. <i>sq. yd.</i>
$30\frac{1}{4}$ square yards	= 1 square rod. <i>sq. rd.</i>
160 square rods.	= 1 acre <i>A.</i>
640 acres.	= 1 square mile <i>sq. mi.</i>

<i>sq. in.</i>	<i>sq. ft.</i>	<i>sq. yd.</i>	<i>sq. rd.</i>	<i>A. sq. mi.</i>
144 =	1			
1,296 =	9	1		
39,204 =	$272\frac{1}{4}$	$30\frac{1}{4}$	1	
6,272,640 =	43,560	4,840	160	1
4,014,489,600 =	27,878,400	3,087,600	102,400	640 = 1

12. Square measure is used in estimating the *area* of surfaces. In commercial use, the square yard is the largest unit employed, the square rod, acre, and square mile being used for measuring land. The unit of square measure is a *square* whose sides are equal in length to the linear unit. The units of square measure are *derived units*; that is, they depend for their value upon the values of some other units, which,

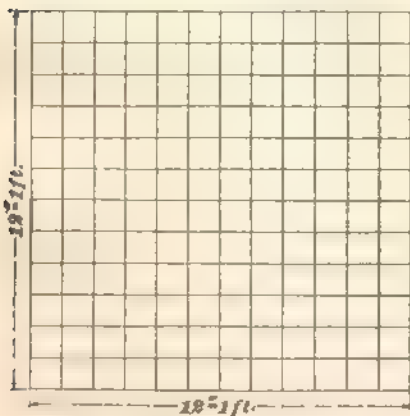


FIG. 1.

in this case, are the units of linear measure. If the large square in Fig. 1 measures 1 foot on each side, as indicated

by the dimension lines, then the space which the square covers, on a flat surface, is *1 square foot*. If horizontal and vertical lines be drawn 1 inch apart, as shown, the small squares so formed will measure 1 inch on each side; and if these small squares be counted, they will be found to number 144. Hence, there are 144 square inches in 1 square foot. In the same way, it can be shown that 1 square yard contains 9 square feet. The *acre* is the only unit that forms an exception—it cannot be expressed as an exact square of any unit. A piece of land 208.71 feet square contains almost exactly an acre.

Roofers, plasterers, and carpenters frequently call 100 square feet a **square**.

SURVEYOR'S SQUARE MEASURE.

TABLE IV.

625 square links (<i>sq. li.</i>)	= 1 square rod....	<i>sq. rd.</i>
16 square rods	= 1 square chain .	<i>sq. ch.</i>
10 square chains	= 1 acre.....	<i>A.</i>
640 acres	= 1 square mile...	<i>sq. mi.</i>
36 square miles (6 miles square)	= 1 township	<i> Tp.</i>

13. Surveyor's square measure is used only by civil engineers and surveyors. For this reason, no further remarks will be made concerning this measure.

CUBIC MEASURE.

TABLE V.

1,728 cubic inches (<i>cu. in.</i>)	= 1 cubic foot.....	<i>cu. ft.</i>
27 cubic feet	= 1 cubic yard	<i>cu. yd.</i>
128 cubic feet		= 1 cord of wood.	

cu. in.	cu. ft.	cu. yd.
1,728	= 1	
46,656	= 27	= 1

14. Cubic measure is used in measuring the volumes of solids or bodies which have length, breadth, and thickness. The units of cubic measure are also derived units, since they depend upon linear measurements for their values.

The unit of cubic measure is a cube whose edges are equal in length to the corresponding linear unit. Fig. 2 represents a cube whose sides are all 3 feet long. By dividing it into equal parts, as shown, it is readily seen that 27 small cubes,

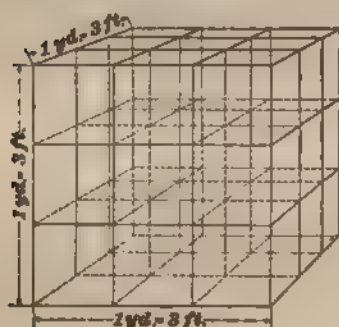


FIG. 2.

measuring 1 foot on each edge, will be formed; hence, 1 cubic yard contains 27 cubic feet. In a similar manner, it can be shown that 1 cubic foot contains 1,728 cubic inches.

15. The cord is used in measuring wood. A pile of wood 8 feet long, 4 feet wide, and 4 feet high contains 1 cord, since $8 \times 4 \times 4 = 128$ cubic feet.

A cord foot is 1 foot in length of such a pile; that is, it is 1 foot long, 4 feet wide, and 4 feet high. The cord foot contains $1 \times 4 \times 4 = 16$ cubic feet.

16. Masons use what is termed a perch. A perch of masonry is 1 rod ($16\frac{1}{2}$ ft.) long, $1\frac{1}{2}$ feet thick, and 1 foot high, and contains $16\frac{1}{2} \times 1\frac{1}{2} \times 1 = 24\frac{3}{4}$ cubic feet. The perch is going out of use, the cubic yard being used instead; but, when used, it is generally considered to be 25 cubic feet.

MEASURES OF WEIGHT.

AVOIRDUPOIS WEIGHT.

TABLE VI.

16 ounces (oz).....	= 1 pound.....	<i>lb.</i>
100 pounds.....	= 1 hundredweight.....	<i>cwt.</i>
20 hundredweight {	= 1 ton.....	<i>T.</i>
2,000 pounds.....		

oz.	lb.	cwt.	T.
16 =	1		
1,600 =	100 =	1	
32,000 =	2,000 =	20 =	1

17. Avoirdupois weight is used in nearly all commercial transactions. The ton of 2,000 pounds is always understood, unless the *long* or *gross* ton is specified. Formerly, the long ton of 2,240 pounds was used exclusively, and is still used in Great Britain, by U. S. Custom Houses, in ocean freights, and by wholesale dealers in coal and iron, and iron ores. The **long ton table** is as follows:

LONG TON TABLE.

TABLE VII.

16 ounces (oz.)	= 1 pound	lb.
28 pounds	= 1 quarter	qr.
4 quarters	= 1 hundredweight	cwt.
20 hundredweight	{	= 1 ton
2,240 pounds				

oz.	lb.	qr.	cwt.	T.
16 =	1			
448 =	28 =	1		
1,792 =	112 =	4 =	1	
35,840 =	2,240 =	80 =	20 =	1

18. Formerly, the ounce was divided into 16 parts, called drams. The dram is now seldom or never used.

TROY WEIGHT.

TABLE VIII.

24 grains (gr.)	= 1 pennyweight	pwt.
20 pennyweights	= 1 ounce	oz.
12 ounces	= 1 pound	lb.

gr.	pwt.	oz.	lb.
24	1		
480	20 =	1	
5,760	240 =	12 =	1

19. Troy weight is used by jewelers and in weighing of gold, silver, coins, and jewelry. The **Troy pound** contains 5,760 grains, while the **avoirdupois pound** contains 7,000 grains.

5760 000
11520 000

7,000 grains. Hence, the Troy ounce contains $5,760 \div 12 = 480$ grains, and the avoirdupois ounce, $7,000 \div 16 = 437\frac{1}{2}$ grains. The student will find it useful to remember these facts. When it is stated that an ounce of gold is worth, say, twenty dollars, the Troy ounce, which contains 480 grains, is meant.

APOTHECARIES' WEIGHT.

TABLE IX.

20 grains (<i>gr</i>)	=	1 scruple	<i>sc.</i> or \mathfrak{d}
3 scruples	=	1 dram	<i>dr.</i> or \mathfrak{z}
8 drams	=	1 ounce	<i>oz.</i> or \mathfrak{z}
12 ounces	=	1 pound	<i>lb.</i> or \mathfrak{lb}

gr. \mathfrak{d} \mathfrak{z} \mathfrak{z} \mathfrak{lb} .

20 = 1

60 = 3 = 1

480 24 = 8 = 1

5,760 288 = 96 = 12 = 1

20. Apothecaries' weight is used by physicians and druggists in prescribing and compounding medicines not in the liquid state. When the *symbols* are used instead of the abbreviations, they are placed before the figures denoting the number of units. Thus, $\mathfrak{ss} \mathfrak{d}2$ means 5 drams 2 scruples. The pound, ounce, and grain are the same as in Troy weight.

Drugs and medicines are sold, when in large quantities, by avoirdupois weight.

MEASURES OF CAPACITY.

LIQUID MEASURE.

TABLE X.

4 gills (<i>gi.</i>)	=	1 pint	<i>pt.</i>
2 pints	=	1 quart	<i>qt.</i>
4 quarts	=	1 gallon	<i>gal.</i>
31½ gallons	=	1 barrel	<i>bbL</i>
2 barrels	}	=	1 hogshead	<i>hhd.</i>
63 gallons					

gi	pt.	qt.	gal.	hhd.
4 =	1			
8 =	2	1		
32 =	8	4 =	1	
1,008 =	252	126 =	31½ =	1
2,016 =	504 =	252 =	63	2 = 1

21. Liquid measure is used for measuring liquids. The standard gallon used in the United States is the *wine gallon*, so called to distinguish it from the *beer gallon*. The beer gallon was formerly used for measuring beer, milk, etc., but has now passed out of use. The wine gallon contains 231 cubic inches; the beer gallon contained 282 cubic inches. The gallon used in Great Britain is called the **British imperial gallon**; it contains 277.274 cubic inches. One wine gallon of water weighs 8.355 pounds, and 1 cubic foot contains 7.481 wine gallons. One British imperial gallon weighs 10 pounds. The student should carefully memorize these facts; he will find them very useful.

22. The barrel and hogshead are used in estimating the capacity of tanks, cisterns, reservoirs, etc. The gallon is the unit most commonly used when estimating in large quantities. The ordinary barrels and hogsheads used in commerce vary greatly in size, and their contents can be determined only by gauging or actual measurement.

APOTHECARIES' FLUID MEASURE.

TABLE XI

60 minims, or drops, (m.)	=	1 fluidrachm	..	f℥.
8 fluidrachms	..	=	1 fluidounce f℥.
16 fluidounces	..	=	1 pint.	... O.
8 pints	..	=	1 gallon Cong.

23. Apothecaries' fluid measure is used in prescribing and compounding medicines. The gallon and pint are the same as the wine gallon and pint. As in apothecaries' weight, the symbols precede the numbers to which they refer. For example, Cong. 2 O 7 f℥ 12 means 2 gallons 7 pints 12 fluidounces. *Cong.* is the abbreviation of the Latin word *congius*, gallon; *O.* is the abbreviation of the Latin word *octarius*, one-eighth.

DRY MEASURE.

TABLE XII.

2 pints (<i>pt</i>)	= 1 quart.....	<i>qt</i> .
8 quarts	= 1 peck.....	<i>pk</i> .
4 pecks	= 1 bushel.....	<i>bu</i> .

pt.	qt.	pk.	bu.
2	1		
16	8	1	
64	32	4	1

24. Dry measure, as its name implies, is used in measuring dry articles, as fruits, vegetables, grain, etc. The unit is the so called Winchester bushel, which contains 2,150.42 cubic inches, or very nearly 9.31 wine gallons. A box 14 inches long, 12.8 inches wide, and 12 inches deep (all inside measurements) contains almost exactly 1 bushel.

25. It is becoming the custom to sell by weight many articles that would ordinarily be sold by dry measure. The number of pounds of various commodities that are taken as equivalent to one bushel, varies greatly in different states; but the Boards of Trade of the principal cities of the United States use the equivalents given in the following table:

AVOIRDUPOIS POUNDS IN A BUSHEL.

TABLE XIII.

Commodities.	Lb.	Commodities.	Lb.	Commodities.	Lb.
Barley	48	Corn (shelled)	56	Potatoes,	60
Beans	60	Corn (in the ear)	70	Rye	58
Buckwheat	48	Malt	44	Timothy Seed..	45
Clover Seed	60	Oats	32	Wheat	60

26. The following units are also in commercial use:

1 Quintal of fish	= 100 lb.
1 Barrel of flour	= 196 lb.
1 Barrel of pork or beef	= 200 lb.
1 Gallon of petroleum	= 6½ lb.
1 Keg of nails	= 100 lb.

27. **Stricken measure** means measuring the vessel even full, and striking off the surplus with a stick. Grain, seeds, berries, etc. are sold by stricken measure. The standard bushel mentioned above is a stricken bushel. The **heaped bushel** means the contents of the measuring vessel heaped up in the form of a cone. Corn in the ear, large fruits and vegetables, coal, lime, and other bulky articles, are sold by the heaped bushel. It is customary to allow 5 stricken bushels for 4 heaped ones.

In San Francisco and some other markets, produce is bought and sold by the cental, a **cental** being 100 pounds. In computing freight charges the hundredweight of 100 pounds is taken as the unit.

MEASURES OF TIME.

TABLE XIV.

60 seconds (<i>sec.</i>).....	=	1 minute.....	<i>min.</i>
60 minutes.....	=	1 hour.....	<i>hr.</i>
24 hours.....	=	1 day.....	<i>da.</i>
7 days.....	=	1 week.....	<i>wk.</i>
4 weeks.....	=	1 month.....	<i>mo.</i>
12 months.....	=	1 year.....	<i>yr.</i>
100 years.....	=	1 century.....	<i>C.</i>

sec.	min.	hr.	da.	wk.	yr.
60 =	1				
3,600 =	60 =	1			
86,400 =	1,440 =	24 =	1		
604,800 =	10,080 =	168 =	7 =	1	
31,556,936 =	525,948 =	8,765 =	365 =	52 =	1

28. The divisions of time are peculiar. The only units that may be called *natural* are the day, the lunar month, and the year, the other divisions being artificial. The time in which the earth makes one complete revolution around the sun is called a **solar year**, and it equals 365 days, 5 hours, 48 minutes, 49.7 seconds, or $365\frac{1}{4}$ days, very nearly. A **solar day** is the interval between two consecutive returns of the sun to the meridian. On account of the earth moving in an

elliptical path around the sun, the length of the solar day varies; hence, for civil purposes, the average of all the days in the year is taken as a unit. The months contain from 28 to 31 days.

In order to make the *calendar*, or *civil*, year agree as nearly as possible with the solar year, it is customary to add 1 day to the year (making 366 days) every fourth year. Years containing 366 days are called **leap years**. Every leap year is exactly divisible by 4. Thus 1896, which was a leap year, is divisible by 4 without a remainder. Any year which ends with two or more ciphers, as 1800, 1900, 2000, etc., is called a **secular year**. Since the year falls short of $365\frac{1}{4}$ days by 11 minutes and 10.3 seconds, the addition of 1 day every 4 years makes about $\frac{1}{4}$ of a day too much in a century; hence, to correct this, secular years are not leap years except when exactly divisible by 400. Thus, 1900 will not be a leap year but 2000 will. This last correction makes an error of about 1 day in 5,400 years.

29. The following is a list of months, in regular order, with the number of days which each contains:

	Days		Days
1. January (Jan.).....	31	7. July.....	31
2. February (Feb.)	28	8. August (Aug) . . .	31
3. March (Mar.).....	31	9. September (Sept)...	30
4. April (Apr.).....	30	10. October (Oct.).....	31
5. May.	31	11. November (Nov.)... 30	
6. June	30	12. December (Dec.)... 31	

In leap years, one day is added to February, giving it 29 days. The following lines will assist the student in remembering the number of days in each month:

"Thirty days hath September,
April, June, and November,
All the rest have thirty-one,
Save February, which alone
Hath but twenty-eight, in fine,
But leap year gives it twenty-nine."

In many business transactions, the year is regarded as 360 days, or 12 months of 30 days each.

MEASURES OF ANGLES OR ARCS.

CIRCULAR MEASURE.

TABLE XV.

60 seconds (")	= 1 minute.. . . '
60 minutes.	= 1 degree..... °
360 degrees.. . . .	= 1 circle.....	⊙
60' =	1'	
3,600' =	60' =	1°
1,296,000" =	21,600' =	360° = 1⊙

30. A **quadrant** is one-fourth of a circle, or 90° ; a **sextant** is one-sixth of a circle, or 60° . A **right angle** (\perp) contains 90° . The unit of measurement is the degree, or $\frac{1}{360}$ of the circumference of a circle.

31. **Circular or angular measure** is used principally by surveyors, navigators, astronomers, and by technical men generally, for measuring angles and arcs of circles.

LONGITUDE AND TIME.

32. The earth is very nearly spherical in shape, and has a rotary movement about an imaginary line (called the **axis**) which is assumed to pass through the **center**. The two points where the axis emerges from the earth are called **poles**—one the **north pole**, the other the **south pole**. The time required for the earth to make one rotation, or complete turn, about this axis, is called one **day** (see Art. 28). Midway between the poles, the earth is supposed to have a circle passing around it, called the **equator**. The equator is divided into 360 equal parts called **degrees** (see Table XV); each degree is divided into 60 equal parts, called **minutes**; and each minute is again divided into 60 equal parts, called **seconds**. Through each point of division a circle, called a **meridian of longitude**, is supposed to pass, the meridian also passing through the poles, as shown in Fig 3. Other circles, called **parallels of latitude**, are supposed to be drawn parallel to the equator, and between it and the poles. By means of these imaginary circles the position of any place on the surface of the earth may be determined; this will be made plain by observing where the

parallels and meridians intersect in Fig 3. It is evident that if we know what parallel and what meridian pass through the place, the position of the place is determined by their point of intersection. The distance between the equator and either pole is divided into 90° . The **latitude** of a place is its distance from the equator, measured in degrees,

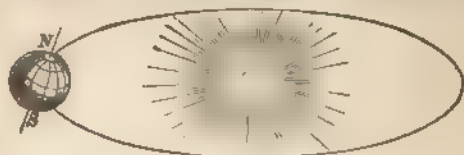


FIG 3.

minutes, and seconds along the meridian passing through the place. The **longitude** of a place is its distance from some fixed meridian (called the **prime meridian**) in degrees, minutes, and seconds, measured along the parallel passing through the place. There are three prime meridians: that of Greenwich, that of Paris, and that of Washington, that of Greenwich being the one principally used by navigators. Longitudes reckoned east of the meridian passing through Greenwich to 180° are called **east longitudes**, and those reckoned west, **west longitudes**. Latitudes reckoned north from the equator are called **north latitudes**, and those reckoned south from the equator, **south latitudes**.

The astronomical day begins at noon, when the meridian of any place comes in direct line with the sun; but, for business and other reasons, the civil day begins twelve hours earlier, bringing what we call **noon** in the middle of the day. A little consideration of these facts will make it evident to the student that for any place situated on a meridian east of where he lives, **noon** will occur earlier, and for any place west of him, later, than for the place in which he is. If the longitudes of any two places are known, their difference of time, as it is called, is easily determined. For, since the earth rotates from *west* to *east* 360° in 24 hours, the sun appears to go from *east* to *west* $\frac{1}{24}$ of 360° , or 15° , in 1 hour of time. Hence, in 1 minute of time it seems to go $\frac{1}{60}$ of 15° , or $15'$, and in 1 second of time, $\frac{1}{60}$ of $15'$, or $15''$. This is shown in the following table:

360° in longitude correspond to 24 hours of time.

15° in longitude correspond to 1 hour of time.

15' in longitude correspond to 1 minute of time.

15" in longitude correspond to 1 second of time

33. Rule. -To find the difference of time between two places, find the difference in longitude* in ° ' ", and divide by 15; the result will be their difference in time in hours, minutes, and seconds.

34. Rule. -To find the difference in longitude of two places, multiply their difference in time (solar time) expressed in hours, minutes, and seconds by 15; the result will be their difference in longitude in ° ' ".

35. In the following table is given the difference in time between some important cities and Greenwich; also, the longitudes of those cities reckoned from the prime meridian passing through Greenwich:

	hr.	min.	sec.	Longitudes.
Albany.	4	54	59.9	73° 44' 48" W.
Ann Arbor.	5	34	55.1	83° 43' 46.5" W.
Boston.	4	44	15.3	71° 3' 49.5" W.
Berlin.	0	53	34.9	13° 23' 43.5" E
Calcutta.	5	55	20.7	88° 50' 10.5" E.
New York.	4	55	53.6	{ Columbia College. 73° 58' 24" W.
New Orleans.	6	0	12.9	{ U. S. Mint. 90° 3' 13.9" W.
Paris.	0	9	20.9	2° 20' 13.5" E.
Philadelphia.	5	0	38.5	75° 9' 37.5" W.
Rome.	0	49	54.7	12° 28' 40.5" E
Cincinnati.	5	37	41.3	84° 25' 19.5" W.
Chicago.	5	50	26.7	87° 36' 40.5" W.
Jefferson City.	6	8	36	92° 9' 0" W.
London.	0	0	22.5	5° 37.5" W.
City of Mexico.	6	36	26.7	99° 6' 40.5" W.
Richmond.	5	9	44	77° 26' 0" W.
San Francisco.	8	9	38.1	122° 24' 31.5" W.
St. Paul.	6	12	20	93° 5' 0" W.
St. Louis.	6	0	41.1	90° 10' 16.5" W.
Washington.	5	8	12	77° 3' 0" W.

Examples relating to the above will be given later. See Arts. 82-84.

*To add, subtract, multiply, or divide compound numbers, see rules, Arts. 60, 63, 73, and 78.

MEASURES OF MONEY.

UNITED STATES MONEY.

TABLE XVI.

10 mills (m.).....	= 1 cent.	<i>ct.</i>
10 cents	= 1 dime	<i>d.</i>
10 dimes	= 1 dollar	<i>ℓ.</i>
10 dollars.....	= 1 eagle	<i>E.</i>

m.	ct.	d.	ℓ.	E.
10 =	1			
100 =	10	1		
1,000 =	100	10	1	
10,000 =	1,000	100	10	1

36. The unit of value is the gold dollar, which weighs 25.8 grains. Since gold and silver are so soft that they would rapidly lose in weight if circulated as money in their natural state, they are alloyed with 1 part of alloy to every 9 parts of the pure metal. In other words, a gold or silver coin contains only .9 of its weight of pure gold or silver. Since 9 parts in 10 are equivalent to 900 parts in every thousand, a gold dollar is stated to contain 25.8 grains of gold 900 *fine*, the word "fine" meaning the number of parts of the pure metal in 1,000 parts by weight of the coin or alloyed metal. In business operations, the terms dollar and cent only are used, the terms eagle and dime being merely names of coins. The mill is not coined, and is rarely used except in referring to tax rates.

37. The term **legal tender** is applied to money which may be legally offered in payment of debts. All coins are legal tender up to a certain amount, depending upon the value of the coins. By legal tender is meant that if the debtor offers to pay his obligation in legal-tender money, the creditor *must* accept it. All gold coins are legal tender for their face value to any amount, provided their weight has not diminished more than $\frac{1}{100}$. Silver dollars are also legal tender to any amount; but silver coins of lower denomination than one dollar are legal tender only for sums not exceeding \$10. Nickel and copper coins are legal tender for sums not exceeding 25 cents.

38. The legal coins of the United States are:

GOLD.

Weight in grains.

1-dollar piece,	25.8
2½-dollar piece, or {	64.5
quarter-eagle,	
3-dollar piece,	77.4
5-dollar piece, or {	129.0
half-eagle,	
10-dollar piece, or {	258.0
eagle,	
20-dollar piece, or {	516.0
double-eagle,	

SILVER.

Weight.

Standard dollar,	412½ grains.
Half-dollar, or {	192.9 grains, or 12½ grains.
50-cent piece,	
Quarter-dollar, or {	96.45 grains, or 6½ grains.
25-cent piece,	
Dime, or {	38.58 grains, or 2½ grains.
10-cent piece,	

COPPER AND NICKEL.

5-cent piece,	77.16 grains, or 5 grams.
3-cent piece,	30.00 grains.
1-cent piece,	48.00 grains.

39. United States money is expressed and read decimally. Eagles and dimes are never read; only dollars and hundredths, or dollars and cents are read. Thus, 5 eagles 4 dollars 7 dimes and 3 cents would be written \$54.73 and read "fifty-four dollars and seventy-three cents." If there are more than two decimal places, the extra ones are read as decimal parts of a cent. Thus, \$54.7329 is read "fifty-four dollars and seventy-three and twenty-nine hundredths cents."

When the number of cents is less than ten, a cipher must be written in the first place to the right of the decimal point. Thus, four dollars and five cents is written \$4.05.

In checks, notes, drafts, etc., to prevent forgery and mistakes, the cents are written as hundredths of a dollar in the form of a fraction. Thus, \$13.47 would be written \$13 $\frac{47}{100}$.

ENGLISH MONEY.

TABLE XVII

4 farthings (<i>far.</i>).....	= 1 penny.....	<i>d.</i>
12 pence	= 1 shilling.....	<i>s.</i>
20 shillings.....	= 1 pound, or	
	sovereign...	<i>£.</i>

<i>far.</i>	<i>d.</i>	<i>s.</i>	<i>£.</i>
4 =	1		
48 =	12 =	1	
960 =	240 =	20 =	1

40. The unit of English money is the **pound sterling**, the value of which in United States money is \$4.8665. The fineness of English silver is .925; of the gold coins, .916 $\frac{2}{3}$. What is called sterling silver when applied to *solid* silver articles has the same fineness. Hence the name—sterling silver.

The other coins of Great Britain are the *florin* (= 2 shillings), the *crown* (= 5 shillings), the *half-crown* (= 2 $\frac{1}{2}$ shillings), and the *guinea* (= 21 shillings). The largest silver coin is the crown, and the smallest, the sixpence ($\frac{1}{2}$ shilling). The shilling is worth 25ct. (24.3+ ct) in United States money. The guinea is no longer coined. The abbreviation *£* is written before the number, while *s.* and *d.* follow. Thus, £25 4 s. 6 d. = 25 pounds 4 shillings 6 pence.

MISCELLANEOUS TABLES.

41. The following table is used in counting certain articles:

TABLE XVIII.

12 of anything.....	= 1 dozen.	<i>doz.</i>
12 dozen	= 1 gross. . . .	<i>gr.</i>
12 gross. . . .	= 1 great gross...	<i>g. gr.</i>
20 of anything	= 1 score.	

units	doz.	gr.	g. gr.
12 =	1		
144 =	12 =	1	
1,728 =	144 =	12 =	1

42. The following table is used in the paper trade:

TABLE XIX.

24 sheets.....	= 1 quire.....	<i>qr.</i>
20 quires.....	= 1 ream.....	<i>rm.</i>
2 reams.....	= 1 bundle.....	<i>bdl.</i>
5 bundles.....	= 1 bale.....	<i>B.</i>

sheets	qr.	rm.	bdl.	B.
24 =	1			
480 =	20 =	1		
960 =	40 =	2 =	1	
4,800 =	200 =	10 =	5 =	1

It is now becoming customary to consider 500 sheets as a ream, and to discard the higher denominations.

REDUCTION OF COMPOUND NUMBERS.

43. Reduction of compound numbers is the process of changing their denomination without changing their value. Reduction is divided into two cases. In one case, we reduce the number to units lower than the highest named in the number; in the other case, we reduce units of a low denomination to a higher one. The first case is called *reduction descending*, and the second, *reduction ascending*.

REDUCTION DESCENDING.

44. Reduction descending, or reducing units of a higher to those of a lower denomination, is performed by multiplication. Thus, to reduce 4 miles (4 mi.) to rods, yards, feet, and inches, successively, it is known that there are 320 rd. in 1 mi. (see Table I); hence, in 4 mi., there are evidently $4 \times 320 = 1,280$ rd. Again, since there are $5\frac{1}{2}$ yd. in 1 rd., in 1,280 rd. there are $1,280 \times 5\frac{1}{2} = 7,040$ yd. In 1 yd. there are 3 ft.; hence, in 7,040 yd., there are $7,040 \times 3$

= 21,120 ft. And, since in 1 ft. there are 12 in., in 21,120 ft., there are $21,120 \times 12 = 253,440$ in. All examples in reduction descending are performed in the same manner.

45. If more than one unit is given, reduce the highest unit to the next lower denomination mentioned, and then add the units of this next lower denomination to the result previously obtained. So proceed until the lowest denomination is reached. An example, which will serve as a model for the student, is here given:

EXAMPLE.—Reduce 5 mi. 47 rd. 3 yd. $1\frac{1}{2}$ ft. to feet.

SOLUTION.—

mi.	rd.	yd.	ft.
5	47	3	$1\frac{1}{2}$
<hr/>			
320			
<hr/>			
1600	rd.		
<hr/>			
47			
<hr/>			
1647	rd.		
<hr/>			
5	$\frac{1}{2}$		
<hr/>			
828	$\frac{1}{2}$		
<hr/>			
8285			
<hr/>			
9058	$\frac{1}{2}$ yd.		
<hr/>			
8			
<hr/>			
9061	$\frac{1}{2}$ yd.		
<hr/>			
8			
<hr/>			
1	$\frac{1}{2}$		
<hr/>			
27188			

9,061½ by 3, and adding the 1½ to the product, will give the required result, or 27,186 ft. Ans.

46. Rule.—*Multiply the number of units of the highest denomination in the given compound number by the number of units of the lower denomination required to make one unit of the higher denomination, and to the product add the given number of units of the lower denomination. Proceed in this manner until the given compound number is reduced to the required denomination.*

47. In order to avoid mistakes, if any denomination be omitted between the highest and lowest denominations of the given number, represent it by a cipher.

EXAMPLE.—Reduce 40 A. 21 sq. yd. 6 sq. ft. 93 sq. in. to square inches.

SOLUTION.—

A.	sq. rd.	sq. yd.	sq. ft.	sq. in.
40	0	21	6	93
160				
6400	sq. rd.			
30½				
1600				
192000				
193600	sq. yd.			
21				
193621	sq. yd.			
9				
1742589	sq. ft.			
6				
1742595	sq. ft.			
144				
6970380				
6970380				
1742595				
250933680	sq. in.			
93				
250933773	sq. in.			Ans.

The absence of any square rods is denoted by the cipher, as shown.

EXAMPLES FOR PRACTICE.

48. Reduce:

(a) 7 mi. 4 rd. 2 yd. 2 ft. to feet.	(a) 87,094 ft.
(b) 4 bu. 2 pk. 2 qt. to quarts	(b) 146 qt.
(c) 8 lb. 4 oz. 6 pwt. 12 gr. to grains.	(c) 48,156 gr.
(d) 52 hhd. 24 gal. 1 pt. to pints.	(d) 26,401 pt.
(e) $5 \odot 16^{\circ} 20' 46''$ to seconds.	(e) 6,538,846".
(f) 14 bu. to quarts.	(f) 448 qt.
(g) 11 T. 9 cwt. 57 lb. to pounds.	(g) 22,957 lb.
(h) £9 13 s. 10 d. to pence.	(h) 2,328 d.
(i) 16 cd. 112 cu. ft. to cubic feet.	(i) 2,160 cu. ft.
(j) 97 sq. rd. to square feet	(j) 26,408½ sq. ft.
(k) 4 yr. 8 mo. 1 wk. 3 da. to days.	(k) 1,578 da.
(Count 12 mo. to the year, and 4 wk. to the month)	
(l) 19 perches to cubic feet.	(l) 470½ cu. ft.
(m) 21 cu. yd. to cubic feet.	(m) 567 cu. ft.
(n) Cong. 2 O. 6 f 3 10 f 3 5 to fluidrachms.	(n) f 3 2,901.
(o) 13 long tons to pounds.	(o) 29,120 lb.
(p) Ml 3 9 3 6 3 to grains.	(p) 10,480 gr.
(q) Find the actual number of days in example (k).	(q) 1,711 da.
(Count 1 leap year and 30 days to the month.)	

Ans.

REDUCTION ASCENDING.

49. Reduction ascending, or reducing units of a lower to those of a higher denomination, is performed by division. Thus, to reduce 253,440 in. to feet, we must evidently divide by 12, since there are 12 in. in 1 ft. That is, $253,440 \text{ in.} \div 12 = 21,120 \text{ ft.}$ To reduce this last number to yards, we divide by 3; or, $21,120 \text{ ft.} \div 3 = 7,040 \text{ yd.}$ Continuing the process, $7,040 \text{ yd.} \div 5\frac{1}{2} = 1,280 \text{ rd.}$ $1,280 \div 320 = 4 \text{ mi.}$ It will be noticed that this is the reverse of the reduction of 4 mi. to inches, as in Art 44.

50. If, in reducing a number, as in the last article, to a higher denomination there is a remainder, reserve it and divide the quotient obtained by the division by the number of units required to make one unit of the next higher denomination. Or, carry the division farther by annexing ciphers to the remainder, thus obtaining a decimal part of the next higher unit.

EXAMPLE.—Reduce 27,186 feet to miles, rods, etc.

SOLUTION.—Instead of giving all the work of division, we shall use the short method of division to express the various steps. This will save space and make the work plainer.

$$\begin{array}{r}
 3 \overline{) 27186} \text{ ft.} \\
 5 \frac{1}{2} \overline{) 9062} \text{ yd.} \\
 320 \overline{) 1647} \text{ rd.} + 3 \frac{1}{2} \text{ yd.} \\
 \quad \quad 5 \text{ mi.} + 47 \text{ rd.}
 \end{array}
 \qquad
 3 \frac{1}{2} \text{ yd.} = 3 \text{ yd. } 1 \frac{1}{2} \text{ ft.}$$

Hence, 27,186 ft. = 5 mi. 47 rd. 3 yd. $1 \frac{1}{2}$ ft. Ans. (See Art. 45.)

EXPLANATION.—Since there are 3 ft. in 1 yd., divide 27,186 by 3 and obtain 9,062 yd. Dividing this by $5 \frac{1}{2}$ to reduce yards to rods, the result is, 1,647 rd. and $3 \frac{1}{2}$ yd. remaining. Write as shown. Dividing 1,647 rd. by 320, the number of rods in a mile, the result is 5 mi. and 47 rd. remaining. Hence, 27,186 ft. = 5 mi. 47 rd. $3 \frac{1}{2}$ yd. But $\frac{1}{2}$ yd. = $\frac{1}{2} \times 3 = 1 \frac{1}{2}$ ft.; consequently, instead of the above, 5 mi. 47 rd. 3 yd. $1 \frac{1}{2}$ ft. may be written.

51. When dividing a whole number by a mixed number, as in the above example, where 9,062 was divided by $5 \frac{1}{2}$, a simple method is the following: *Multiply both dividend and divisor by the denominator of the fraction, and then divide the new dividend by the new divisor.* This will make the divisor a whole number and avoid decimals. In the above case, the denominator of the fraction was 2; $5 \frac{1}{2} \times 2 = 11$, and $9,062 \times 2 = 18,124$. Then, $18,124 \div 11 = 1,647 + 7$ remainder. This remainder, however, is 2 times too large; since, when the dividend was multiplied by 2, any remainder that might occur in the division was also multiplied by 2. Therefore, the true remainder is $7 \div 2 = 3 \frac{1}{2}$, as may be proved by dividing 9,062 by 5.5. That this method of division is correct may be easily shown. Thus, $9,062 \div 5 \frac{1}{2} = \frac{9,062}{5 \frac{1}{2}}$,

which corresponds exactly to a fraction having a mixed number for a denominator. Since multiplying both numerator and denominator by the same number does not alter the *value* of the fraction, it follows that the method is correct.

52. Instead of retaining the lower units, the preceding number may be reduced to miles and decimals of a mile, as in the following example:

EXAMPLE.—Reduce 27,186 feet to miles and decimals of a mile.

SOLUTION.—The process is essentially the same as the preceding.

$$\begin{array}{r}
 3 \overline{) 27186 \text{ ft.}} \\
 5 \frac{1}{2} \overline{) 9062} \\
 11 \overline{) 18124} \\
 320 \overline{) 1647686 \frac{2}{3}} \\
 \hline
 5.14886 \frac{4}{11} \text{ mi. Ans.}
 \end{array}$$

EXPLANATION.—The work should be evident from what has preceded. Before dividing by $5\frac{1}{2}$ we multiply both dividend and divisor by 2. (See Art. 51.) When obtaining a decimal it is not necessary to remember that the remainder 7 is $\frac{1}{2}$ yd. We simply annex a cipher to the 7 and continue the division. This is evidently correct; for $\frac{7}{5\frac{1}{2}}$ is the same as $\frac{7}{11}$. The quotient contains the repeating decimal $\dot{6}3$; and, when dividing by 320, we bring down first a 6 and next a 3, etc. instead of ciphers.

53. Rule.—Divide the number of units of the denomination given by the number of units of that denomination that are required to make one unit of the next higher denomination; the remainder (if any) will be of the same denomination.



SOLUTION.—

$$144 \overline{) 250933773} \text{ sq. in.}$$

$$9 \overline{) 1742595} \text{ sq. ft.} + 93 \text{ sq. in.}$$

$$30\frac{1}{4} \overline{) 193621} \text{ sq. yd.} + 6 \text{ sq. ft.}$$

$$121 \overline{) 774484}$$

$$160 \overline{) 6400} \text{ sq. rd.} + \frac{84}{4} \text{ sq. yd.} = 21 \text{ sq. yd.}$$

$$40 \text{ A. } 21 \text{ sq. yd. } 6 \text{ sq. ft. } 93 \text{ sq. in.} \quad \text{Ans.}$$

EXPLANATION.—Multiplying the third divisor and the third dividend by 4 to get rid of the fraction, the remainder, after division, is 84; that is, it is $\frac{84}{4}$ sq. yd. = 21 sq. yd. The remainder of the work should be clear from the preceding explanations.

EXAMPLE.—Reduce f 32,901 to gallons.

SOLUTION.—Understanding by the wording of the example that gallons and decimals of a gallon are required, the solution is as follows:

$$8 \overline{) f 32901}$$

$$16 \overline{) f 3362.625}$$

$$8 \overline{) O. 226640625}$$

$$\text{Cong. } 2.8330078125, \text{ say Cong. } 2.833. \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE.

55. Reduce all answers to the “Examples for Practice,” in Art. 48, to units of higher denomination. Also solve the following, reducing as required, and expressing the result in decimals, as in Arts. 52 and 54 (second example).

Reduce the following:

- (a) 875 pt. to gallons.
- (b) 10,000 pt. to bushels.
- (c) 147,368 cu. in. to cubic yards.
- (d) 10,000 gr. to Troy pounds.
- (e) 8,375 d. to pounds.
- (f) 28,140 sq. yd. to acres.
- (g) 49,175 in. to miles.
- (h) 380,421" to degrees.

$$\text{Ans.} \left\{ \begin{array}{ll} (a) & 109.375 \text{ gal.} \\ (b) & 156.25 \text{ bu.} \\ (c) & 3.1586+ \text{ cu. yd.} \\ (d) & 1.7361 \text{ lb.} \\ (e) & £34.8958+. \\ (f) & 5.814+ \text{ A.} \\ (g) & 0.776+ \text{ mi.} \\ (h) & 105.6725^\circ. \end{array} \right.$$

56. In solving examples (a) to (h), Art. 55, the table of equivalents, which accompanies every table of measures,

will be found very useful. Thus, in solving (*g*), for instance, it will be seen, by referring to Table I (lower half), that 1 mi. contains 63,360 in. Hence, (*g*) may be solved by simply dividing 49,175 by 63,360. Similarly, to solve (*f*) we find, by Table III, that 1 A. contains 4,840 sq. yd.; hence, divide 28,140 sq. yd. by 4,840. The student should practise both methods.

ADDITION OF COMPOUND NUMBERS.

57. Addition of compound numbers is similar in every respect to addition of whole numbers or of decimals, so far as the principles involved are concerned. The points of difference arise from the use of a varying scale of notation instead of a uniform scale of 10. An example will serve to show the process.

EXAMPLE.—Find the sum of 4 T. 3 cwt. 46 lb. 13 oz.; 8 cwt. 12 lb. 13 oz.; 2 T. 12 cwt. 50 lb. 13 oz., 1 T. 27 lb. 4 oz.

SOLUTION.—	T.	cwt.	lb.	oz.
	4	3	46	12
		8	12	13
	2	12	50	13
	1		27	4
	7	23	135	42
or	8 T.	4 cwt.	37 lb.	10 oz. Ans.

EXPLANATION.—Write the units of the same denomination in the same column, as shown, with the abbreviation of the name of the unit at the head of each column, and decreasing in order from left to right. Beginning with the right-hand column, the sum of the numbers in that column is 42, i. e., 42 oz. The sum of the numbers in the second column is 135 lb.; in the third column, 23 cwt.; in the fourth column, 7 T. Now, since 42 oz. are more than 1 lb., reduce the 42 oz. to pounds and ounces, obtaining 2 lb. 10 oz. Reserve the 10 oz., add the 2 lb. to the 135 lb. in the second column, obtaining 137 lb.; this reduced to hundredweight and pounds gives 1 cwt. 37 lb. The 1 cwt. added to the 23 cwt. in the next column gives 24 cwt., or 1 T. 4 cwt. Finally, 7 T. + 1 T. = 8 T. Hence, the sum is 8 T. 4 cwt. 37 lb. 10 oz.

58. That the process just described is the same in all respects as the addition of simple numbers will be manifest after a little consideration. Thus, suppose that we represent *thousands* by *thds.*, *hundreds* by *h.*, *tens* by *t.*, and *units* by *u.*; then the addition of 2,046, 812, 2,151, and 1,707 might be performed as follows:

thds.	h.	t.	u.
2		4	6
	8	1	2
2	1	5	1
1	7		7
<hr/>			
5	16	10	16
or 6 thds.	7 h.	1 t.	6 u., which equals 6,716.

It is easily seen that the same principles govern both cases.

59. Instead of writing the sum of each column separately and then reducing, as in the preceding example (Art. 57), it is customary to add the right-hand column, and if the sum contains more units than are required to make one unit of the next higher denomination, to reduce it to the next higher denomination, placing the remainder (if any) under the right-hand column and carrying the quotient to the next column. The student will recognize this as corresponding exactly to the ordinary process of addition.

EXAMPLE.—What is the sum of 2 rd. 3 yd. 2 ft. 5 in.; 6 rd. 1 ft. 10 in.; 17 rd. 1 yd. 11 in.; 1 rd. 4 yd. 1 ft.?

SOLUTION.—	rd.	yd.	ft.	in.
	2	3	2	5
	6		1	10
	17	1		11
	1	4	1	
<hr/>				
	27	4½	0	2
or	27 rd.	4 yd.	1 ft.	8 in. Ans.

EXPLANATION.—The sum of the units in the first column is 26 in., or 2 ft. 2 in. Writing the 2 in. under the right-hand column, and carrying the 2 ft. to the next column, the result is 2 (carried) + 1 + 1 + 2 = 6 ft. = 2 yd. 0 ft. Carrying the 2 yd. to the third column, the sum is 10 yd. = 1 rd. 4½ yd. Carrying the 1 rd. to the fourth column, the sum is 27 rd.

Now, to avoid fractions, the $\frac{1}{2}$ yd. is reduced to feet and inches, giving 1 ft. 6 in., which added to the 0 ft. and 2 in. gives for the answer 27 rd. 4 yd. 1 ft. 8 in.

60. Rule.—Place the numbers in vertical columns so that like denominations are under each other. Begin at the right-hand column, and add. If the sum contains more units than are necessary to make one unit of the next higher denomination, reduce the sum to the next higher denomination, placing the remainder under the column added, and carrying the unit (or units) of the next higher denomination so obtained to the second column. Continue in this manner until the required denomination is reached. Should fractions of a unit be obtained, or should they occur in the numbers to be added, reduce them to units of lower denomination and add them to the sum first obtained.

EXAMPLES FOR PRACTICE.

61. Find the sum of the following:

(a) 25 lb. 7 oz. 15 pwt. 23 gr.; 17 lb. 16 pwt.; 15 lb. 4 oz. 12 pwt.; 18 lb. 16 gr., 10 lb. 2 oz. 11 pwt. 16 gr.

(b) 9 mi. 13 rd. 4 yd. 2 ft.; 16 rd. 5 yd 1 ft. 5 in.; 16 mi. 2 rd 3 in.; 14 rd. 1 yd. 9 in.

(c) £16 5 s. 4 d.; £12 8 s. 9 d., £13 14 s. 8 d., £42 7 d.; 18 s. 6 d.

(d) 18 cwt. 46 lb. 12 oz.; 12 cwt 9½ lb.; 2½ cwt 21½ lb.

(e) 4 bu. 3 pk. 6 qt. 1 pt.; 10 bu. 2 pk. 7 qt. 1 pt.; 11 bu. 3 pk. 1 qt. 1 pt.; 9 bu. 2 pk. 5 qt. 1 pt.

(f) 10 yr 8 mo. 5 wk. 3 da.; 43 yr. 6 mo. 7 da.; 7 yr 5 mo. 18 wk. 4 da.; 17 yr. 17 da.

(g) 14 sq. yd. 8 sq. ft. 19 sq. in.; 105 sq. yd. 16 sq. ft. 240 sq. in.; 42 sq. yd. 28 sq. ft. 165 sq. in.

(h) 16 gal. 3 qt. 1 pt., 45 gal. 2 qt.; 17 gal. 1 qt. 1 pt.; 4 gal. 3 qt.; 15 gal. 1 pt.; 24 gal. 3 qt. 1 pt.

(i) 16 hr. 48 min. 48 sec.; 3 hr. 12 min. 40 sec.; 1 hr. 40 min. 18 sec.; 5 hr. 19 sec.

Ans.

(a) 86 lb. 3 oz. 16 pwt. 7 gr

(b) 25 mi 47 rd 1 ft 5 in

(c) £35 7 s. 10 d.

(d) 1 T. 8 cwt. 2 lb. 14 oz.

(e) 37 bu 5 qt.

(f) 78 yr. 1 mo 8 wk 3 da.

(g) 5 sq. rd. 15 sq. yd. 7 sq. ft. 100 sq. in.

(h) 124 gal 2 qt.

(i) 26 hr. 48 min.

SUBTRACTION OF COMPOUND NUMBERS.

62. The operation of subtraction is the same in principle for compound numbers as for simple numbers, the difference being due wholly to the varying scales.

EXAMPLE.—From 21 rd. 2 yd. 2 ft. $6\frac{1}{4}$ in. take 9 rd. 4 yd. $10\frac{1}{4}$ in.

SOLUTION.—	rd.	yd.	ft.	in.
	21	2	2	$6\frac{1}{4}$
	9	4		$10\frac{1}{4}$
	<hr/>			
	11	$3\frac{1}{2}$	1	$8\frac{1}{4}$

or 11 rd. 3 yd. 2 ft. $14\frac{1}{4}$ in. = 11 rd. 4 yd. $2\frac{1}{4}$ in. Ans.

EXPLANATION.—Writing the numbers as for addition, with units of like denomination under each other, we begin with the right-hand column and subtract. Since $10\frac{1}{4}$ inches cannot be subtracted from $6\frac{1}{4}$ inches, 1 foot (= 12 inches) is borrowed from the column of feet, reduced to inches, and added to the inches in the minuend, giving $18\frac{1}{4}$ inches. Then, $18\frac{1}{4}$ in. — $10\frac{1}{4}$ in. = $8\frac{1}{4}$ in. Since 1 foot was borrowed from the 2 feet, only 1 foot remains, and, as there are no feet in the subtrahend, the 1 foot is brought down in the remainder. As 4 yards cannot be taken from 2 yards, 1 rod is borrowed from the rod column, reduced to yards, and added to the 2 yards, giving $7\frac{1}{2}$ yards; then, $7\frac{1}{2}$ yd. — 4 yd. = $3\frac{1}{2}$ yd. Finally, 20 rd. — 9 rd. = 11 rd., and the remainder is 11 rd. $3\frac{1}{2}$ yd. 1 ft. $8\frac{1}{4}$ in. Reducing the $\frac{1}{2}$ yd., the remainder becomes 11 rd. 3 yd. 2 ft. $14\frac{1}{4}$ in.; or, since $14\frac{1}{4}$ in. = 1 ft. $2\frac{1}{4}$ in., and 2 ft. + 1 ft. = 3 ft. = 1 yd., the remainder is 11 rd. 4 yd. $2\frac{1}{4}$ in.

63. Rule.—*Place the less quantity under the greater quantity, with units of like denomination under each other. Beginning at the right, subtract successively the units in the subtrahend in each denomination from those above, and place the several remainders beneath, as in simple subtraction. If the units of any denomination in the minuend are fewer than units of the same denomination in the subtrahend, borrow one unit from the next higher denomination in the minuend, reduce it to the next lower denomination, and add it to the proper number in the minuend; then subtract as before. Reduce fractional results in the remainder if there are any.*

64. EXAMPLE.—From lb 31 take lb 35 32 16 gr.

SOLUTION.	lb	3	3	3	gr.
	4	0	0	1	0
	1	5	0	2	16
	2	6	7	1	4 Ans.

EXPLANATION.—Since 16 grains cannot be taken from 0 grain, 31 is borrowed from the second column. 31 = 20 gr., and 20 gr. — 16 gr. = 4 gr. As 32 cannot be taken from 30, and there are no drams or ounces, lb1 is borrowed from the fifth column. lb1 = 311 37 33; hence, subtracting 32 from 33, 30 from 37, 35 from 311, and lb1 from lb3, the remainder, or answer, is lb2 36 37 31 4 gr.

65. The only case of subtraction that need cause any trouble is the finding of the difference between two dates. If great exactness is not required, put down the year, the number of the month, and the day of the month in the case of both minuend and subtrahend. Then, counting 30 days to the month, subtract as above.

EXAMPLE How many years, months, and days between September 28, 1868, and June 15, 1891?

SOLUTION.—	yr.	mo.	da.
	1891	6	15
	1868	9	28
	22	8	17 Ans.

EXPLANATION.—June of the minuend is the 6th month, and September of the subtrahend is the 9th month. Writing the minuend and subtrahend as shown, we subtract as above. The result is 22 years, 8 months, 17 days Ans.

66. Had it been required to find the exact number of days between the two given dates, the above method would not give the correct result.

67. The following method is employed by most banks and by the United States government:

From Sept. 28, 1868, to Sept. 28, 1890 =	Ans. {	22 yr.
From Sept. 28, 1890, to June 15, 1891 in days is		
Sept. Oct Nov Dec. Jan Feb Mar Apr May June		
2 + 31 + 30 + 31 + 31 + 28 + 31 + 30 + 31 + 1 =		260 da.

EXPLANATION.—The exact whole number of years between the dates is first found; it equals 22 years. The remaining days are then found as shown.

68. The following table will be of great assistance in determining the actual number of days between two dates. The table gives the number of days between the same dates of any two months. Thus, to find the number of days between Mar. 12 and Sept. 12 of any year, we find opposite Mar. in the left-hand column and in the column headed Sept. the number 184, the required number of days. Had it been required to find the number of days between Mar. 12 and

TABLE XX.

	Jan.	Feb.	Mar.	Apr.	May	June	July.	Aug.	Sept.	Oct.	Nov.	Dec.
Jan.	365	31	59	90	120	151	181	212	243	273	304	334
Feb.	334	365	28	59	89	120	150	181	212	242	273	303
Mar.	306	337	365	31	61	92	122	153	184	214	245	275
Apr.	275	306	334	365	30	61	91	122	153	183	214	244
May.	245	276	304	334	365	31	61	92	123	153	184	214
June.	214	245	273	304	334	365	30	61	92	122	153	183
July.	184	215	243	274	304	335	365	31	62	92	123	153
Aug.	153	184	212	243	273	304	334	365	31	61	92	122
Sept.	122	153	181	212	242	273	303	334	365	30	61	91
Oct.	92	123	151	182	212	243	273	304	335	365	34	61
Nov.	61	92	120	151	181	212	242	273	304	334	365	30
Dec.	31	62	90	121	151	182	212	243	274	304	335	365

Sept. 25, we should have found the number of days between Mar. 12 and Sept. 12, or 184 days; then subtracting 12 from 25, the difference, 13, must be added to 184, obtaining 197 days, the number of days between Mar. 12 and Sept. 25. Had it been required to find the number of days between

Sept. 25 and Mar. 12, we should find opposite Sept., and in the column headed Mar., 181; then subtracting 12 from 25, we subtract the difference from 181, because 181 days is the number of days between Sept. 25 and Mar. 25, instead of Mar. 12, which occurs 13 days earlier. Hence, there are $181 - 13 = 168$ days between Sept. 25 and Mar. 12. Had Mar. 12 occurred in a leap year, there would have been one day more, or 169 days between the two dates, on account of Feb. 29.

The table will also be useful in those cases where a certain number of days is to be added to a given date. Thus, to find the date of 90 days after Feb. 18, we see, on referring to the table, that 89 days after Feb. 18 is May 18; hence, 90 days after Feb. 18 is May 19, or, if it is a leap year, May 18. Again, 127 days after Feb. 19 is June 26; because, referring to the table, 120 days after Feb. 19 is June 19, and $127 - 120 + 19 = 26$.

If it is desired to subtract a certain number of days from a given date, the process is simply reversed. To find, for example, the date 120 days previous to Sept. 21, we look down the column headed Sept. and find opposite May the number 123; hence, from May 21 to Sept. 21 is 123 days, and therefore from May 24 to Sept. 21 is 120 days.

EXAMPLES FOR PRACTICE.

69. Solve the following examples:

- (a) From £10 6 s. 4 d. take £8 15 s. 3 d.
- (b) From 125 lb. 8 oz. 14 pwt. 18 gr. take 96 lb. 9 oz. 10 pwt. 4 gr.
- (c) From 16 yr. 8 mo. 10 da. take 12 yr. 5 mo. 8 da.
- (d) From 126 hhd. 27 gal. take 104 hhd. 14 gal. 1 qt. 1 pt.
- (e) Find the exact number of days between Sept. 20, 1895, and Mar. 17, 1897, inclusive.
- (f) From 65 T. 14 cwt. 64 lb. 10 oz. take 16 T. 11 cwt. 14 oz.
- (g) From 148 sq. yd. 16 sq. ft. 102 sq. in. take 132 sq. yd. 136 sq. in.
- (h) Subtract 2 bu. 2 pk. 5 qt. 1 pt. from 100 bu.
- (i) Subtract 3 mi. 27 rd. 11 yd. 4 ft. 10 in. from 14 mi. 84 rd. 16 yd.
- (j) Subtract $27^{\circ} 18' 43''$ from $136^{\circ} 37' 28''$.

(*k*) Referring to Art. 35, find difference of longitude between Boston and San Francisco.

(*l*) Find difference of longitude between Albany and Calcutta.

Answers.—(*a*) £1 11s. 1d.; (*b*) 28 lb. 11 oz. 4 pwt. 14 gr.; (*c*) 4 yr. 3 mo. 2 da.; (*d*) 22 hhd. 12 gal. 2 qt. 1 pt.; (*e*) 545 da.; (*f*) 49 T. 3 cwt. 63 lb. 13 oz.; (*g*) 16 sq. yd. 15 sq. ft. 110 sq. in.; (*h*) 71 bu. 1 pk. 2 qt. 1 pt.; (*i*) 11 mi. 8 rd. 2 yd. 1 ft. 7 in.; (*j*) $99^{\circ} 17' 36''$; (*k*) $51^{\circ} 20' 42''$; (*l*) $162^{\circ} 34' 58.5''$.

MULTIPLICATION OF COMPOUND NUMBERS.

70. The multiplication of compound numbers is similar in all respects to multiplication of simple numbers. The process will be illustrated by an example.

EXAMPLE.—A merchant divided his syrup into 12 equal parts, each containing 2 gal. 2 qt. $1\frac{1}{2}$ pt.; how much did he have altogether?

SOLUTION.—He evidently had 12 times 2 gal. 2 qt. $1\frac{1}{2}$ pt., or

gal.	qt.	pt.	
2	2	$1\frac{1}{2}$	
		12	
24	24	18	= 32 gal. 1 qt. Ans.

EXPLANATION.—Multiplying the units of each denomination by 12, and reducing the units of each denomination of the products to higher denominations, the result is 32 gal. 1 qt.

71. Such examples are usually solved as follows:

gal.	qt.	pt.	
2	2	$1\frac{1}{2}$	
		12	
32	1	0	Ans.

EXPLANATION.— $1\frac{1}{2}$ pt. $\times 12 = 18$ pt. = 9 qt.; reserve this, and add to the quarts product. 2 qt. $\times 12 + 9$ qt. = 33 qt. = 8 gal. 1 qt. Write the 1 qt. and reserve the 8 gal. 2 gal. $\times 12 + 8$ gal. = 32 gal. Hence, the answer is 32 gal. 1 qt.

72. When the multiplier contains a fraction, it is usually easier to reduce the multiplicand to the *lowest* denomination before multiplying, and then reduce the product to higher denominations. In any case, the method used in Art. 70 is to be preferred to that given in Art. 71, when the multiplier contains a fraction. All three methods will be applied to an example to illustrate the point.

EXAMPLE.—Multiply 7 lb 5 oz. 13 pwt. 15 gr. by $5\frac{1}{2}$.

SOLUTION.—*First Method*—

	lb.	oz.	pwt.	gr.	
	7	5	13	15	
				$5\frac{1}{2}$	
	<hr/>				
	$38\frac{1}{2}$	$27\frac{1}{2}$	$71\frac{1}{2}$	$82\frac{1}{2}$	
or	38	38	81	$94\frac{1}{2}$	
or	41 lb.	1 oz.	4 pwt.	$22\frac{1}{2}$ gr.	Ans.
<i>Second Method</i> —					
	lb.	oz.	pwt.	gr.	
	7	5	13	15	
				$5\frac{1}{2}$	
	<hr/>				
	$40\frac{1}{2}$	$6\frac{1}{2}$	$14\frac{1}{2}$	$10\frac{1}{2}$	
or	40	12	24	$22\frac{1}{2}$	
or	41 lb.	1 oz.	4 pwt.	$22\frac{1}{2}$ gr.	Ans.

Third Method— 7 lb. 5 oz. 13 pwt. 15 gr. = 43,047 gr. (if the lowest denomination given had not been grains, we should still have reduced to grains). Hence, $43,047 \text{ gr.} \times 5.5 = 236,758.5 \text{ gr.} = 41 \text{ lb. } 1 \text{ oz. } 4 \text{ pwt. } 22\frac{1}{2} \text{ gr.}$ Ans.

73. Rule.—Multiply the units of the lowest denomination by the multiplier and reduce the product to the next higher denomination. Multiply the units of the next higher denomination by the multiplier, and add to the product the result obtained by the first operation. So continue with the remaining units. The last result, together with the various remainders, is the entire product. Should the entire product contain fractions, reduce the fractions of a unit to lower denominations, as in Art 72.

(Or, reduce the multiplicand to its lowest denomination; perform the multiplication, and then reduce the product to higher denominations.)

EXAMPLES FOR PRACTICE.

74. Solve the following:

- Multiply £17 10s. 8d. by 7; by 9; by 15.
- How many cords of wood in 12 loads, each load containing 2 cd. 108 cu. ft.?
- Find the weight of 2 dozen silver spoons, each spoon weighing 1 oz. 13 pwt. What would they cost at 6 cents per pennyweight?
- If 15 men perform a certain piece of work in 3 da. 16 hr. 52 min., how long would it take one man to perform the work?

- (e) Multiply 3 T. 15 cwt. 90 lb. by 5.
 (f) Multiply 4 hhd. 3 gal. 1 qt. 1 pt. by 12.
 (g) At \$2.16 per gallon what would be the cost of Cong. 2 O. 6 f 3 10 of a certain drug?
 (h) What would be the cost of 5 bu. 3 pk. 6 qt. of potatoes at 48 cents per bushel?
 (i) Multiply 6 A. 114 sq. rd. 19 sq. yd. 53 sq. ft. by 13.

$$\text{Ans. } \left\{ \begin{array}{l} (a) \left\{ \begin{array}{l} £122 \text{ 14 s. 8 d.} \\ £157 \text{ 16 s.} \\ £263. \end{array} \right. \\ (b) \quad 34 \text{ cd. 16 cu. ft.} \\ (c) \left\{ \begin{array}{l} 3 \text{ lb. 3 oz. 12 pwt.} \\ \$47.52. \end{array} \right. \\ (d) \quad 55 \text{ da. 13 hr.} \\ (e) \quad 18 \text{ T. 19 cwt. 50 lb.} \\ (f) \quad 48 \text{ hhd. 1 bbl. 9 gal.} \\ (g) \quad \$6.11. \\ (h) \quad \$2.85. \\ (i) \quad 87 \text{ A. 52 sq. rd. 21 sq. yd. } \frac{1}{2} \text{ sq. ft.} \end{array} \right.$$

DIVISION OF COMPOUND NUMBERS.

75. There are two cases of division of compound numbers. In the first case, the divisor is an abstract number; in the second case, the divisor is itself a compound number. When the divisor is an abstract number, the division may be conveniently performed as in the following examples, Arts. 76–78, inclusive.

76. EXAMPLE.—Divide 48 lb. 11 oz. 6 pwt. by 8.

$$\begin{array}{r} \text{SOLUTION.—} \quad \text{lb.} \quad \text{oz.} \quad \text{pwt.} \quad \text{gr.} \\ 8 \overline{) 48 \quad 11 \quad 6 \quad 0} \\ \underline{6 \text{ lb.} \quad 1 \text{ oz.} \quad 8 \text{ pwt.} \quad 6 \text{ gr.}} \quad \text{Ans.} \end{array}$$

EXPLANATION.—After placing the quantities as above, proceed as follows: 8 is contained in 48 six times without a remainder. 8 is contained in 11 oz. once with 3 oz. remaining. $3 \times 20 = 60$; $60 + 6 = 66$ pwt.; $66 \text{ pwt.} \div 8 = 8 \text{ pwt.}$ and 2 pwt. remaining; $2 \times 24 \text{ gr.} = 48 \text{ gr.}$; $48 \text{ gr.} \div 8 = 6 \text{ gr.}$ Therefore, the entire quotient is 6 lb. 1 oz. 8 pwt. 6 gr. **Ans.**

EXAMPLE.—A silversmith melted up 2 lb. 8 oz. 10 pwt. of silver, which he made into 6 soup ladles; what was the weight of each?

SOLUTION.—

	lb.	oz.	pwt.	
6)	2	8	10	
		5 oz.	8 pwt. 8 gr.	Ans.

EXPLANATION.—Since we cannot divide 2 lb. by 6, we reduce them to ounces. 2 lb. = 24 oz., and 24 oz. + 8 oz. = 32 oz.; 32 oz. ÷ 6 = 5 oz. and 2 oz. over. 2 oz. = 40 pwt. 40 pwt. + 10 pwt. = 50 pwt., and 50 pwt. ÷ 6 = 8 pwt. and 2 pwt. over. 2 pwt. = 48 gr., and 48 gr. ÷ 6 = 8 gr. Hence, each ladle weighs 5 oz. 8 pwt. 8 gr. Ans.

77. EXAMPLE.—Divide 820 rd. 4 yd. 2 ft. by 112.

SOLUTION.—

	rd.	
112)	820	(7 rd.
	36 rd.	
	5 ½	
	198 yd.	
	4	
112)	202	yd. (1 yd.
	90 yd.	
	3	
	270 ft.	
	2	
112)	272	ft. (2 ft.
	48 ft.	
	12	
112)	576	in. (5 ½ = 5 ¼ in.
	16 in.	

EXPLANATION.—We divide as in long division, using the short method. The first quotient is 7 rd. with a remainder of 36 rd., which = 198 yd. 198 yd. + 4 yd. = 202 yd.; 202 yd. ÷ 112 = 1 yd. with a remainder of 90 yd., which = 270 ft. 270 ft. + 2 ft. = 272 ft.; 272 ft. ÷ 112 = 2 ft. with a remainder of 48 ft., which = 576 in. 576 in. ÷ 112 = 5 ¼ in.

If desired, ¼ in. may be reduced to a decimal in the manner already explained, = .1428 + in. The common fractional form is, however, better than the decimal form, since if the quotient be multiplied by the divisor the result will then be *exactly* the same as the dividend.

78. Rule.—*Find how many times the divisor is contained in the first or highest denomination of the dividend. Reduce the remainder (if any) to the next lower denomination, and add to it the number in the dividend expressing that denomination. Divide this new dividend by the divisor. The quotient will be the next denomination in the quotient required. Continue in this manner until the lowest denomination is reached. The successive quotients will constitute the entire quotient.*

EXAMPLES FOR PRACTICE.

79. Divide:

- (a) 376 mi. 276 rd. by 22; (b) 1,137 bu. 3 pk. 4 qt. 1 pt. by 10;
 (c) 84 cwt. 48 lb. 49 oz. by 16; (d) 78 sq. yd. 18 sq. ft. 41 sq. in. by 18;
 (e) 148 mi. 64 rd. 24 yd. by 12; (f) 100 T. 16 cwt. 18 lb. 11 oz. by 15;
 (g) 36 lb. 18 oz. 18 pwt. 14 gr. by 8; (h) 112 mi. 48 rd. by 100.

Ans. { (a) 17 mi. 41 rd. 3 yd. 1 ft. 6 in.
 (b) 113 bu. 3 pk. 1 qt. $\frac{1}{2}$ pt.
 (c) 5 cwt. 28 lb. $3\frac{1}{8}$ oz.
 (d) 4 sq. yd. 4 sq. ft. $2\frac{5}{8}$ sq. in.
 (e) 12 mi. 112 rd. 2 yd.
 (f) 6 T. 14 cwt. 41 lb. $3\frac{1}{2}$ oz.
 (g) 4 lb. 8 oz. 7 pwt. $7\frac{3}{4}$ gr.
 (h) 1 mi. 38 rd. 4 yd. 2 ft. 6.24 in.

80. When the divisor is a compound number, the easiest and best way to perform the division is to reduce both numbers to the lowest denomination given in either the dividend or divisor; then divide as in whole numbers.

81. EXAMPLE.—How many bottles, each holding 1 qt. $\frac{1}{2}$ pt., can be filled from a cask holding 21 gal. 3 qt.?

SOLUTION.—In 1 qt. $\frac{1}{2}$ pt. there are $2\frac{1}{2}$ pt.; in 21 gal. 3 qt. there are 174 pt. Then, $174 \text{ pt.} \div 2\frac{1}{2} \text{ pt.} = 69 \text{ bottles and } 1\frac{1}{2} \text{ pt. left over.}$ Ans.

LONGITUDE AND TIME—(Continued.)

82. We are now prepared to find the difference of time, or the difference of longitude, between two places. For this purpose we use the rules given in Arts. 33 and 34.

EXAMPLE.—What is the difference of time between New York and Chicago?

SOLUTION.—In Art. 33 the longitude of New York is stated to be $73^{\circ} 58' 24''$ W., and of Chicago $87^{\circ} 38' 40\frac{1}{2}''$ W. Applying the rule given in Art. 33,

$$\begin{array}{r} 87^{\circ} \quad 38' \quad 40\frac{1}{2}'' \text{ W.} \\ 73^{\circ} \quad 58' \quad 24'' \text{ W.} \\ \hline 13^{\circ} \quad 38' \quad 16\frac{1}{2}'' \text{ W.} \end{array}$$

Dividing $13^{\circ} 38' 16\frac{1}{2}''$ by 15 the result is

$$\begin{array}{r} 15 \overline{) 13^{\circ} \quad 38' \quad 16\frac{1}{2}''} \\ \hline \quad 54 \text{ min. } 33\frac{1}{2} \text{ sec.} \end{array}$$

EXAMPLE.—Find the difference of time between Albany and Rome.

SOLUTION.—Applying rule, Art. 33, difference of longitude equals

$$\begin{array}{r} 73^{\circ} \quad 44' \quad 48'' \text{ W.} \\ 12^{\circ} \quad 38' \quad 40\frac{1}{2}'' \text{ E.} \\ \hline 86^{\circ} \quad 13' \quad 28\frac{1}{2}'' \end{array} \quad \begin{array}{r} 15 \overline{) 86^{\circ} \quad 13' \quad 28\frac{1}{2}''} \\ \hline \quad 5 \text{ hr. } \quad 44 \text{ min. } 53\frac{1}{2} \text{ sec.} \end{array} \quad \text{Ans.}$$

We must evidently *add* to find the difference, since Rome is *east* of Greenwich, or the prime meridian, and Albany is *west*. Were both places on the *same* side, both east or both west, we should subtract.

33. The longitude of a place is determined by means of a very accurate watch, called a *chronometer*, and by observation of the sun or stars. The watch is set for Greenwich time, and an observation of the sun is taken for the place whose longitude it is desired to find. By aid of suitable instruments it can be determined when it is exactly noon at any place, and by looking at the watch, the difference of time between noon and the time indicated by the watch will be the difference of time between the place at which the observation is taken and Greenwich. If the watch appears to be *slow*, the longitude is *east*; if *fast*, it is *west*. Knowing the difference of time, the longitude is easily found by the rule given in Art. 34.

34. EXAMPLE. A watch set to Greenwich time appeared to be 6 hr 10 min. 41 sec. fast at a certain place. What was the longitude of the place?

SOLUTION.—Applying rule, Art. 34,

$$\begin{array}{r} 6 \text{ hr. } \quad 10 \text{ min. } \quad 41 \text{ sec.} \\ \quad \quad \quad 15 \\ \hline 92^{\circ} \quad 40' \quad 15'' \end{array}$$

Since the watch appeared fast, the longitude was $92^{\circ} 40' 15''$ W.

EXAMPLES FOR PRACTICE.

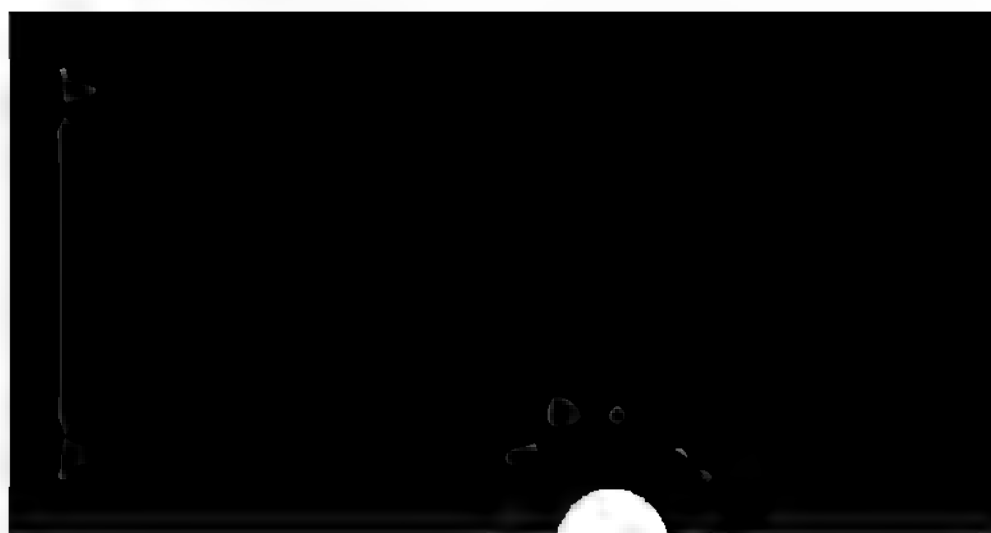
85. Find the difference of time between:

- | | |
|---------------------------------------|--|
| (a) London and City of Mexico. | Ans. (a) 6 hr. 36 min. $4\frac{1}{2}$ sec. |
| (b) Paris and Philadelphia. | Ans. (b) 5 hr. 9 min. $59\frac{1}{2}$ sec. |
| (c) Richmond, Va., and San Francisco. | Ans. (c) 2 hr. 59 min. $54\frac{1}{10}$ sec. |
| (d) Boston and Ann Arbor. | Ans. (d) 50 min. $39\frac{1}{2}$ sec. |
| (e) London and Calcutta. | Ans. (e) 5 hr. 55 min. $43\frac{1}{2}$ sec. |

Find the longitude when the watch is apparently

- | | | |
|--------------------------------|--------|-------------------------------|
| (f) Fast 7 hr. 43 min. 11 sec. | Ans. { | (f) $115^{\circ} 47' 45''$ W. |
| (g) Slow 1 hr. 0 min. 49 sec. | | (g) $15^{\circ} 12' 15''$ E. |
| (h) Slow 4 hr. 37 min. 6 sec. | | (h) $69^{\circ} 16' 30''$ E. |
| (i) Fast 8 hr. 19 min. 24 sec. | | (i) $124^{\circ} 51'$ W. |

The student should verify the table of times given in the table, Art. 35.



ARITHMETIC.

THE METRIC SYSTEM.

1. In the metric system, a uniform scale of 10 is used throughout, as in the ordinary scale of numbers and in United States money. The name is derived from the meter (from the Greek word *metron*, a measure), the unit from which all the other units are derived. The use of the metric system was made legal in the United States in 1866, but has not yet been made compulsory; it is used by scientists throughout the world.

2. The **meter** is very nearly one ten-millionth part of the distance from the equator to the pole, and has been officially declared by the United States Government to equal 39.37 inches, which corresponds very nearly to the distance above mentioned.

3. The metric system has three principal units, the only ones we shall consider, which are: the **meter** (pronounced meeter), the unit of length; the **liter** (pronounced leeter), the unit of capacity; and the **gram**, the unit of weight. Each of the units has its multiples and subdivisions.

4. The names of the denominations higher than the leading unit are obtained by prefixing to the name of the unit the Greek names *dek'a*, *hek'to*, *kil'o*, and *myr'ia*. Thus, taking the meter as the unit, we write

§ 5

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Dek'a me'ter, 10 meters, deka meaning 10;
 Hek'to me'ter, 100 meters, hekto meaning 100;
 Kil'o me'ter, 1,000 meters, kilo meaning 1,000;
 Myr'ia me'ter, 10,000 meters, myria meaning 10,000.

5. The names of the denominations lower than the leading unit are obtained by prefixing to the name of the unit the Latin names *dec'i*, *cent'i*, and *mil'li*. Thus,

Dec'i me'ter, $\frac{1}{10}$ meter, deci meaning $\frac{1}{10}$;
 Cent'i me'ter, $\frac{1}{100}$ meter, centi meaning $\frac{1}{100}$;
 Mil'li me'ter, $\frac{1}{1000}$ meter, milli meaning $\frac{1}{1000}$.

The prefixes and their meanings, as given in this and the preceding article, should be carefully committed to memory.

MEASURES OF EXTENSION.

MEASURES OF LENGTH.

TABLE I.

10 millimeters (<i>mm.</i>)	= 1 centimeter...	<i>cm.</i>	= .3937 in.
10 centimeters = 1 decimeter....	<i>dm.</i>	= 3.937 in.
10 decimeters = 1 meter.....	<i>m.</i>	= 3.28 ft
10 meters = 1 dekameter...	<i>Dm.</i>	= 32.8 ft.
10 dekameters = 1 hektometer...	<i>Hm.</i>	= 328.09 ft.
10 hektometers = 1 kilometer. ...	<i>Km.</i>	= .62137 mi.
10 kilometers = 1 myriameter...	<i>Mm.</i>	= 6.2137 mi.

It will be noticed that the abbreviations of the names of the units of the higher denominations begin with a *capita* ' letter, and those of the

7. The *approximate* length of the meter is 40 inches, or, more closely, $39\frac{3}{8}$ inches (39 $\frac{1}{4}$ inches is a closer approximation). When great exactness is not required, 40 inches is near enough. Hence, 1 decimeter equals $40 \times \frac{1}{16} = 4$ inches; 1 centimeter = .4 inch, and 1 millimeter = .04 inch. The length of 1 kilometer = $\frac{5}{8}$ mile, approximately.

8. Since meters, centimeters, and millimeters have a decimal scale like dollars, cents, and mills, they may be read in a similar way. Thus, 41.457 meters may be read as 41 and 457-thousandths meters, or as 41 meters 45 centimeters 7 millimeters. It might also be read 41 meters 45 and 7-tenths centimeters, or 41 meters 457 millimeters.

9. To express metric numbers decimally in terms of a given unit:

EXAMPLE. — Express 4 Km. 2 Hm. 1 m., 5 cm. and 9 mm. in meters.

SOLUTION. — We write the meters in units place, at the left of the decimal point, then, writing the other units in order, supplying the missing denominations (if any) with ciphers, we have 4301.059 meters.

Ans.

Had it been required to express the above in kilometers, the result would have been 4.301059 Km.

10. Rule. *Write the number of given units; then, the numbers of the higher denominations on the left, as integers, and those of the lower denominations on the right, as decimals, supplying any missing denomination with a cipher.*

11. To reduce a metric number to higher or lower denominations, all that is necessary is to move the decimal point. Thus, 74.1026 Km. = 741.026 Hm. = 7,410.26 Dm. = 74,102.6 m. = 741,026 dm = 7,410,260 cm = 74,102,600 mm. Also, to reduce 97,452 cm. to kilometers, write 97,452 cm. Now, beginning with the decimal point (which, of course, follows the 2), count centi, deci, meter, deka, hekto, kilo, placing the decimal point after the last-named unit, and supplying any missing ones with ciphers. The result is 0.97452 Km.

EXAMPLES FOR PRACTICE.

12. Solve the following:

- (a) Reduce 25.7 Km. to meters.
 (b) Reduce 43.4 m. to millimeters.
 (c) Reduce 4,823.6 m. to hektometers.
 (d) Reduce 48,689 cm. to meters.
 (e) Reduce 738.4 Dm. to centimeters.

Read the following:

- (f) 14.5 m.
 (g) 47.8 Dm.
 (h) 568 Hm.
 (i) 434.5 Km.
 (j) 27.4 mm.
 (k) 92.76 cm.

Answers.—(a) 25,700 m.; (b) 43,400 mm.; (c) 48.236 Hm.; (d) 486.89 m.; (e) 738,400 cm.; (f) 14 and 5-tenths meters; (g) 47 and 8-tenths dekameters; (h) 568 hektometers; (i) 434 and 5-tenths kilometers; (j) 27 and 4-tenths millimeters, (k) 92 and 76-hundredths centimeters.

SQUARE MEASURE.

TABLE II.

100 square millimeters (<i>sq. mm. or mm².</i>) =	1 square centimeter .. <i>sq. cm. or cm².</i>
100 square centimeters.... =	1 square decimeter ... <i>sq. dm. or dm².</i>
100 square decimeters..... =	{ 1 square meter <i>sq. m. or m².</i> or centare..... <i>ca.</i>
100 square meters { (<i>hectares</i>) { =	{ 1 square dekameter <i>sq. Dm. or Dm².</i> or are1

13. In square measure, the scale is 100 (10×10); hence, in reducing units of any denomination to a lower or higher denomination, the decimal point must be moved two places for each denomination. Thus, to reduce 42.09872 sq. m. to square centimeters, begin at the decimal point, point off two places to the right and say square decimeters, then two more places and say square centimeters, obtaining 420,987.2 sq. cm. Had square millimeters been desired, it would have been necessary to annex a cipher; thus, 42.09872 sq. m. = 42,098,720 sq. mm. Reduction to higher denominations is performed in the same manner, moving the decimal point to the left. Thus, 42,098,720 mm². = 0.4209872 ares.

14. The square meter is used in measuring floors, ceilings, and other ordinary surfaces; the are and hektare in measuring land, and the square kilometer in measuring states and territories.

EXAMPLES FOR PRACTICE.

15. Solve the following:

- (a) Write 78.29 ares as centares; also as hektares.
- (b) Write 9 m². as square decimeters; also as square centimeters.
- (c) In 3,246 ca. how many ares?
- (d) Express 7,041.6 sq. dm. in ares.

Answers.—(a) 7,829 ca. or .7829 Ha.; (b) 900 dm². or 90,000 cm².; (c) 32.46 A.; (d) .70416 A.

CUBIC MEASURE.

TABLE III.

1,000 cubic millimeters (cu. mm. or mm ³ .) =	
	1 cubic centimeter.... cu. cm. or cm ³ .
1,000 cubic centimeters..... =	1 cubic decimeter.... cu. dm. or dm ³ .
1,000 cubic decimeters..... =	1 cubic meter..... cu. m. or m ³ .

COMMON EQUIVALENTS.

1 cu. cm. = .06102 cu. in.

1 cu. dm. = 61.023 cu. in.

1 cu. m. = 61,023.4 cu. in. = 35.3145 cu. ft. = 1.308 cu. yd.

In measuring wood, the cubic meter is called a *stere*.

16. Units higher than the cubic meter are not used, except in denoting the volume of planets.

In cubic measure, the scale is 1,000 ($10 \times 10 \times 10$); hence, to reduce units from one denomination to another, apply the method given in Art. 13, moving the decimal point three places each time, instead of two places, as in square measure.

MEASURES OF CAPACITY.

TABLE IV.

10 mil'li li'ters (<i>ml.</i>)	=	1 cen'ti li'ter	<i>cl.</i>
10 centiliters	=	1 dec'i li'ter	<i>dl.</i>
10 deciliters	=	1 li'ter	<i>l.</i>
10 liters	=	1 dek'a li'ter	<i>Dl.</i>
10 dekaliters	=	1 hek'to li'ter	<i>Hl.</i>
10 hektoliters	=	1 kil'o li'ter	<i>Kl.</i>
10 kiloliters	=	1 myr'ia li'ter	<i>Ml.</i>

COMMON EQUIVALENTS.

1 liter	=	61.023 cu. in.
1 liter	=	1.0567 liquid quarts.
1 liter	=	0.9078 dry quarts.
1 hektoliter	=	3.53144 cu. ft.
1 hektoliter	=	26.417 gallons.
1 hektoliter	=	2.8378 bushels.

17. The **liter** is equal in volume to 1 cubic decimeter, i. e., to a cube whose edges measure 1 decimeter on a side. The liter is the principal unit in measures of capacity, and is used for both *dry* and *liquid* measure; it is very nearly equal to a liquid quart.

18. One milliliter is equal in volume to 1 cubic centimeter, since $1 \text{ cu. cm.} = \frac{1}{1000} \text{ cu. dm.} = \frac{1}{1000} \text{ liter} = 1 \text{ milliliter.}$

The centiliter is a little less than $\frac{1}{4}$ gill; it is used for measuring small quantities of liquids, as medicines. The liter is used for the same purposes as the quart, and the hektoliter for the same purposes as the gallon and the bushel.

The units are reduced from one denomination to another in the same way as measures of length. See Art. 11.

EXAMPLES FOR PRACTICE.

19. Solve the following:

- (a) Express 8.53 l. as centiliters; as deciliters.
- (b) Express 4.64 Kl. as liters; as hektoliters.
- (c) How many deciliters in 8 liters? In 9.35 liters?
- (d) How many liters in 6.358 cl.?
- (e) In 8,500 liters how many kiloliters?

Answers.—(a) 853 cl. or 85.3 dl.; (b) 4,640 l. or 46.4 Hl.; (c) 80 dl.; 93.5 dl.; (d) .06358 l.; (e) 8.5 Kl

MEASURES OF WEIGHT.

TABLE V.

10 milligrams (<i>mg.</i>)..	=	1 centigram	<i>cg.</i>
10 centigrams	=	1 decigram	<i>dg.</i>
10 decigrams	=	1 gram	<i>g.</i>
10 grams	=	1 dekagram.....	<i>Dg.</i>
10 dekagrams.....	=	1 hektogram.....	<i>Hg.</i>
10 hektograms.....	=	1 kilogram, or kilo...	<i>Kg.</i> or <i>K.</i>
10 kilograms.....	=	1 myriagram	<i>Mg.</i>
100 myriagrams.....	=	1 tonneau, or ton	<i>T.</i>

COMMON EQUIVALENTS.

1 gram	=	$\left\{ \begin{array}{l} 1 \text{ cu. cm., or} \\ 1 \text{ ml. of water.} \end{array} \right.$
1 kilogram. . .	=	$\left\{ \begin{array}{l} 1 \text{ cu. dm., or} \\ 1 \text{ liter of water.} \end{array} \right.$
1 metric ton. =	$\left\{ \begin{array}{l} 1 \text{ cu. m., or} \\ 1 \text{ kiloliter of water.} \end{array} \right.$	
1 gram	=	15.432 gr. Troy.
1 gram	=	0.03527 oz. avoirdupois.
1 kilogram ..	=	2.2046 lb. avoirdupois.
1 metric ton .	=	1.1023 tons of 2,000 lb.

20. The gram is the principal unit of weight; it is the weight of 1 cubic centimeter of pure distilled water at its temperature of maximum density, or 39.2° Fahrenheit. The gram is used in weighing gold, silver, letters (for postage), and in mixing medicines.

The kilogram is generally called the *kilo*, and is used for the same purposes as the pound avoirdupois. The tonneau

(usually called the *metric ton*) is nearly equal in weight to the long ton of 2,240 pounds.

21. Referring to Art. 38, § 4, it will be noticed that the 5-cent piece (nickel) weighs 5 grams, that the half-dollar weighs $12\frac{1}{2}$ grams, etc.

22. The units of weight most commonly used are the milligram (in chemical analysis), the gram, the kilo, and the ton. It will be noticed that $1,000 \text{ mg.} = 1 \text{ g.}$, $1,000 \text{ g.} = 1 \text{ K.}$, and $1,000 \text{ K.} = 1 \text{ ton.}$ The milligram is, approximately, equal to $\frac{1}{63\frac{1}{2}}$ of a grain; the gram, to $\frac{1}{7\frac{1}{8}}$ oz. avoirdupois; the kilo, to $2\frac{1}{4}$ pounds; and the metric ton, to $1\frac{1}{4}$ (one-half of $2\frac{1}{4}$) short tons. Unless precise equivalents are desired, the values here given are accurate enough for all practical purposes. It should be firmly kept in mind that the weight of 1 liter of water is 1 kilogram, and that 1 liter of water will fill a space of 1 cubic decimeter.

OPERATIONS WITH METRIC UNITS.

23. The rules for adding, subtracting, multiplying, and dividing metric numbers are the same as for the corresponding operations on decimals. Be sure, however, that all numbers are expressed in the same units.

EXAMPLE.—What is the sum of 45.69 Dm., 68.4 Hm., and 6,845 cm.?

SOLUTION.—Reducing all the numbers to the same unit, say meters, and adding as in decimals, the sum is 6,865.25 m.

$$\begin{array}{r} 45.69 \\ 68.40 \\ \underline{6.845} \\ 6865.25 \text{ m. Ans.} \end{array}$$

EXAMPLE.—From 5.462 kilos take 7 Hg. 4 g. 9 cg.

SOLUTION.—Expressing both numbers in kilos, and subtracting, the result is 4.75791 Kg.

$$\begin{array}{r} 5.462 \\ \underline{.70409} \\ 4.75791 \text{ Kg. Ans.} \end{array}$$

EXAMPLES FOR PRACTICE.

24. Solve the following:

- (a) What is the difference between 8.5 Kg. and 976 grams ?
 Ans. 7,524 Kg.
- (b) How much silk is contained in $12\frac{1}{2}$ pieces, if each piece contains 48.75 m ?
 Ans. 609.375 m.
- (c) If 735 kilos of flour are equally distributed among 35 persons, how many kilos will each person receive ?
 Ans. 21 Kg.
- (d) In the last example, how many pounds did each person receive, counting $2\frac{1}{2}$ pounds to a kilo ?
 Ans. 46.2 lb.
- (e) Add 74 HL, 147 2 L, 5,006.3 cl., and 6.5421 KL, expressing the result in liters.
 Ans. 14,139 363 l.
- (f) What is the weight in pounds of 197,862 cl. of water at 39.2° Fahrenheit ?
 Ans. 4,353 lb., nearly.

FORMULAS.

25. A **formula** is an abridged statement of a general rule, in which symbols are used.

The symbols used are the letters of the alphabet, which represent numbers, and the signs $+$, $-$, \times , \div , $\sqrt{}$, etc., which have the same meaning as in arithmetic.

To illustrate, let the following example be taken: If a person exchanges 10 books, worth \$3 per volume, for cloth at \$2 per yard, how many yards will he obtain? A rule for solving this example, and all others like it, may be stated as follows: Multiply the number of books by the price per volume, and divide the product by the price of the cloth. The result will be the number of yards of cloth.

A more concise way of stating the rule is by using letters. Thus,

Let A = number of books;
 B = price per volume;
 C = price of the cloth;
 D = number of yards of cloth.

Then, according to the rule,

$$\frac{\text{number of books} \times \text{price per volume}}{\text{price of cloth}} = \text{number of yards of}$$

cloth, or
$$\frac{A \times B}{C} = D.$$

26. This last expression is a formula; the letters A , B , C , and D stand for the numbers given in the particular example to which it is applied; and the sign of multiplication (\times), and the horizontal dividing line of the fraction, which indicates division, show what operations must be performed upon the numbers to produce the answer D . In the example in question, $A = 10$, the number of books; $B = 3$, the price per volume; $C = 2$, the price of the cloth. Hence, writing for A , B , and C their values, 10, 3, and 2; D , the number of yards $= \frac{10 \times 3}{2} = 15$. Ans.

In modern technical works the rules for solving examples are commonly given by formulas, and it is important to understand how to use them. Having become accustomed to them, they will be found more convenient than rules written out in words.

27. *The multiplication sign, \times , is generally omitted in formulas, multiplication being indicated by simply writing the letters or expressions together.* Thus, the formula $\frac{A \times B}{C} = D$, given above, would ordinarily be written $\frac{AB}{C} = D$. The expression $4ab$ means the same as $4 \times a \times b$. Evidently, the sign cannot be omitted between *two figures*, as addition, instead of multiplication, would be indicated. Thus, 32 means $30 + 2$, not 3×2 .

28. Formulas are usually written with the letter whose value is to be obtained standing alone at the left of the sign of equality. *To apply a formula, therefore, we have simply to substitute the given values for the letters on the right of the sign of equality and then perform the operations indicated by the signs.*

EXAMPLE.—What is the value of v , in $v = \frac{a + bc}{d}$ when $a = 5$, $b = 10$, $c = 4$, and $d = 20$?

SOLUTION.—Writing for a , b , c , and d their values,

$$v = \frac{5 + 10 \times 4}{20} = \frac{45}{20} = 2\frac{1}{4}. \text{ Ans.}$$

EXAMPLE.—What is the value of D , if $D = \frac{Pc}{6a}$, and $P = 5$, $c = 300$, and $a = 10$?

SOLUTION.—Substituting the values of the letters,

$$D = \frac{5 \times 300}{6 \times 10} = 25. \text{ Ans.}$$

EXAMPLE.—The letters having the same values as before, what does x equal in the formula $x = \frac{c}{2Pa}$?

SOLUTION.—Substituting, $x = \frac{300}{2 \times 5 \times 10} = \frac{300}{100} = 3. \text{ Ans.}$

EXAMPLE.—When $A = 10$, $B = 8$, $C = 5$, and $D = 4$, what is the value of E in the following:

$$(a) E = \frac{BCD}{A\left(2 + \frac{D}{C}\right)}? \quad (b) E = \frac{A - \frac{1}{2}D + \frac{4B}{A+C}}{A - \frac{2B}{A+22}}?$$

SOLUTION.—(a) Substituting,

$$E = \frac{8 \times 5 \times 4}{10\left(2 + \frac{4}{5}\right)}.$$

To simplify the denominator, notice that $2 + \frac{4}{5}$ is equivalent to the mixed number $2\frac{4}{5}$; hence, $10\left(2 + \frac{4}{5}\right) = 10 \times 2\frac{4}{5} = 10 \times \frac{14}{5} = \frac{140}{5}$

Therefore, $E = \frac{8 \times 5 \times 4}{\frac{140}{5}} = \frac{160 \times 5}{140} = 5\frac{5}{7}. \text{ Ans.}$

(b) Substituting the values of the letters,

$$E = \frac{10 - \frac{1}{2} \times 4 + \frac{4 \times 8}{10+5}}{10 - \frac{2 \times 8}{10+22}} = \frac{10 - 2 + \frac{32}{15}}{10 - \frac{16}{32}} = \frac{8\frac{2}{15}}{9\frac{1}{2}} = \frac{\frac{122}{15}}{\frac{19}{2}} = \frac{244}{285}. \text{ Ans.}$$

EXAMPLES FOR PRACTICE.

29. Find the numerical values of x in the following formulas, when $a = 9$, $b = 8$, $c = 2$, $d = 10$, and $e = 3$:

$$(a) x = \frac{d+ce}{bd-40}. \quad \text{Ans. } x = \frac{1}{2}.$$

$$(b) x = \frac{\frac{2}{3}(a+e)}{ce}. \quad \text{Ans. } x = 1\frac{1}{2}.$$

$$(c) x = \frac{ad}{2c} + abc. \quad \text{Ans. } x = 166.5.$$

$$(d) x = \frac{acd}{bc} + 4\frac{1}{8}. \quad \text{Ans. } x = 21.$$

INVOLUTION.

30. A **power** of a number is the result obtained by taking the number any number of times as a factor.

Thus, 4 is the *second* power of 2, since $2 \times 2 = 4$.

27 is the *third* power of 3, since $3 \times 3 \times 3 = 27$.

The *second* power is called the *square*, and the *third* power the *cube*.

The number itself is called the *first* power of the number.

31. **Involution** is the process of finding any required power of a number.

32. An **exponent** is a small figure placed to the right and a little above a number to denote how many times the number is to be used as a factor.

Thus, $4^3 = 4 \times 4 \times 4 = 64$; $2^4 = 2 \times 2 \times 2 \times 2 = 16$.

An exponent is sometimes called the index of the power.

33. The student should memorize the *squares* of numbers from 1 to 25, and their *cubes* as far at least as to 12.

34. To find any power of a number:

EXAMPLE.—Find the cube or third power of 35.

SOLUTION.— $35 \times 35 = 1,225 = 35^2$.
 $1,225 \times 35 = 42,875 = 35^3$. Ans.

EXAMPLE.—What is the fourth power of 15?

SOLUTION.— $15 \times 15 = 225 = 15^2$.
 $225 \times 15 = 3,375 = 15^3$.
 $3,375 \times 15 = 50,625 = 15^4$. Ans.

The same result can be obtained by multiplying the second power by itself

Thus, $225 \times 225 = 50,625 = 15^2 \times 15^2$.

EXAMPLE.—What is the value of 1.2^3 ?

SOLUTION.— $1.2 \times 1.2 \times 1.2 = 1.728$. Ans.

EXPLANATION. The operation is performed as in ordinary multiplication, disregarding the decimal point, and then the result is pointed off in accordance with the rule for multiplication of decimals.

$13^2 = 169$
 $14^2 = 196$
 $15^2 = 225$
 $16^2 = 256$
 $17^2 = 289$
 $18^2 = 324$
 $19^2 = 361$

$2^3 = 8$
 $3^3 = 27$
 $4^3 = 64$
 $5^3 = 125$
 $6^3 = 216$

$7^3 = 343$
 $8^3 = 512$
 $9^3 = 729$
 $10^3 = 1000$
 $11^3 = 1331$
 $12^3 = 1728$
 $13^3 = 2197$

$8^2 = 64$
 $9^2 = 81$
 $10^2 = 100$
 $11^2 = 121$
 $12^2 = 144$
 $13^2 = 169$

35. EXAMPLE.—What is the cube of $\frac{3}{8}$?

$$\text{SOLUTION. — } \left(\frac{3}{8}\right)^3 = \frac{3^3}{8^3} = \frac{3 \times 3 \times 3}{8 \times 8 \times 8} = \frac{27}{512} \quad \text{Ans.}$$

EXPLANATION.—To raise a fraction to any power, it is necessary to raise the numerator and the denominator separately to the power desired.

EXAMPLE.—Find the fourth power of $1\frac{7}{8}$.

SOLUTION.—

$$\left(1\frac{7}{8}\right)^4 = \left(\frac{15}{8}\right)^4 = \frac{15^4}{8^4} = \frac{15 \times 15 \times 15 \times 15}{8 \times 8 \times 8 \times 8} = \frac{50,625}{4,096} \quad \text{Ans.}$$

36. Rule.—I. To raise a whole number or a decimal to any power, use it as a factor as many times as there are units in the exponent.

II. To raise a fraction to any power, raise both the numerator and denominator to the power indicated by the exponent.

EXAMPLES FOR PRACTICE.

37. Raise the following to the powers indicated:

(a) 85^2 .	Ans. {	(a) 7,225.
(b) $(\frac{1}{4})^3$.		(b) $\frac{1}{64}$.
(c) 0.5^2 .		(c) 42,25.
(d) 14^4 .		(d) 38,416.
(e) $(\frac{3}{4})^2$.		(e) $\frac{9}{16}$.
(f) $(\frac{1}{2})^3$.		(f) $\frac{1}{8}$.
(g) $(\frac{1}{2})^2$.		(g) $\frac{1}{4}$.
(h) 1.4^4 .		(h) 5,37824.

EVOLUTION.

38. The process of finding the root of a number is called **evolution**, and consists of finding one of the equal factors which, when multiplied together, will give a product equal to the given number—it is the reverse of involution.

39. When the given number is resolved into *two* equal factors, one of them is called the **square root**; when resolved into *three* equal factors, one of them is called the

cube root; when resolved into *four* equal factors, one of them is called the **fourth root**; etc. For example, the number 36 is equal to the product of the two equal factors 6 and 6; hence, 6 is the square root of 36. Again, the number 343 is equal to the product of the three equal factors 7, 7, and 7; hence, 7 is the cube root of 343. Also, 32 is equal to the product of the five equal factors 2, 2, 2, 2, and 2; hence, 2 is the fifth root of 32.

40. To indicate that some root of a number is to be extracted, a sign ($\sqrt{}$), called the **radical sign**, is placed before the number; and to indicate what root is to be extracted, a small figure, called the **Index**, is placed within the sign. Thus, the square root of 36, the cube root of 243, and the fifth root of 32 would be, respectively, indicated by $\sqrt{36}$, $\sqrt[3]{243}$, and $\sqrt[5]{32}$. It is usual, however, to combine the vinculum with the radical sign, in which case the above expressions would become $\sqrt[2]{36}$, $\sqrt[3]{243}$, and $\sqrt[5]{32}$, respectively. Square root is always indicated by simply placing the radical sign before the number and omitting the index; thus, $\sqrt{36}$.

41. The roots most commonly required to be found are the square root and cube root, and, occasionally, the fifth root. The fourth root is very seldom required. Hence, we shall describe only the methods to be pursued in extracting square and cube roots.

SQUARE ROOT.

42. The greatest number that can be written with *one* figure is 9, and $9^2 = 81$; the greatest number that can be written with *two* figures is 99, and $99^2 = 9,801$; with *three* figures 999, and $999^2 = 998,001$; with four figures 9,999, and $9,999^2 = 99,980,001$; etc.

In each of the above it will be noticed that the square of the number contains just twice as many figures as the given number. Hence, in order to find the square root of a number, first find how many figures there will be in the root:

this is done by pointing off the number into *periods* of *two* figures each, *beginning with the unit figure*. The number of periods will indicate the number of figures in the root.

Thus, the square root of 83,740,801 must contain four figures, since, pointing off the periods, we get 83'74'08'01, or four periods; consequently, there must be four figures in the root. In like manner, the square root of 50,625 must contain three figures, since there are three (5'06'25) periods.

43. The square of any number wholly decimal always contains twice as many figures as the number. For example, $.1^2 = .01$; $.13^2 = .0169$; $.751^2 = .564001$; etc. Hence, in pointing off a decimal, begin at the decimal point and proceed to the right. If the last period contains but one figure, complete it by annexing a cipher; thus, .43'42'94'40.

44. Hereafter, when referring to any digit of a number, the digit occupying the highest order in the number will be called the first digit, or figure; that occupying the next highest order will be called the second digit, or figure; etc. So, also, when a number is divided into periods, the extreme left-hand period will be called the first period; the one next to it, the second period; etc.

45. The process of extracting the square root of a number is best illustrated by means of an example.

EXAMPLE.—Extract the square root of 517.92.

SOLUTION.—First point off the number into periods of two figures each, beginning at the decimal point; this gives 5'17'.92. Now find the whole number whose square most nearly equals the first period. In the present case this is 2, since $2^2 = 4$, and $3^2 = 9$, and the difference between 5 and 4 is less than the difference between 5 and 9. Call this number (2 in the present case) the *trial root*. Divide the first period (which call the *trial dividend*) by twice the trial root, and to the quotient add one-half the trial root. Thus, $\frac{5}{2 \times 2} + \frac{2}{2} = 1.25 + 1 = 2.25$. Call this result the new trial root; retain two figures of it, increasing the second by 1 if the third figure is 5 or greater, thus obtaining for the above result 2.3 or 23, not regarding the decimal point for the present.

Repeat the process, using the first and second periods of the given number for a trial dividend, and 23 for a trial root.

Thus, $\frac{5'17}{2 \times 23} + \frac{28}{2} = 22.74$, or 22.7 to three figures.

Repeat the process once more, using 227 for the trial root, and the first three periods of the given number for the trial dividend.

Thus, $\frac{5'17.92}{2 \times 227} + \frac{227}{2} = 227.58$, using five figures.

To determine the correct position of the decimal point, apply the principle that *there must be as many places in the whole-number part of the root as there are periods in the whole-number part of the given number*. In 517.92 there are two periods to the left of the decimal point; hence, $\sqrt{517.92} = 22.758$ Ans.

46. The method as outlined in the above example will always give the first five figures of the root, and will generally give the first six figures. These are all that are usually required. If, however, a greater number of figures is required, repeat the process once more, using the answer as obtained above for the trial root, and using all the figures of the given number for the trial dividend. The result then obtained should be correct to from 10 to 13 figures.

47. Rule.—I. *Begin at the decimal point and point off the number into periods of two figures each. Having done this, neglect the decimal point until required to decide its position in the root.*

II. *Find the number whose square is nearest in value to the first period; call this number the trial root. Using the first period of the given number as a trial dividend, divide it by twice the trial root, add to the quotient one half the trial root, and retain two figures of the result, increasing the second figure by 1 if the third figure is 5 or greater.*

III. *Use the first two periods for a trial dividend (the second period being a cipher period if the number contains but one period) and the result obtained in II for a trial root. Repeat the process, retaining three figures of the result, which use as a trial root when three periods are used for a trial dividend. The first five figures of this last result will be the first five figures of the root. Increase the fifth figure by 1 if the sixth figure is 5 or greater, and then locate the decimal point.*

IV. *If the number contains but one or two periods, additional periods may be obtained by annexing ciphers.*

48. The following formula will assist the student in remembering this rule:

$$\text{Root} = \frac{D}{2R} + \frac{R}{2},$$

in which D = the trial dividend, and R = the trial root.

EXAMPLE.—Extract the square root of 3; that is, find the value of $\sqrt{3}$.

SOLUTION.—The number whose square is nearest 3 is 2; hence, we use 2 for the first trial root.

Then,
$$\frac{3}{2 \times 2} + \frac{2}{2} = 1.75, \text{ say } 1.8.$$

Substituting 18 for R ,

$$\frac{3'00}{2 \times 18} + \frac{18}{2} = 17.8.$$

Substituting 173 for R ,

$$\frac{3'00'00}{2 \times 173} + \frac{173}{2} = 173.205.$$

Hence, $\sqrt{3} = 1.73205$, which is correct to six figures. Ans.

EXAMPLE.— $\sqrt[4]{.00217} = ?$

SOLUTION.—Pointing off the number into periods of two figures each, it becomes .00'21'70, annexing a cipher to complete the last period. Treating 21 as the first period, the number whose square is nearest to 21 is 5, since $4^2 = 16$ and $5^2 = 25$, and there is a greater difference between 16 and 21 than there is between 21 and 25.

Hence,
$$\frac{21}{2 \times 5} + \frac{5}{2} = 4.6.$$

Substituting 46,
$$\frac{21'70}{2 \times 46} + \frac{46}{2} = 46.6.$$

Substituting 466,
$$\frac{21'70'00}{2 \times 466} + \frac{466}{2} = 465.83.$$

We locate the decimal point as follows: Since the number is wholly decimal, the root must also be wholly decimal; and since there are two ciphers following the decimal point, there must be *one cipher after the decimal point in the root*, for if any number have *one* cipher following the decimal point, its square will have two ciphers following the decimal point. Hence, $\sqrt[4]{.00217} = .046583$. Ans.

EXAMPLE.— $\sqrt[4]{.8} = ?$

SOLUTION.—Annexing a cipher to complete the period (since the number is wholly decimal), we obtain $\sqrt[4]{.80} = ?$ The number whose square is nearest 80 is 5.

Substituting 5,	$\begin{array}{r} 30 \\ 2 \times 5 + 2 \end{array} \approx 5.5.$
Substituting 55,	$\begin{array}{r} 30'00 \\ 2 \times 55 + 2 \end{array} = 54.8.$
Substituting 548,	$\begin{array}{r} 30'00'00 \\ 2 \times 548 + 2 \end{array} = 547.72.$

Since the number is wholly decimal, the root is wholly decimal, and $\sqrt[4]{.8} = .54772$. Ans.

When there are ciphers between the decimal point and the first digit of a number that is wholly decimal, and it is required to ascertain the position of the decimal point in the root, count the ciphers between the decimal point and the first digit and divide the result by 2. The quotient in each case, neglecting any remainder that may be obtained, will denote the number of ciphers between the decimal point and the first digit of the root. Thus, the square root of .00000027 will have $6 \div 2 = 3$ ciphers between the decimal point and the first digit of the root; likewise, the square root of .0000027 will have $5 \div 2 = 2$ ciphers between the decimal point and the first figure of the root. Another way to ascertain the position of the decimal point is to point off the given number into periods, and write one cipher in the root for each cipher period between the decimal point and the first digit of the root. Thus, the square root of .00'00'00'25 = .0005; and the square root of .00'04 = .02.

49. If the first period is a perfect square, the result obtained by the first approximation will be the same as the trial root. In such cases, we must proceed as follows:

Use the first two periods of the given number for the trial dividend; annex a cipher to the trial root and proceed as before. Or, find the number composed of two figures whose square is nearest in value to the number represented by the first two periods of the given number, and use it for the trial root.

For example, to extract the square root of 190.72, it is

seen that the first period, 1, is a perfect square; hence, use the first two periods. By the first method,

$$\frac{1'90}{2 \times 10} + \frac{10}{2} = 14.5; \quad \frac{1'90'72}{2 \times 145} + \frac{145}{2} = 138.3;$$

$$\frac{1'90'72'00}{2 \times 1,383} + \frac{1,383}{2} = 1,381.01,$$

or 13.8101 after locating the decimal point. By the second method, we take 14 for the trial root and obtain:

$$\frac{1'90}{2 \times 14} + \frac{14}{2} = 13.8; \quad \frac{1'90'72}{2 \times 138} + \frac{138}{2} = 138.101,$$

or 13.8101 after locating the decimal point.

Although the first method is a little longer, it saves the labor of trying to find the number whose square is nearest to the number represented by the first two periods. When using the first method, the substituting of three figures for R in the trial root will not generally give the first five figures of the root in the result. Hence, it is necessary to substitute four figures of the result for R , and repeat the process. The result thus obtained will usually be correct to six figures.

50. When the given number whose root it is desired to find is a perfect square, the process just described does not give the exact root; but the exact root may always be found as shown in the following examples:

EXAMPLE.— $\sqrt{15,625} = ?$

SOLUTION.—Pointing off into periods, we get 1'56'25. Since the first period is a perfect square, it is necessary to employ one of the methods described in Art. 49. Using the first method,

$$\frac{1'56}{2 \times 10} + \frac{10}{2} = 12.8.$$

Substituting 128 for R ,
$$\frac{1'56'25}{2 \times 128} + \frac{128}{2} = 125.035.$$

Substituting 1,250 for R ,
$$\frac{1'56'25'00}{2 \times 1,250} + \frac{1,250}{2} = 1,250.$$

Now, since this last result is the same as the trial root, we know that the last trial dividend is a perfect square. Hence, locating the decimal point, $\sqrt{15,625} = 125$. Ans.

EXAMPLE.— $\sqrt{64,128,064} = ?$

SOLUTION.—Pointing off into periods, we get 64'12'80'64. If we assume that $R = 8$, the first result will be 8, since 64 is a perfect square. Thus,

$\frac{64}{2 \times 8} + \frac{8}{2} = 8$. Hence, we use the first two periods for a trial dividend, and 80 for R , getting,

$$\frac{64'12}{2 \times 80} + \frac{80}{2} = 80.075, \text{ say } 80.1.$$

Substituting 801 for R , $\frac{64'12'80}{2 \times 801} + \frac{801}{2} = 800.8$.

Substituting 8,008 for R , $\frac{64'12'80'64}{2 \times 8,008} + \frac{8,008}{2} = 8,008$.

Hence, $\sqrt{64,128,064} = 8,008$. Ans.

EXAMPLE.— $\sqrt{5,688,327,241} = ?$

SOLUTION.—Pointing off into periods, we get 56'88'32'72'41.

Substituting 7 for R , $\frac{56}{2 \times 7} + \frac{7}{2} = 7.5$.

Substituting 75 for R , $\frac{56'88}{2 \times 75} + \frac{75}{2} = 75.42$.

Substituting 754 for R , $\frac{56'88'32}{2 \times 754} + \frac{754}{2} = 754.21$.

Since the first four figures of this result are the same as those of the preceding operation, we may be sure that the fifth figure of this result is also correct; and, since the given number contains five periods, and the square of the fifth figure is the same as the last figure of the fifth period of the given number, the given number is probably a perfect square. Hence, substituting 75,421 for R , and using all five periods,

$$\frac{56'88'32'72'41}{2 \times 75,421} + \frac{75,421}{2} = 75,421.$$

Therefore, $\sqrt{5,688,327,241} = 75,421$. Ans.

EXAMPLES FOR PRACTICE.

51. In the following examples, — after the answer indicates that the last figure was increased by 1 in consequence of the next figure being 5 or a greater digit; + after the answer indicates that the next figure was less than 5. When the answer is followed by neither + nor —, the root is exact.

(a) What is the square root of 90 ?	Ans. {	(a) 9.4868+
(b) What is the square root of 54.3 ?		(b) 7.3689-
(c) What is the square root of 2,796 ?		(c) 52.877+
(d) What is the square root of 3,510.76 ?		(d) 59.252-
(e) What is the square root of 1,485,961 ?		(e) 1,219.
(f) What is the square root of 2,164.1104 ?		(f) 46.52.
(g) What is the square root of 47.80754449 ?		(g) 6.9148.

CUBE ROOT.

52. The first step in finding the cube root of a number is to find how many figures there will be in the root. Proceeding as in Art. 42, we have $9^3 = 729$; $99^3 = 970,299$; $999^3 = 997,002,999$. It appears that in each of the above cases, the cube of the number contains three times as many figures as the number. Therefore, if a number is pointed off into periods of *three* figures each, beginning at the right, the cube root of the number will contain as many figures as there are periods. Thus, the cube root of 1,767,172,329 must contain four figures, since the number contains four periods.

53. The cube of any number wholly decimal contains three times as many figures as the number cubed. Thus, $.2^3 = .008$, $.02^3 = .000008$, $.007^3 = .000000343$, $.317^3 = .031855013$. The first step in finding the cube root of a decimal is to separate it into periods of three figures each, beginning at the decimal point. The location of the decimal point in the root is readily determined from the number of ciphers between the decimal point and the first digit; the whole number obtained by dividing this number by 3 will denote the number of ciphers immediately following the decimal point in the root. The cube root of .000076, neglecting the decimal point, is 42358. To locate the decimal point, we observe that there are four ciphers following the decimal point in the given number; $\frac{4}{3} = 1$ and 1 remainder.

Therefore, there will be one cipher following the decimal point in the root; or $\sqrt[3]{.000076} = .042358$. In dividing the number of ciphers by 3, no attention is paid to the remainder, if there be one.

54. The process of extracting the cube root of a number may be shown by an example.

EXAMPLE.—Extract the cube root of 19,397.82.

SOLUTION.—The number is first divided into periods of three figures, beginning at the decimal point; this gives 19'397'820. The first period is 19, and the nearest perfect cube is 27, therefore, $\sqrt[3]{27} = 3$ is taken as the first trial root. The first period is divided by *three times the square of the trial root*, and to the quotient is added two-thirds of the trial root. Thus, in the present case, $\frac{19}{3 \times 3^2} + \frac{2}{3} \times 3 = .703 + 2 = 2.703$.

Two figures of this quotient are retained, the decimal point being neglected as in the process of extracting square root, this gives 27 as the second trial root. Using the trial root and the first two periods as a trial dividend, the process is repeated. Thus, $\frac{19'397}{3 \times 27^2} + \frac{2}{3} \times 27 = 8.86 + 18 = 26.86$. Retaining three figures, the third trial root is 269. The process is repeated, using three periods. Thus, $\frac{19'397'820}{3 \times 269^2} + \frac{2}{3} \times 269 = 89.35669 + 179.33333 = 268.69002$. Since there are two periods in the whole-number part of the given number, there must be two places in the whole-number part of the root. Therefore, the required root is 26.869. The result thus obtained is correct to six figures, since the first seven figures of the root are 26.86897.

55. Instead of always using the cube root of the perfect cube that is nearest in value to the first period, for the first trial root, a better way is to proceed as follows: Find the two perfect cubes between which the first period lies, and divide their difference by the index of the root (in this case 3); then add the result (neglecting any remainder) to the smaller perfect cube. If the first period is less than the result thus obtained, use the cube root of the *smaller* perfect cube for the first trial root; but if the first period is equal to or greater than this result, use the cube root of the greater perfect cube.

For example, suppose that the first period is 267; it lies between $6^3 = 216$ and $7^3 = 343$. To determine whether to use 6 or 7 for the trial root, proceed as above directed. $343 - 216 = 127$; $127 \div 3 = 42$; and $216 + 42 = 258$. Hence, 7 would be used for the trial root.

Again, suppose the first period was 35. $3^3 = 27$; $4^3 = 64$;

$64 - 27 = 37$; $37 \div 3 = 12$; $27 + 12 = 39$. Hence, 3 would be used for the first trial root.

In the same manner, if the first period were 4, the trial root used would be 2, for $1^3 = 1$; $2^3 = 8$; $(8 - 1) \div 3 = 2$; and $1 + 2 = 3$, which is less than 4.

If the trial root is determined as explained in this article, the third approximation (when three figures are used for the trial root) will usually give six figures of the root exactly; if four figures are used for the trial root, the answer will be correct to eight figures.

56. Rule.—I. *Beginning at the decimal point, separate the number into periods of three figures each. Having done this, neglect the decimal point until it is required to determine its position in the root.*

II. *Find the first trial root as described in Art. 55. Using the first period as a trial dividend, divide it by three times the square of the trial root, and add to the quotient two-thirds of the trial root. Retain two figures of the result, increasing the second figure by 1 if the third figure is 5 or greater.*

III. *Use the first two periods for a trial dividend and the result obtained in II for a trial root, and repeat the process described in II. Retain three figures of the result as a new trial root, and, using three periods as a trial dividend, repeat the process once more, retaining five or six figures of the result, and locate the decimal point.*

IV. *If the number contains but one or two periods, the additional periods may be obtained by annexing ciphers.*

57. The above process of extracting cube root is expressed by the following formula:

$$\text{Root} = \frac{D}{3R^2} + \frac{2R}{3},$$

in which D = the trial dividend, and R = the trial root.

EXAMPLE.—Extract the cube root of 75.

SOLUTION.— $4^3 = 64$; $5^3 = 125$; $(125 - 64) \div 3 + 64 = 84$. Hence, 4 will be used for the first trial root.

First approximation, $\frac{75}{3 \times 4^2} + \frac{2 \times 4}{3} = 4.23$, or 4.2.

Second approximation, $\frac{75'000}{3 \times 42^2} + \frac{2 \times 42}{3} = 42.17$, or 42.2.

Third approximation, $\frac{75'000'000}{3 \times 422^2} + \frac{2 \times 422}{3} = 421.71652$.

Locating the decimal point, $\sqrt[3]{75} = 4.2173$ to five figures. Ans.
The exact root to eight figures is 4.2171633.

58. If the first period is a perfect cube, proceed in a manner similar to that described in Art. 49. Also, if the given number is a perfect cube, the exact root may be found by a method similar to that described in Art. 50.

EXAMPLE.—Extract the cube root of 1,728.

SOLUTION.—Pointing off into periods of three figures each, it is seen that the first period, 1, is a perfect cube; hence, use 10 for the trial root in the first approximation.

First approximation, $\frac{1'728}{3 \times 10^2} + \frac{2 \times 10}{3} = 12.48$, or 12.4.

Second approximation, $\frac{1'728'000}{3 \times 124^2} + \frac{2 \times 124}{3} = 120.13$, or 120.1.

Third approximation, $\frac{1'728'000'000}{3 \times 1,201^2} + \frac{2 \times 1,201}{3} = 1,200.0008$,

or 12 after fixing the decimal point.

Substituting 12 for R ,

$\frac{1,728}{3 \times 12^2} + \frac{2 \times 12}{3} = 12$; hence, 12 is the cube root of 1,728. Ans.

EXAMPLE.—Extract the cube root of 50,000,000,000.

EXAMPLES FOR PRACTICE.

59. In the following examples, — after the answer indicates that the last figure was increased by 1 in consequence of the next figure being 5 or a greater digit; + after the answer indicates that the next figure was less than 5. When the number is followed by neither + nor —, the root is exact.

- | | |
|---|---|
| (a) What is the cube root of 80 ? | $\left\{ \begin{array}{l} (a) \quad 4.8089—. \\ (b) \quad 2.0369—. \\ (c) \quad 586.3. \\ (d) \quad 84.7726—. \\ (e) \quad 17.1. \end{array} \right.$ |
| (b) What is the cube root of 8.451 ? | |
| (c) What is the cube root of 201,539,270.647 ? Ans. | |
| (d) What is the cube root of 609,210 ? | |
| (e) What is the cube root of 5,000.211 ? | |

60. To assist the student in determining the first trial root for square and cube roots, we have prepared the following table. The table is not a necessity, it is merely a help.

SQUARE ROOT.

- If the first period is less than 8, use 1 for the trial root.
 If the first period is less than 7, use 2 for the trial root.
 If the first period is less than 18, use 3 for the trial root.
 If the first period is less than 21, use 4 for the trial root.
 If the first period is less than 31, use 5 for the trial root.
 If the first period is less than 43, use 6 for the trial root.
 If the first period is less than 57, use 7 for the trial root.
 If the first period is less than 73, use 8 for the trial root.
 If the first period is less than 91, use 9 for the trial root.
 If the first period is between 91 and 100, use 10 for the trial root.

CUBE ROOT.

- If the first period is less than 8, use 1 for the trial root.
 If the first period is less than 14, use 2 for the trial root.
 If the first period is less than 39, use 3 for the trial root.
 If the first period is less than 84, use 4 for the trial root.
 If the first period is less than 155, use 5 for the trial root.
 If the first period is less than 258, use 6 for the trial root.
 If the first period is less than 399, use 7 for the trial root.
 If the first period is less than 584, use 8 for the trial root.
 If the first period is less than 819, use 9 for the trial root.
 If the first period is between 819 and 1,000, use 10 for the trial root.

EXAMPLE.—Extract the cube root of 990.

SOLUTION.—Referring to the table, we see that 10 should be used for the trial root.

First approximation,

$$\frac{990}{8 \times 10^3} + \frac{2 \times 10}{8} = 9.967.$$

We must use 99.7 for the next trial root, since if we increase the number expressed by the first two figures by 1 in consequence of the third figure being 6, the result is 10.0, the same as the first trial root. This indicates that the first approximation is correct to at least three figures.

Second approximation,

$$\frac{990'000}{8 \times 99.7^3} + \frac{2 \times 99.7}{8} = 99.9655.$$

Hence,

$$\sqrt[3]{990} = 9.9655. \text{ Ans.}$$

The exact root to eight figures is 9.965549.

61. To extract the square root or the cube root of a fraction, reduce the fraction to a decimal and extract the required root of the decimal.

EXAMPLE.—Extract the cube root of $\frac{1}{12}$.

SOLUTION.—Reducing $\frac{1}{12}$ to a decimal, it becomes .2916.

The first period is 291 (the others are all 6's, since the 6 repeats); hence, referring to the table. Art. 60, 7 should be used for the trial root.

$$\text{First approximation, } \frac{291}{8 \times 7^3} + \frac{2 \times 7}{8} = 6.64+.$$

$$\text{Second approximation, } \frac{291'666}{8 \times 66^3} + \frac{2 \times 66}{8} = 66.81+.$$

$$\text{Third approximation, } \frac{291'666'666}{8 \times 663^3} + \frac{2 \times 663}{8} = 663.176+.$$

$$\text{Therefore, } \sqrt[3]{\frac{1}{12}} = \sqrt[3]{.2916} = .663176. \text{ Ans.}$$

ARITHMETIC.

MENSURATION.

1. Mensuration treats of the measurement of lines, angles, surfaces, and solids.

LINES AND ANGLES.

2. A straight line is one that does not change its direction throughout its whole length—it is the shortest distance between two points. To distinguish one straight line from another, two of its points are designated by letters. The line shown in Fig. 1 would be called the line *AB*.



FIG. 1.

3. A curved line changes its direction at every point. (Fig. 2.)



FIG. 2.

4. Parallel lines are equally distant from each other at all points. (Fig. 3.)

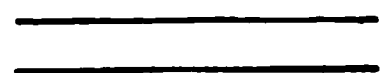


FIG. 3.

5. A line is perpendicular to another when it meets that line so as not to incline towards it on either side. (Fig. 4.)

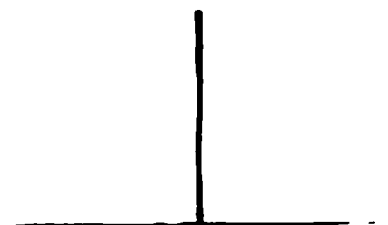


FIG. 4.

6. A horizontal line is a line parallel to the horizon or water level. (Fig. 5.)

7. A vertical line is a line perpendicular to a horizontal line; consequently, it has the direction of a plumb-line. (Fig. 5.)

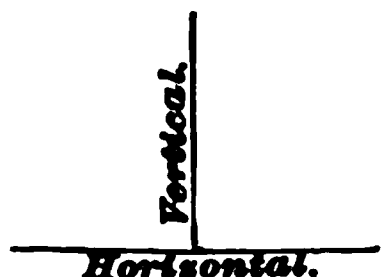


FIG. 5.

8. An **angle** is the amount of divergence between two lines that intersect, or meet; the point of meeting is called the **vertex** of the angle. Thus, in Fig. 6, the two lines form an angle whose vertex is at B .

Angles are distinguished by naming the vertex and a point on each line. Thus, in Fig. 6, the angle formed by the lines AB and CB is called the angle ABC , or the angle CBA ; the letter at the vertex is always placed at the middle. When an angle stands alone so that it cannot be mistaken for any other angle, only the vertex letter need be used. Thus, the angle referred to might be designated simply as the angle B .



FIG. 6.

9. A **right angle** is one of the angles formed by the intersection of two lines which are perpendicular to each other. In Fig. 7, the line AB is perpendicular to the line CD ; therefore, the angles ABC and ABD are right angles.



FIG. 7.

10. An **acute angle** is less than a right angle. The angle ABC , Fig. 8, is an acute angle.

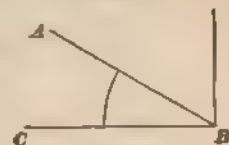


FIG. 8.



FIG. 9.

11. An **obtuse angle** is greater than a right angle. The angle ABD , Fig. 9, is an obtuse angle.

QUADRILATERALS.

12. A **plane figure** is any part of a plane, or flat, surface, bounded by straight or curved lines.

13. A **quadrilateral** is a plane figure bounded by four straight lines.

14. A **parallelogram** is a quadrilateral whose opposite sides are parallel.

There are four kinds of parallelograms: the **rectangle**, the **square**, the **rhomboid**, and the **rhombus**.



FIG. 10.

15. A **rectangle** is a parallelogram whose angles are all right angles. (Fig. 10.)

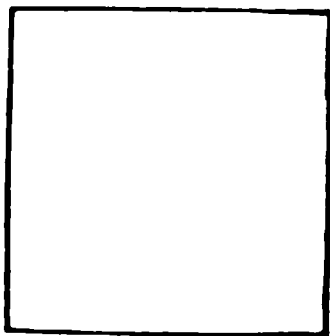


FIG. 11.

16. A **square** is a rectangle whose sides are all of the same length. (Fig. 11.)



FIG. 12.

17. A **rhomboid** is a parallelogram whose opposite sides are equal, and whose angles are not right angles. (Fig. 12.)

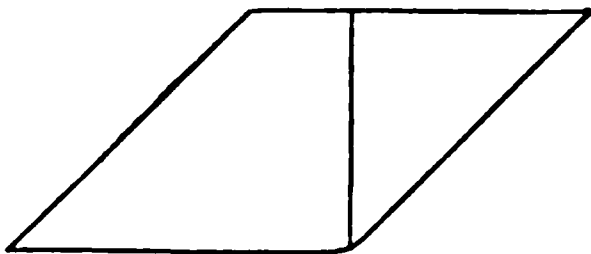


FIG. 13.

18. A **rhombus** is a rhomboid having equal sides. (Fig. 13.)

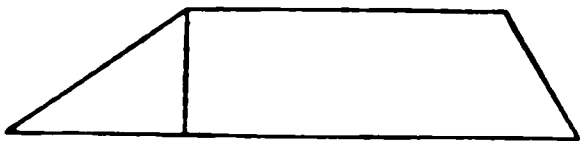


FIG. 14.

19. A **trapezoid** is a quadrilateral having only two of its sides parallel. (Fig. 14.)

20. The **altitude** of a parallelogram or trapezoid is the perpendicular distance between the parallel sides. The length of the dotted lines in Figs. 12, 13, and 14 is the altitude.

21. The **base** of a quadrilateral is the side on which it is supposed to stand. Any side may be taken as the base.

22. The **area** of a plane figure is the number of square units contained in its surface. The square unit may be a square inch, square foot, square yard, square meter, etc., as is most convenient.

23. The area of a parallelogram is equal to the product of the base and the altitude. This can be shown readily in

the case of the rectangle. Suppose, for example, the leaf of a book is 6 inches wide and 9 inches long (Fig. 15). It is a



FIG. 15.

rectangle with a base of 9 inches and an altitude of 6 inches. Suppose the base to be divided into 9 equal parts, each 1 inch in length, and assume lines to be drawn through each point of division, parallel to the short sides of the rectangle. In a similar

manner, suppose the altitude, or short side, to be divided into 6 equal parts, each 1 inch long, and through these points of division let lines be drawn parallel to the base. The rectangle is divided by these two sets of lines into little squares, as shown in Fig. 15. The area of one of the small squares is 1 square inch, since each of its sides is 1 inch in length. There are 9 of the squares in each horizontal row, and there are 6 rows. Hence, the total number of the little squares is $6 \times 9 = 54$, and the area of the surface is 54 square inches.

24. Rule.—*To find the area of a rectangle, multiply the base by the altitude.*

25. In ordinary language, the base and altitude of a rectangular surface are spoken of as length and breadth; the area of the surface is obtained by multiplying together the length and breadth. In applying the above rule, care must be taken that the base and altitude, or length and breadth, are reduced to the same kind of units. For example, if the base is given in feet and the altitude in inches, they cannot be multiplied together unless both are feet or both inches. This principle is of great importance, and holds good throughout the subject of Mensuration.

It must not be understood from the foregoing that *feet can be multiplied by feet or inches by inches*. In multiplication the multiplier is *always abstract*. In Fig. 15 there are 9 square inches in 1 row, and 6 times as many in 6 rows.

The operation in reality is $9 \text{ sq. in.} \times 6 = 54 \text{ sq. in.}$, or $6 \text{ sq. in.} \times 9 = 54 \text{ sq. in.}$

26. *EXAMPLE.*—What is the area of a floor 16 feet long and $13\frac{1}{2}$ feet wide?

SOLUTION.—The base is 16 feet and the altitude is $13\frac{1}{2}$ feet.

Area = base \times altitude $= 16 \times 13\frac{1}{2} = 216 \text{ sq. ft.}$ Ans.

27. The area of any parallelogram is equivalent to the area of a rectangle of the same base and altitude. In Fig. 16, the plane figure $ABDC$ is a rhomboid. Suppose the corner ACB is cut off, as shown, and placed at the other end in the position BDF . If the cutting line AE is perpendicular to the base CD , the new figure $ABFE$ is a rectangle. It is plain that the base and altitude of the rectangle are the same as the base and altitude of the rhomboid, and that the areas of the two figures are the same.

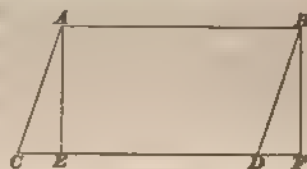


FIG. 16.

28. *To find the area of any parallelogram, multiply the base by the altitude.*

EXAMPLE.—A piece of cloth 1 yard wide is "cut on the bias," that is, it has the shape shown in Fig. 16. If the length of the strip is 8 feet, what is its area?

SOLUTION. The altitude is 1 yd. = 3 ft., and the base is 8 ft. Hence,

Area = base \times altitude $= 8 \times 3 = 24 \text{ sq. ft.}$ Ans.

29. *Rule.* *To find the area of a trapezoid, multiply one-half the sum of the parallel sides by the altitude.*

30. The reason for this rule will appear from an examination of Fig. 17. If E and F be the middle points of the sides that are not parallel, and if $AE3$ and $BF2$ be cut off below by $4-3$ and $1-2$, perpendicular to AB , and placed

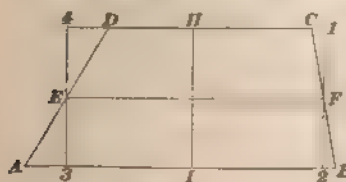


FIG. 17.

above, as shown, we have a rectangle whose area is equal to $EF \times HI$. But EF is as much less than AB as it is greater than DC . In other words,

EF = half the sum of the parallel sides of the trapezoid.

Hence, area of trapezoid = $\frac{DC + AB}{2} \times HI$.

EXAMPLE.—A piece of land has the form of a trapezoid. The parallel sides are, respectively, 40 rd. and 56 rd. long, and the perpendicular distance between them is 35 rods. How many acres are contained in the piece?

SOLUTION.—One-half the sum of the parallel sides = $\frac{40 + 56}{2}$ = 48 rd.

Area = 48×35 = 1,680 sq. rd. = $\frac{1,680}{160}$ = 10.5 acres. Ans.

31. The perimeter of a quadrilateral is the sum of the lengths of its four sides.

EXAMPLE.—A room is 23 ft. long and 18 ft. wide. What is its perimeter?

SOLUTION. Perimeter = $23 + 23 + 18 + 18$ = $23 \times 2 + 18 \times 2$ = 82 ft.
Ans.

PLASTERING, PAINTING, AND KALSOMINING.

32. Plastering, painting, and kalsomining are usually estimated by the square yard. Allowances for doors, windows, etc. are not regulated by any established usage. Sometimes no deduction is made for them, sometimes one-half their extent is deducted; but this is a matter usually specified in the contract.

33. EXAMPLE.—At 22 cents per square yard, what will it cost to plaster a room 65 ft. long, 22 ft. wide, and 15 ft. high deducting in full for 8 doors 4 ft. 6 in. wide and 11 ft. 6 in. high; 10 windows 3 ft. 6 in. wide and 8 ft. high; and a baseboard 6½ in. high extending around the room?

SOLUTION.—Perimeter of the room = $65 \times 2 + 22 \times 2$ = 174 ft.

Area of walls..... = 174×15 = 2,610 sq. ft.

Area of ceiling..... = 65×22 = 1,430 sq. ft.

Total..... = 4,040 sq. ft.

Area of doors..... = $4\frac{1}{2} \times 11\frac{1}{2} \times 8$ = 414 sq. ft.

Area of windows..... = $3\frac{1}{2} \times 8 \times 10$ = 280 sq. ft.

Area of baseboard = (perimeter less the width

of 8 doors) $\times \frac{6\frac{1}{2}}{12}$ = $(174 - 41\frac{1}{2} \times 8) \times \frac{6\frac{1}{2}}{12}$ = 74½ sq. ft.

Total, after reduction..... = 3,271½ sq. ft.

Area in square yards..... = $3,271\frac{1}{2} \div 9$ = 363½ sq. yd.

Cost..... = $\$22 \times 363\frac{1}{2}$ = \$7990 Ans.

34. Rule.—*Multiply the perimeter of the room by the height of the ceiling for the area of the walls. To this add the area of the ceiling, and from the sum make such deductions as are specified. Reduce the results to square yards, and multiply the price per square yard by the number denoting the area in square yards.*

EXAMPLES FOR PRACTICE.

35. Solve the following examples:

1. What will it cost to plaster a room 24 ft. by 30 ft., the ceiling being 9 ft. 6 in. high, at 25 cents a square yard, if no deductions are made for openings? Ans. \$48.50.

2. At 12 cents per square yard, what will it cost to paint the walls and ceiling of a hall 60 ft. long, 45 ft. wide, and 15 ft. high, deducting one-half for 4 doors, 11 ft. high and 8 ft. wide, and 8 windows, 9 ft. high and 4 ft. wide? Ans. \$73.73.

3. What must be paid, at 5 cents per square yard, for kalsomining 3 rooms, each having a ceiling 8 ft. 9 in. high, and the following dimensions, respectively: 18 ft. by 20 ft., 21 ft. by 27 ft., and 24 ft. by 30 ft., there being no deductions? Ans. \$22.76.

PAPERING.

36. Wall paper as made in the United States is 18 inches ($\frac{1}{2}$ yard) wide, and is sold in *single rolls* and *double rolls*; a single roll is 8 yards long, and a double roll is 16 yards long. When cutting the paper, paper hangers divide the rolls into strips of sufficient length to reach from the baseboard to a short distance (say 6 inches) above the lower edge of the border. There is always considerable waste in cutting, owing to the matching of the figures forming the design, and the fact that there is a part of a strip left over after cutting up the roll. The parts of strips thus left over are used for the surface above doors, and above and below windows, and other irregular places. Although double rolls are usually counted as two single rolls, there is a choice between them in certain cases. Thus, suppose the strips were required to be 9 feet (3 yards) long; only 2 strips could be cut from a single roll, or 4 strips from 2 single rolls, while 5 strips could be cut from a double roll. The length of a roll of border is the same as the length of a roll of paper.

37. On account of the waste in cutting, the varying sizes and shapes of rooms, the number of windows, doors, etc., it is difficult to estimate exactly the number of rolls required. We give herewith two rules, both of which are used in practice:

Rule I. *From the perimeter of the room subtract the widths of openings (windows and doors), and reduce the result to half-yards; the number of half-yards so obtained will be the total number of strips required. Find the number of strips that can be cut from a roll and divide the first result by the second; the quotient will be the number of rolls required.*

Rule II.—*Divide the number of half-yards in the perimeter of the room by the number of strips that can be cut from a roll; the quotient will be the number of rolls required.*

38. If computed by the first rule, the number of rolls obtained may be too small, and if computed by the second rule, too large. But, since paper dealers will usually take back all rolls that are intact, the second rule will generally give the best results, as it will prevent the loss of time required to send to the dealer for extra rolls, in case they are required.

EXAMPLE Find how much paper will be needed to cover the walls and ceiling of a room 15 ft. by 20 ft., the border for both walls and ceiling to be 18 inches wide. The baseboard is 8 inches high, and the height of walls from floor to ceiling is 9 feet.

SOLUTION Since the widths of the openings are not specified, it will be necessary to use rule II.

Perimeter of room = $2 \times 15 + 2 \times 20 = 70$ ft. = $23\frac{1}{2}$ yd. = 46½ half-yards, or 47 strips. Assuming that the strips extend the height of the baseboard above the bottom edge of the border, the length of a strip is (since 18 in. = $1\frac{1}{2}$ ft.) $9 - 1\frac{1}{2} = 7\frac{1}{2}$ ft. = $2\frac{1}{4}$ yd. Hence, the number of strips in a single roll is $8 \div 2\frac{1}{4} = 3$ strips, and the number of rolls required is $47 \div 3 = 15\frac{2}{3}$ or 16 rolls.

In papering the ceiling, the direction in which the strips are to run must be considered. If the strips run lengthwise of the room, the distance between the edges of the border is $20 - 2 \times 1\frac{1}{2} = 17$ ft., and the length of the strips must be at least 18 ft., or 6 yd. long; hence, but one strip can be cut from a single roll, and but two from a double roll. The width of the room in half-yards is $(15 + 3) \div 2 = 10$ hence, allowing for the border 9 strips, or 9 single rolls will be required.

If the strips run crosswise of the room, the length of a strip between the edges of the border will be $15 - 2 \times 1\frac{1}{2} = 12$ ft., and the length of

a strip must be at least 13 ft., or $4\frac{1}{2}$ yd.; hence, 1 strip may be obtained from a single roll, or $16 \div 4\frac{1}{2} = 3$ strips from a double roll. The length of the room in half-yards is $(30+3) \times 2 = 13\frac{1}{2}$, hence, allowing the paper to extend 6 in. beyond the inner edge of the border at both ends of the room, 12 strips will be required. The number of double rolls required will be $12 \div 3 = 4$ double rolls. Consequently, there is less waste, in this case, when the paper runs crosswise than when it runs lengthwise.

Since the perimeter of the room is 70 ft., or $23\frac{1}{2}$ yd., $23\frac{1}{2} \div 3 = 8$ single rolls of border for the walls, and the same amount for the ceiling will be required. Therefore, 16 single rolls of paper are required for the walls, 4 double rolls for the ceiling, 8 single rolls of border for the walls, and 8 single rolls for the ceiling. Ans.

CARPETING.

39. Carpet is made of various widths. Ingrain carpet is usually 36 inches, or 1 yard wide; Brussels carpet is 27 inches, or $\frac{3}{4}$ yard wide. Carpet borders are $22\frac{1}{2}$ inches, or $\frac{5}{8}$ yard wide. A linear yard of ingrain carpet contains a square yard, and a linear yard of Brussels carpet contains $\frac{3}{4}$ of a square yard. If no allowance is made for cutting and matching the strips of carpet, the number of linear yards of carpet required for a room is found by dividing the area of the room in square yards by the area of a linear yard of the carpet.

EXAMPLE How many yards of Brussels carpet are required to cover a floor 36 ft. long and 21 ft. wide, making no allowance for cutting and matching?

SOLUTION — Area of floor = $36 \times 21 = 756$ sq. ft. = $\frac{756}{9} = 84$ sq. yd.

A linear yard of Brussels carpet has an area of $\frac{3}{4}$ sq. yd. Hence, the number of linear yards required is $84 \div \frac{3}{4} = 112$ yd. Ans.

40. In practice, there is usually considerable loss due to cutting and matching. To find the number of yards required for a room, when allowance is made for loss, the width of the room is divided by the width of a single strip. The quotient is the number of strips required, supposing them to run lengthwise of the room. The number of strips multiplied by the length in yards of a single strip, making allowance for the loss required for matching, is the number of linear yards required.

EXAMPLE—How many yards of Brussels carpet are required to cover a room 23 ft. long and 15 ft. wide, making an allowance of 1 ft. on each strip for matching? The carpet is supposed to run lengthwise.

SOLUTION.—Width of room = 15 ft. 180 in. Width of carpet = 27 in. Number of strips $180 \div 27 = 6\frac{1}{2}$. Hence, 7 strips must be used, the excess, 9 in., being cut off or turned under. Allowing 1 foot for matching, length of strip = $23 + 1 = 24$ ft. = 8 yd. Number of linear yards required = $7 \times 8 = 56$ yd. Ans.

41. The number of linear yards of carpet border required for a room is equal to the perimeter of the room in yards.

EXAMPLE How many yards of border are required in carpeting a room 42 ft. long and 26½ ft. wide?

SOLUTION.—Perimeter of room = $42 \times 2 + 26\frac{1}{2} \times 2 = 137$ ft. $\frac{137}{3} = 45\frac{1}{3}$ yd. Ans.

BOARD MEASURE.

42. In measuring lumber, the unit is the **board foot**, which is a board 1 foot long, 1 foot wide, and 1 inch (or less) thick. One board foot is equal to $\frac{1}{12}$ of a cubic foot. Hence, to find the number of board feet in any piece of lumber:

Rule.—*Multiply the length in feet by the breadth in feet, and this product by the thickness in inches, if it be more than one inch; or, otherwise, multiply the length in feet by the breadth in inches, and this product by the thickness in inches, and then divide by 12.*

EXAMPLE How many board feet are contained in a joist 18 feet long, 14 inches wide, and 12 inches thick?

SOLUTION — $\frac{18 \times 14 \times 12}{12} = 252$ board feet. Ans.

43. Lumber is sold by the thousand (M) feet, the term foot being always used instead of the longer term, board foot. Hence, to find the cost, divide the number of feet by 1,000, and multiply by the cost per M.

EXAMPLE What will be the cost of 19 boards, 14 feet long, 15 inches wide, and 1½ inches thick, at \$21.50 per M?

SOLUTION.—Number of thousand feet $\frac{19 \times 14 \times 15 \times 1\frac{1}{2}}{12 \div 1,000} = .498\frac{1}{2}$.
 $.498\frac{1}{2} \times 21.50 = \11.72 Ans.

44. When expressing the size of anything that is rectangular, it is customary to write the dimensions and connect them by the sign of multiplication. Thus, to express the size of a room that is 12 feet long and 10 feet wide, it would be written $12' \times 10'$, and read *12 feet by 10 feet*. In such cases the abbreviations (') and (") are generally used instead of feet and inches. If three dimensions are to be expressed, all three are connected by the cross (read *by*), the length being written first, then the breadth, and, lastly, the thickness or height. Thus, a room 18 feet long, 14 feet wide, and 10 feet high would be expressed as a room $18' \times 14' \times 10'$. Hence, the joist in the example, Art. 42, would be expressed as $18' \times 14" \times 12"$.

45. Shingles are sold in bundles of 250 ($\frac{1}{4}$ M). The lengths of all shingles in bundle are the same (usually 12", 14", or 16"), but the width varies. The *average* width, however, is generally 4", the width of all bundles being alike. When laying shingles, 4" are usually exposed to the weather, the remaining portions being concealed by the other shingles. Hence, to find the number of shingles required to cover a roof:

46. Rule. *Compute the total area of the roof in square inches, and divide this area by the product of the average width of the shingle and the length that is exposed to the weather.*

EXAMPLE. What would it cost to shingle a roof, each side measuring $40' \times 16'$, if the shingles cost \$4.50 per M?

SOLUTION. Since the size of the exposed portion is not stated, it will be assumed as $4" \times 4"$. Then, for one side, $\frac{40 \times 16 \times 144}{4 \times 4} = 5,760$ shingles will be required, and for both sides, $5,760 \times 2 = 11,520$ shingles. Therefore, the cost will be $11.52 \times 4.50 = \$51.84$. Ans.

We multiply by 144 in order to reduce the square feet (40×16) to square inches. Allowance should also be made for waste.

47. If the exposed portion is $4" \times 4"$, it will take 9 shingles for each square foot; hence, in such cases it is only necessary to find the total area in square feet and multiply by

9 to find the number of shingles. Thus, in the last example, the total area in square feet is $40 \times 16 \times 2 = 1,280$ sq. ft., and $1,280 \times 9 = 11,520$ shingles, the same result as before.

EXAMPLES FOR PRACTICE.

48. Solve the following:

1. How many shingles are required for a roof which measures $45' \times 17'$ on one side and $45' \times 24'$ on the other side, the exposed portion of the shingles being $4" \times 5"$? Ans. 13,284 shingles.

2. (a) How many thousand feet of lumber are contained in a pile having 42 layers of boards 16 feet long, the width of the layers being 11 feet, and the thickness of the boards, 1 inch? (b) What would be its cost at \$18.75 per M? Ans. (a) 7,392 M.
(b) \$138.60

3. What is the area in square feet of a parallelogram whose base is $58\frac{1}{2}"$ and altitude is $23\frac{1}{2}"$? Ans. 9,556.6 sq. ft.

4. How many square yards of oilcloth will cover a floor $15' \times 13\frac{1}{2}'$? Ans. $22\frac{1}{2}$ sq. yd.

5. If Brussels carpet costs 95 cents per yard, what will be the cost of carpeting a room $13\frac{1}{2}' \times 18'$, allowing 1 ft. on each strip for waste in matching? Ans. \$36.10.

6. How many sheets of tin $20" \times 14"$ are required to cover a roof $56' \times 30'$? Ans. 804 sheets.

7. At 18 cents per square yard, what will be the cost of plastering the ceiling and walls of a room 23 ft. long, 16 ft. wide, and 12 ft. high, making allowance for 3 doors, 3 ft. 6 in. wide by 7 ft. 6 in. high, 5 windows, 3 ft. 6 in. wide by 5 ft. 4 in. high, and a baseboard 8 in. high? Ans. \$21.74

8. At \$2.50 per square yard, what is the cost of paving a street $\frac{1}{4}$ mile long and 60 feet wide? Ans. \$44,000.

9. How many double rolls of paper and border are required to cover the walls of the room of example 7, assuming that the border, which is 18 in. wide, extends the height of the baseboard over the paper? Use rule I, Art. 37. Ans. 19 rolls for walls,
12 rolls for border.

10. How many board feet in a stick of timber $27' \times 9' \times 8"$? Ans. 162 ft.

11. How many single rolls of paper would be required to paper the ceiling of the room of example 7, assuming that there is no border and that the paper overlaps on the walls at least 2 in.? Ans. 11 rolls.

THE TRIANGLE.

49. A **triangle** is a plane figure having three sides.

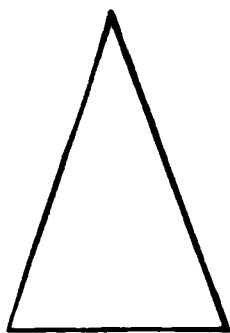


FIG. 18.

50. An **isosceles** triangle is one having two of its sides equal, as in Fig. 18.

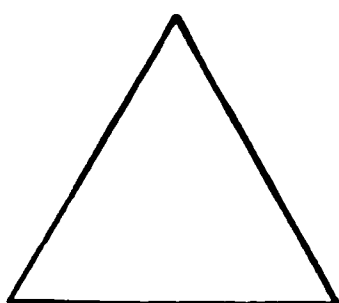


FIG. 19.

51. An **equilateral** triangle is one having all of its sides equal. (Fig. 19.)



FIG. 20.

52. A **scalene** triangle is one having no two of its sides equal. (Fig. 20.)

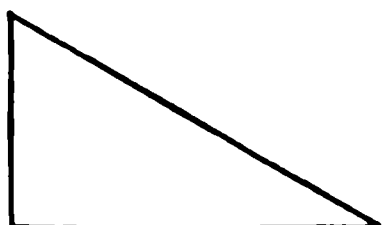


FIG. 21.

53. A **right-angled** triangle is any triangle having one right angle. The side opposite the right angle is called the **hypotenuse**. (Fig. 21.) A right-angled triangle may be isosceles or scalene.

54. The **altitude** of any triangle is a line drawn from the vertex of the angle opposite the base perpen-

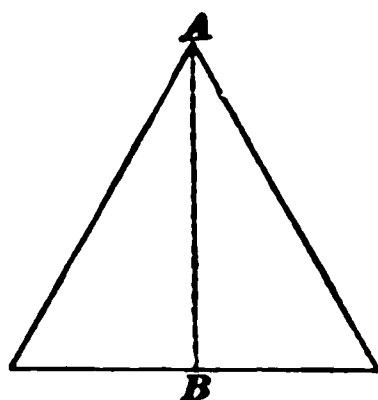


FIG. 22.

dicular to the base, or to the base extended. In Figs. 22 and 23 the vertical dotted line AB is the altitude of the triangle.

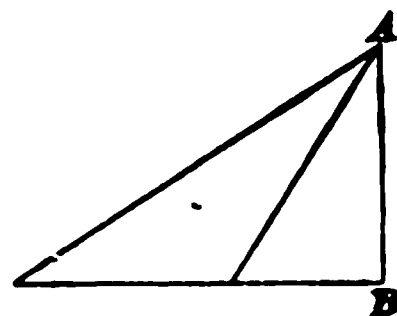


FIG. 23.

The **perimeter** of a triangle is the sum of the lengths of the three sides.

55. If in any parallelogram a straight line, called the **diagonal**, is drawn, connecting two opposite corners, the parallelogram is divided into two equal triangles, as DAB and DCB , Fig. 24. The area of each triangle, therefore, is equal to one-half the area of the parallelogram, or to one-half the product of the base and the altitude. Any side of a triangle may be taken as the base.

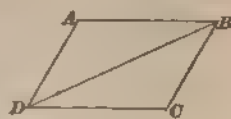


FIG. 24.

56. Rule. — *To find the area of a triangle, multiply the base by the altitude and divide the product by 2.*

EXAMPLE — The base of a triangle is 36 in. long and its altitude is 20½ in. What is the area of the triangle?

$$\text{SOLUTION. — Area} = \frac{\text{base} \times \text{altitude}}{2} = \frac{36 \times 20\frac{1}{2}}{2} = 369 \text{ sq in. Ans.}$$

57. Rule. — *To find the altitude or the base of a triangle, having given the area and the base or the altitude, multiply the area by 2, and divide by the given dimension.*

EXAMPLE — What must be the altitude of a triangular piece of sheet metal to contain 100 sq in., if the base is 10 in. long?

$$\text{SOLUTION} — 100 \times 2 = 200 \quad 200 \div 10 = 20 \text{ in. Ans.}$$

58. The following relations between the sides of a right-angled triangle are frequently useful:

1. *The square of the hypotenuse is equal to the sum of the squares of the two short sides.*

2. *The square of one of the short sides is equal to the difference between the square of the hypotenuse and the square of the other short side.*

59. These relations may be expressed by formulas.

Let a = one short side of the triangle;
 b = other short side of the triangle;
 c = hypotenuse of the triangle.

$$\begin{aligned} \text{Then,} \quad c^2 &= a^2 + b^2; \\ b^2 &= c^2 - a^2; \\ a^2 &= c^2 - b^2. \end{aligned}$$

Since the square of the hypotenuse is equal to the sum of the squares of the short sides, the hypotenuse itself must be equal to the *square root* of the sum of the squares of the short sides.

60. Rule.—*To find the hypotenuse of a right-angled triangle, square each of the short sides; add the squares together and extract the square root of the sum.*

Denoting the sides by the letters a , b , and c , as above, the rule may be expressed by the formula :

$$c = \sqrt{a^2 + b^2}.$$

EXAMPLE.—The short sides of a right-angled triangle are 6 ft. and 8 ft. long, respectively; what is the length of the hypotenuse?

SOLUTION.—In this case, $a = 6$ ft., and $b = 8$ ft. Using the formula,
hypotenuse $= c = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10$ ft. Ans.

61. Rule.—*To find one of the short sides of a right-angled triangle, subtract from the square of the hypotenuse the square of the given short side, and extract the square root of the difference.*

The rule may be expressed by the formula :

$$b = \sqrt{c^2 - a^2}.$$

EXAMPLE.—A ladder 40 feet long is so placed that its top just reaches the top of a house 32 feet from the ground. What is the distance of the foot of the ladder from the house?

SOLUTION.—The ladder, the side of the house, and the ground form three sides of a right-angled triangle, of which the ladder is the hypotenuse c , and the height of the house is the known short side a . Using the formula, the other short side is

$$b = \sqrt{40^2 - 32^2} = \sqrt{1,600 - 1,024} = \sqrt{576} = 24 \text{ ft.}$$

Therefore, the foot of the ladder is 24 feet from the house. Ans.

EXAMPLES FOR PRACTICE.

62. Solve the following examples:

1. Find the number of acres in a triangular field whose base is 184 rd. long, and altitude is 69 rd. Ans. 39.88—A.

2. The area of a triangle is 19.5 sq. in., and its base is 8 in. long. What is the altitude? Ans. $4\frac{1}{2}$ in.

3. The hypotenuse of a right-angled triangle is 13 in. long, and one of the short sides is 5 in. long. (1) Find the length of the other short side, and (2) the area of the triangle.

Ans. $\begin{cases} (a) & 12 \text{ in.} \\ (b) & 30 \text{ sq. in.} \end{cases}$

4. The lengths of the parallel sides of a trapezoid are 22 ft. and 19 ft., respectively, the distance between them is 47 ft. What is the area of the trapezoid?

Ans. 903½ sq. ft.

THE CIRCLE.

63. A circle is a figure bounded by a curved line, called the *circumference*, every point of which is equally distant from a point within, called the *center*. (Fig. 25.) The circumference of a circle is also called its *periphery*.



FIG. 25.

64. NOTE.—When a surface is bounded by straight lines, the length of the bounding line is called the *perimeter*; when the bounding line is a curve, the length of the curve is called the *periphery*. Thus, we speak of the perimeter of a polygon, and the periphery of a circle.

65. The *diameter* of a circle is a straight line passing through the center and terminated at both ends by the circumference. (See *AB*, Fig. 26.)



FIG. 26.

66. The *radius* of a circle is a straight line drawn from the center to the circumference. It is equal in length to one-half the diameter. The plural of radius is *radii*, and we say that all radii of a circle are equal. (*OA*, Fig. 27, is a radius.)

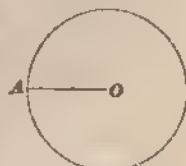


FIG. 27.

67. If a circle is divided by a diameter, each half is called a *semicircle*, and each half-circumference is called a *semi-circumference*.

68. It has been found that the length of the circumference of any circle divided by the length of the diameter

gives a constant number. This number is very nearly 3.1416; it is generally denoted by the Greek letter π (pronounced *pi*).

69. Rule.—*To find the circumference of a circle, multiply the diameter by 3.1416.*

Let C = circumference of circle;
 D = diameter of circle;
 R = radius of circle;
 π = 3.1416.

The above rule may be expressed by the formula,

$$C = \pi D = 3.1416 D.$$

EXAMPLE.—If a car wheel is 36 in. in diameter, what is its circumference?

SOLUTION.— $C = 3.1416 D = 3.1416 \times 36 = 113.0976$ in. Ans.

70. Rule.—*To find the diameter of a circle, divide the circumference by 3.1416.*

Formula: $D = \frac{C}{\pi} = \frac{C}{3.1416}.$

EXAMPLE.—The circumference of a tree is 10 ft. 4 in.; what is the diameter?

SOLUTION.— 10 ft. 4 in. = 124 in. Using the formula,

$$D = \frac{124}{3.1416} = 39.47 \text{ in. Ans.}$$

71. Rule.—*To find the area of a circle, multiply the square of the radius by 3.1416, or multiply the square of the diameter by .7854.*

Formulas: $A = \pi R^2 = 3.1416 R^2,$
 $A = \frac{1}{4} \pi D^2 = .7854 D^2,$

in which A denotes the area of the circle.

EXAMPLE.—If the diameter of a circular piston is 14 in., what is its area?

SOLUTION.—The radius is one-half the diameter (Art. 66), or 7 in.

Hence, $A = 3.1416 \times 7^2 = 3.1416 \times 49 = 153.9384$ sq. in.

or, $A = .7854 \times 14^2 = .7854 \times 196 = 153.9384$ sq. in. Ans.

72. Rule.—*To find the diameter of the circle, the area being given, divide the area by .7854 and extract the square root of the quotient.*

Formula:
$$D = \sqrt{\frac{A}{.7854}}$$

EXAMPLE.—What is the diameter of a circular field whose area is 1,296.64 sq. yd.?

SOLUTION. Substituting the known area in the formula,

$$D = \sqrt{\frac{1,296.64}{.7854}} = \sqrt{1,650.9} = 40.63 \text{ yd.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE.

73. Solve the following examples:

1. Find (a) the circumference and (b) area of a circle 84 ft. in diameter.

$$\text{Ans. } \begin{cases} (a) & 106.814 \text{ ft.} \\ (b) & 907.92 \text{ sq. ft.} \end{cases}$$

2. What is the area of a circle 4 ft. 6½ in. in diameter?

$$\text{Ans. } 2,832.834 \text{ sq. in.}$$

3. If the area of a circular sheet of metal is 130 square inches, what is (a) the diameter? (b) the circumference?

$$\text{Ans. } \begin{cases} (a) & 12.866 \text{ in.} \\ (b) & 40.42 \text{ in.} \end{cases}$$

4. (a) What must be the diameter in rods of a circular race track 1 mile in length? (b) What is the area of the field enclosed?

$$\text{Ans. } \begin{cases} (a) & 101.859 \text{ rd.} \\ (b) & 50.93 \text{ A.} \end{cases}$$

5. Find (a) the circumference and (b) the area of a locomotive driving wheel, the diameter of which is 5 ft. 6½ in.

$$\text{Ans. } \begin{cases} (a) & 208.916 \text{ in.} \\ (b) & 3,473.235 \text{ sq. in.} \end{cases}$$

THE PRISM AND CYLINDER.

74. A solid, or body, has three dimensions: length, breadth, and thickness. The sides that enclose it are called its faces, and the intersections of the sides are called the edges.

75. A prism is a solid whose ends are equal and parallel plane figures, and whose sides are parallelograms. Prisms

take their names from the form of their bases. Thus a triangular prism is one having a triangle for its base.

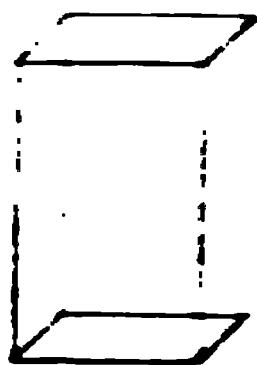


FIG. 26

76. A **parallelepiped** is a prism whose bases and sides are parallelograms. (Fig. 26.)

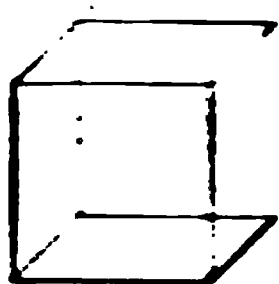


FIG. 27

77. A **cube** is a prism whose faces are equal squares. (Fig. 27.) All the faces of a cube are equal. A cube is also a parallelepiped.

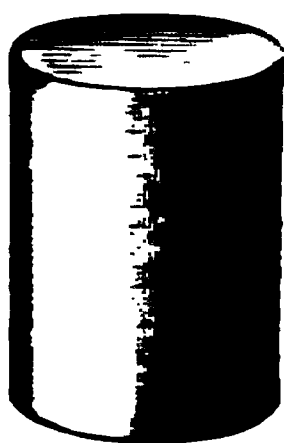


FIG. 28

78. A **cylinder** is a body of uniform thickness throughout its entire length, whose ends are equal parallel circles. (Fig. 28.)

79. The **altitude** of a prism or of a cylinder is the perpendicular distance between its bases.

80. A **right prism** is one whose sides are perpendicular to the bases.

81. A **right cylinder** is one in which the line joining the centers of the two circular bases is perpendicular to those bases.

82. In the case of plane figures we have dealt with perimeters and areas. In the case of solids we have to do with the areas of their outside surfaces and with their contents or volumes.

83. The **entire surface** of any solid is the area of the whole outside of the solid.

84. The **convex surface** of a solid is the same as the entire surface, except that in the case of prisms and cylinders the areas of the ends are not included.

85. Rule.—*To find the convex surface of a prism or cylinder, multiply the perimeter of the base by the altitude.*

EXAMPLE.—A block of marble is 24 in. long and its ends are 9 in. square. What is the area of its convex surface?

SOLUTION.— $9 \times 4 = 36$ in = the perimeter of the base; $36 \times 24 = 864$ sq. in, the convex area. Ans.

86. To find the entire area of the outside surface, add the areas of the two ends to the convex area. Thus, in the last example, the area of the two ends = $9 \times 9 \times 2 = 162$ sq. in.; $864 + 162 = 1,026$ sq. in.

EXAMPLE.—(a) What is the convex surface of a cylindrical tank with flat ends 23 ft. long and 4 ft. 6 in. in diameter? (b) What is the entire surface?

SOLUTION. Perimeter of end = $4\frac{1}{2} \times 3.1416 = 14.137$ ft.

(a) Convex surface $14.137 \times 23 = 325.151$ sq. ft. Ans.

Area of one end = $7854 \times (4\frac{1}{2})^2 = 15.904$ sq. ft.

(b) Entire surface = $325.151 + 2 \times 15.904 = 356.959$ sq. ft. Ans.

87. The **volume** of a solid is the quantity of space it occupies. As shown in Art. 14, § 4, the measuring unit is a cube whose edges are equal in length to a linear unit; it may be a cubic inch, cubic foot, cubic yard, or cubic meter. Fig. 31 represents a rectangular prism 4 ft. long, 3 ft. wide, and 2 ft. thick. Dividing the prism by lines, as shown, it is seen that there are four equal slices, each of which is made up of $2 \times 3 = 6$ cubes. In all, there are $4 \times 2 \times 3 = 24$ cubes, each containing 1 cubic foot; that is,

the volume of the prism is 24 cubic feet. It is seen that the number of cubes in each horizontal layer is just equal to the

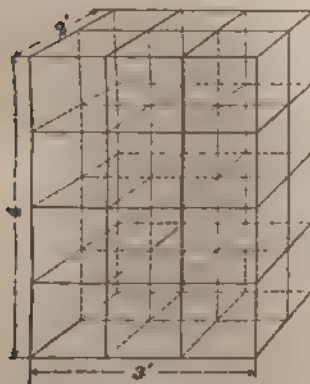


FIG. 31.

number of square feet in the base; and the number of layers is equal to the number of feet in the altitude. The same reasoning holds true for a prism with triangular base, or for a cylinder.

88. Rule.—*To find the volume of a prism or cylinder, multiply the area of the base by the altitude.*

In applying this rule, all dimensions must have the same unit.

EXAMPLE. A packing box is $4\frac{1}{2}$ ft. long, 4 ft. wide, and $3\frac{1}{2}$ ft. deep. What is its volume?

SOLUTION.—Area of base $= 4\frac{1}{2} \times 4 = 18$ sq. ft. Altitude $= 3\frac{1}{2}$ ft.
Volume or cubical contents $= 18 \times 3\frac{1}{2} = 59\frac{1}{2}$ cu. ft. Ans.

EXAMPLE. (a) How many cubic feet of water can be run into a circular cistern 8 ft. in diameter and 10 ft. deep? (b) how many gallons?

SOLUTION. (a) The problem is to find the volume of a cylinder whose altitude is 10 ft., and whose bases are 8 ft. in diameter.

$$\text{Area of base} = 7854 \div 8^2 = 50.265 \text{ sq. ft.}$$

$$\text{Volume} = 50.265 \times 10 = 502.65 \text{ cu. ft. Ans.}$$

(b) According to Art. 21, § 4, 1 gallon contains 231 cu. in. Hence,
the cistern can hold $\frac{502.65 \times 1728}{231} = 3,760$ gal., very nearly. Ans.

89. The dimensions of a rectangular solid are spoken of as length, breadth, and thickness. According to Art. 87, the volume of the solid is the product of these three dimensions.

EXAMPLE. A brick is 8 in. long, 4 in. wide, and 2 in. thick; what is its volume?

SOLUTION. Volume $=$ length \times breadth \times thickness $= 8 \times 4 \times 2 = 64$ cu. in. Ans.

MASONRY.

90. In estimating the cubical contents of stone walls, the perch of $24\frac{3}{4}$ cubic feet is used. As stated in Art. 16, § 4, the perch is often assumed to be 25 cubic feet.

91. Rule.—*To find the number of perches of masonry in a wall, divide the volume of the wall in cubic feet by $24\frac{3}{4}$.*

EXAMPLE.—How many perches in a wall 8 rd. long, $4\frac{1}{2}$ ft. high, and 2 ft. thick?

SOLUTION.—Length of wall $= 8 \times 16\frac{1}{2} = 132$ ft. Cubical contents of wall $= 132 \times 4\frac{1}{2} \times 2 = 1,188$ cu. ft. Number of perches $= 1,188 \div 24\frac{3}{4} = 48$. Ans.

92. In estimating the contents of stone foundations for buildings, the length of the wall is measured on the outside,

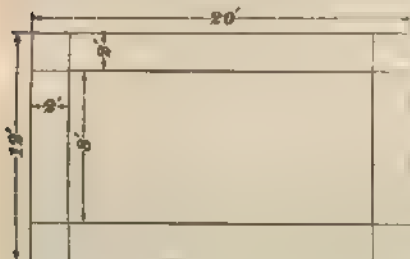


FIG. 32.

thus counting each corner twice. This is illustrated in Fig. 32. If a wall 2 feet thick measures $12' \times 20'$ on the outside, and we assume that the corners are parts of the longer sides, we have two walls each 20 feet long, and two walls each 8 feet long. The actual length is, therefore, $2 \times 20 + 2 \times 8 = 56$ ft. The length estimated on the outside is $2 \times 20 + 2 \times 12 = 64$ ft. To find the actual length of such a wall, subtract four times the thickness of the wall from the length measured on the outside. Thus, in the above case, actual length $= 64 - 4 \times 2 = 56$ ft.

Usually, masons make no allowance for windows or doors in estimating their work. In estimating the quantity of stone required for the wall, such allowance should be made.

EXAMPLE.—(a) How many perches of stone are required to build the walls of a church 60 ft long by 32 ft wide the walls being 24 ft. high and $2\frac{1}{2}$ ft. thick? There are 8 windows each 5 ft wide and 11 ft high, and 2 doors, each 6 ft wide and 9 ft. high. (b) What is the cost of laying the walls at \$3.50 per perch?

SOLUTION.—

Length of wall (outside) $= 2 \times 60 + 2 \times 32 = 184$

Actual length $= 184 - 4 \times 2\frac{1}{2} = 175$ feet.

Actual cubical contents $= 175 \times 24 \times 2\frac{1}{2} = 9,450$ cu. ft.

Allowance for windows $= 5 \times 11 \times 2\frac{1}{2} \times 8 = 900$ cu. ft.

Allowance for doors $= 6 \times 9 \times 2\frac{1}{2} \times 2 = 249$ cu. ft.

Net contents $= 9,450 - (900 + 249) = 8,217$ cu. ft.

(a) Perches required for wall $= 8,217 \div 24\frac{3}{4} = 332$ Ans.

(b) Since in estimating the cost of the work, no allowance is made for corners, doors, and windows,

Cubical contents = $184 \times 24 \times 2\frac{1}{4} = 9,936$ cu. ft.

Perches of stonework = $9,936 \div 24\frac{3}{4} = 401\frac{5}{11}$.

Cost of laying walls = $401\frac{5}{11} \times \$3.50 = \$1,405.09$. Ans.

93. In brickwork, the unit of measurement is one thousand (M) bricks. The dimensions of an ordinary brick are $8'' \times 4'' \times 2''$. In some localities they are made smaller, in others, larger. To allow for mortar, $\frac{1}{4}$ inch is added to the length and to the thickness in making calculations. On this assumption, the ordinary brick with its mortar has a volume of $8\frac{1}{4} \times 4 \times 2\frac{1}{4} = 74\frac{1}{4}$ cubic inches. Since a cubic foot contains 1,728 cubic inches, it takes $1,728 \div 74\frac{1}{4} = 23\frac{3}{11}$ bricks to make a cubic foot of wall.

94. Rule.—*To find the number of ordinary bricks in a wall, multiply its volume in cubic feet by $23\frac{3}{11}$.*

EXAMPLE.—How many bricks are required in a wall 80 ft. long, $16\frac{1}{2}$ ft. high, and 4 ft. thick?

SOLUTION.—

Volume of wall = $80 \times 16\frac{1}{2} \times 4 = 5,280$ cu. ft.

Number of bricks = $5,280 \times 23\frac{3}{11} = 122,880$ or 122.88 M. Ans.

95. In estimating brickwork, it is customary in most localities to use the outside, or gross, length of the wall, and to allow for doors and windows. The practice, however, is not uniform, and in some cases no allowance is made for corners or openings.

EXAMPLE.—What will be the cost of erecting the walls of a building 64 ft. long and 40 ft. wide, the wall being 36 ft. high and 3 bricks (= 1 ft.) thick? Allowance is to be made for 40 windows, each 6 ft. \times 2 ft. 9 in., and 8 doors, each 8 ft. \times 3 ft. 6 in. The bricks cost \$5.75 per M, and the laying costs \$1.40 per M.

SOLUTION.—Outside length of wall = $64 \times 2 + 40 \times 2 = 208$ ft.

Net length of wall = $208 - 4 \times 1 = 204$ ft.

Contents of wall = $204 \times 36 \times 1 = 7,344$ cu. ft.

Deduction for windows = $6 \times 2\frac{3}{4} \times 1 \times 40 = 660$ cu. ft.

Deduction for doors = $8 \times 3\frac{1}{2} \times 1 \times 8 = 224$ cu. ft.

Net contents of wall = $7,344 - (660 + 224) = 6,460$ cu. ft.

Number of bricks = $6,460 \times 23\frac{3}{11} = 150,342 = 150.342$ M.

Cost of bricks = $150.342 \times \$5.75 = \864.47 .

Cost of laying = $150.342 \times \$1.40 = \210.48 .

Total cost of erecting walls = $\$864.47 + \$210.48 = \$1,074.94$. Ans.

EXAMPLES FOR PRACTICE.

96. Solve the following examples:

1. Find the cost of building a stone wall to enclose a rectangular yard 100 ft. long and 108 ft. wide. The wall is 9 ft. high and 2 ft. 6 in. thick, and the price of laying is \$2.25 per perch. Ans. \$1,096 38.

2. How many thousand bricks are required for a house 18 ft. wide, 38 ft. long and 32 ft. high, walls 3 bricks thick, making allowance for 8 doors, each 3 ft. 4 in. by 7 ft. 6 in., and 16 windows, each 3 ft. by 6 ft. ? Ans. 71,983 M.

3. Philadelphia bricks are $8\frac{1}{4}" \times 4\frac{1}{4}" \times 2\frac{1}{4}"$. Allowing $\frac{1}{4}$ in. on length and thickness for mortar, how many of these bricks are required to make a cubic foot ? Ans. 184, nearly.

BINS, CISTERNS, ETC.

97. It is frequently necessary to estimate the capacity of a bin, box, or vessel, in bushels, barrels, or gallons. The volume of the bin or vessel in cubic feet or cubic inches is divided by the number of cubic feet or cubic inches in a bushel, barrel, or gallon, as the case may be.

EXAMPLE. How many bushels of wheat can be put into a bin 35 ft. long, 6 ft. wide, and 8 ft. high ?

SOLUTION. Cubical contents of the bin = $35 \times 6 \times 8 = 1,680$ cu. ft.
 $= 1,680 \times 1,728 = 2,903,040$ cu. in. One bushel contains 2,150.42 cu. in., (Art. 24, § 4). Number of bushels = $2,903,040 \div 2,150.42 = 1,350$ bu., nearly. Ans.

98. For convenience of reference, the following table of capacities is given:

TABLE I. DRY MEASURE.

1 heaped bushel	$2,747.71$ cu. in. $= 1.50$ cu. ft., nearly.
1 stricken bushel	$2,150.42$ cu. in. $= 1.25$ cu. ft., nearly.
1 peck.	537.6 cu. in.
1 quart	67.2 cu. in.
1 pint	33.6 cu. in.

LIQUID MEASURE.

1 hogshead	$= 8.422$ cu. ft.
1 barrel . .	$= 4.211$ cu. ft.
1 gallon . .	$= .231$ cu. in.
1 quart . . .	57.75 cu. in.
1 pint . . .	28.875 cu. in.

99. Rule. *To find the capacity of a bin or other vessel in dry measure or in liquid measure, divide the volume of the bin or vessel in cubic inches by the number of cubic inches in the unit of measure.*

EXAMPLE.—How many liquid quarts are contained in a rectangular pail $8'' \times 5'' \times 4'$?

SOLUTION. Volume of pail = $8 \times 5 \times 4 = 160$ cu. in. In one liquid quart there are 57.75 cu. in. Hence, the capacity of the pail is $160 \div 57.75 = 2.77$ quarts. Ans.

EXAMPLE. How many gallons in a milk can 16 inches in diameter and 30 inches high?

SOLUTION.—Volume of can = area of base \times altitude = $.7854 \times 16^2 \times 30 = 6,031.8$ cu. in. A gallon contains 231 cu. in. Number of gallons = $6,031.8 \div 231 = 26.11$ gal. Ans.

100. The following table of *approximate* capacities is very convenient in rough calculations:

TABLE II.	
1 cu. ft.	63 of a heaped bushel.
1 cu. ft. = .8	of a stricken bushel.
1 cu. ft. = 7.5	liquid gallons.
1 cu. ft. = $\frac{1}{16}$	of a barrel.

The following short rules are approximate, but the results are sufficiently accurate for all practical purposes.

101. Rule.—*To find the capacity of a bin in heaped bushels, multiply the volume in cubic feet by .63.*

102. Rule. *To find the capacity of a bin in stricken bushels, multiply the volume in cubic feet by .8.*

EXAMPLE.—(a) How many stricken bushels in a bin $18' \times 13' \times 7'$?
(b) How many heaped bushels in the same bin?

SOLUTION.—Volume = $18 \times 13 \times 7 = 1,638$ cubic feet

(a) Stricken bushels = $1,638 \times .8 = 1,310.4$ bu. Ans.

(b) Heaped bushels = $1,638 \times .63 = 1,031.94$ bu. Ans.

103. Rule. *To find the number of gallons in a cistern or other vessel, multiply the volume in cubic feet by 7.5.*

104. Rule.—*To find the number of barrels in a cistern, multiply the volume in cubic feet by $\frac{1}{16}$.*

EXAMPLE.—A rectangular cistern 9 ft. 6 in. long, 6 ft. wide, and 4 ft. deep contains (a) how many gallons? (b) how many barrels?

SOLUTION.—Volume of cistern = $9\frac{1}{2} \times 6 \times 4 = 228$ cubic feet.

$$(a) \quad 228 \times 7\frac{1}{2} = 1,710 \text{ gal.} \quad \text{Ans.}$$

$$(b) \quad 228 \times \frac{1}{8} = 54.15 \text{ bbl.} \quad \text{Ans.}$$

105. Rule. *To find the number of gallons in a cylindrical vessel, multiply the square of the diameter in inches by the height in inches, and that product by .0034.*

EXAMPLE.—An oil tank 7 ft. 6 in. long and 24 in. in diameter contains how many gallons?

SOLUTION.—7 ft 6 in. = 90 in. Capacity = $24^2 \times 90 \times .0034 = 176\frac{1}{2}$ gal. Ans.

EXAMPLES FOR PRACTICE.

106. Solve the following examples by the exact methods

1. A wagon body is 14 ft long, 4 ft wide, and 24 in. deep. How many bushels of shelled corn will it hold? Ans. 90 bu.

2. A rectangular can is 30" \times 16" \times 11 $\frac{1}{2}$ " How many more liquid quarts than dry quarts will it hold? Ans. 13.44 liquid quarts.

3. How many barrels are contained in a cylindrical cistern 9 ft. deep and 8 ft. 6 in. in diameter? Ans. 121.28 bbl.

4. A tin cup is 4 in. in diameter and 5 $\frac{1}{2}$ in. deep. (a) How many liquid pints will it hold? (b) how many dry pints?

$$\text{Ans. } \begin{cases} (a) & 2.394 \text{ pt.} \\ (b) & 2.057 \text{ pt.} \end{cases}$$

5. How many dry pecks can be put into a hoghead?

$$\text{Ans. } 27.07 \text{ pk.}$$

Solve the following examples by the approximate rules.

6. A box which holds exactly 14 stricken bushels will hold how many liquid gallons? Ans. 131.25 gal.

7. How many bushels of wheat in a bin 21' \times 6 $\frac{1}{2}$ ' \times 4 $\frac{1}{2}$ '?

$$\text{Ans. } 491\frac{1}{2} \text{ bu.}$$

8. How many barrels are contained in a cistern 11 ft 4 in. deep and 7 ft 6 in. in diameter? Ans. 118.91 bbl.

9. How many heaped bushels of potatoes are contained in a bin 30' \times 18' \times 7 $\frac{1}{2}$ '? Ans. 2,551.5 bu.

10. How many gallons of water can be pumped into a cylindrical stand pipe 12 ft in diameter and 80 ft high? Ans. 67,859 gal.

11. A cubical box holds 100 bushels of wheat. What are its dimensions? Ans. 5' \times 5' \times 5'.

COAL AND HAY.

107. A ton (2,000 lb.) of Lehigh coal, egg size, measures $34\frac{1}{2}$ cubic feet in the bin.

A ton of Schuylkill coal, egg size, measures 35 cubic feet.

A ton of pink gray and red ash coal, egg size, measures 36 cubic feet.

A ton of Wyoming coal, egg size, measures 31 cubic feet.

The bulk of a ton of hay is dependent upon the pressure to which it is subjected. Roughly speaking, a ton of hay lying unpressed measures 500 cubic feet; when in a small stack, 400 cubic feet, and in mows compressed with grain, or in well settled stacks, 300 cubic feet.

EXAMPLES FOR PRACTICE.

108. 1. How many tons of hay are contained in a well compressed mow $30' \times 18' \times 15'$? Ans. 27 tons.

2. How many tons of Lehigh coal will fill a bin 17 ft. long, 13 ft. wide, and 8 ft. high? Ans. 51.2 tons.

3. How many tons of Wyoming coal will fill a car 32 ft. long, $8\frac{1}{2}$ ft. wide, and 4 ft. deep? Ans. 26.84 tons.

THE PYRAMID AND CONE.



FIG. 33.

109. A pyramid is a solid whose base is a plane figure, and whose sides are triangles uniting at a common point, called the **vertex**. (Fig. 33.) If a straight line be drawn on one of the sides of a pyramid from the vertex so as to be perpendicular to one edge of the base, this line is called the **slant height**.



FIG. 34.

110. A cone is a solid whose base is a circle, and whose convex surface tapers uniformly to a point called the **vertex**. (Fig. 34.) If a straight line be drawn on the cone from the vertex to the edge of the base, this line is called the **slant height**.

111. The **altitude** of a pyramid or a cone is the perpendicular distance from the vertex to the base. (See *AB*, Figs. 33 and 34.)

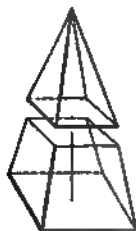


FIG. 35.

112. If a pyramid be cut by a plane parallel to the base, so as to form two parts, the lower part is called the **frustum** of the pyramid. (Fig. 35.)

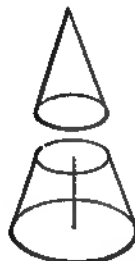


FIG. 36.

113. If a cone be cut in a similar manner, the lower part is called the **frustum** of the cone. (Fig. 36.)

114. The upper end of the frustum of a pyramid or a cylinder is called the **upper base**, and the lower end, the **lower base**. The **altitude** of a frustum is the perpendicular distance between the bases.

115. The **entire area** of a pyramid or cone is the area of the whole outside surface. The **convex area** is the *entire*

of a right-angled triangle whose altitude is the same as the altitude of the cone (18 in.) and whose base is equal to the radius of the base of the cone ($\frac{1}{2} \times 14 = 7$ in.). Hence, slant height $= \sqrt{18^2 + 7^2} = 19.3132$ in.

Therefore, applying rule, convex area $= 43.9824 \times \frac{19.3132}{2} = 424.72$ sq. in. Ans.

(b) Area of base $= 14^2 \times .7854 = 153.94$ sq. in. Hence, entire area $= 424.72 + 153.94 = 578.66$ sq. in. Ans.

117. Rule.—*To find the volume of a pyramid or a cone, multiply the area of the base by one-third the altitude.* ✓

EXAMPLE.—What is the volume of a cone whose altitude is 18 in., and whose base is 14 in. in diameter.

SOLUTION.—Area of the base $= 14^2 \times .7854 = 153.94$ sq. in. Hence, the volume $= 153.94 \times \frac{18}{3} = 923.64$ cu. in. Ans.

EXAMPLE.—Find the volume of a pyramid whose base is a square, measuring 15 in. on a side, and whose altitude is $16\frac{1}{2}$ in.

SOLUTION.—Area of base $= 15^2 = 225$ sq. in. Ans.

Volume $= 225 \times \frac{16\frac{1}{2}}{3} = 1,237\frac{1}{2}$ cu. in. Ans.

118. Rule.—I. *The convex area of a frustum of a pyramid or cone is equal to the sum of the perimeters of the bases multiplied by one-half the slant height.*

II. *The entire area equals the convex area plus the areas of the bases.*

EXAMPLE.—What is (a) the convex and (b) the entire area of a frustum of a square pyramid whose slant height is 22 in., one of the edges of the upper base being 6 in. long and of the lower base 14 in. long?

SOLUTION.—(a) If one of the edges of the upper base is $6 \times 4 = 24$ in., and the perimeter of the lower base is $14 \times 4 = 56$ in., perimeter of upper base is $6 \times 4 = 24$ in., and of the lower base, $14 \times 4 = 56$ in.

Applying rule, $(24 + 56) \times \frac{22}{2} = 880$ sq. in. Ans.

(b) Area of upper base is $6 \times 6 = 36$ sq. in., and of the lower base, $14 \times 14 = 196$ sq. in. Since the entire area equals the convex area plus the area of bases, the entire area is $880 + 36 + 196 = 1,112$ sq. in. Ans.

119. Rule.—*To find the volume of the frustum of a pyramid or a cone, add together the areas of the upper and*

lower bases, and the square root of the product of the two areas; multiply this sum by one third the altitude.

EXAMPLE —Given a frustum of a square pyramid, each edge of the lower base measures 12 in., each edge of the upper base measures 5 in., and its altitude is 16 in., what is its volume?

SOLUTION —Area of upper base $= 5 \times 5 = 25$ sq. in., area of lower base $= 12 \times 12 = 144$ sq. in., the square root of the product of the areas of the two bases $= \sqrt{25 \times 144} = 60$. Adding these three results, and multiplying by one-third the altitude, $25 + 144 + 60 = 229$; $229 \times \frac{16}{3} = 1,221\frac{1}{3}$ cu. in. = the volume. **Ans.**

EXAMPLE. How many gallons of water will a circular tank hold that is 4 ft. in diameter at the top, 5 ft. in diameter at the bottom, and is 8 ft. deep?

SOLUTION —There are 231 cu. in. in a gallon, and the volume of the tank should be found in cubic inches. The tank is in the shape of a frustum of a cone. The diameter of the upper base $= 4 \times 12 = 48$ in. the diameter of the lower base $= 5 \times 12 = 60$ in., and the depth $= 8 \times 12 = 96$ in. Area of upper base $= 48^2 \times .7854 = 1,809.56$ sq. in.; area of lower base $= 60^2 \times .7854 = 2,827.44$ sq. in.;

$$\frac{1}{3} (1,809.56 + 2,827.44 + 2,261.95) = 6,898.95;$$

Whence, $1,809.56 + 2,827.44 + 2,261.95 = 6,898.95$;
 $6,898.95 \times \frac{96}{3} = 220,766.4$ cu. in. = contents. Now since there are 231 cu. in. in 1 gallon, the tank will hold $220,766.4 \div 231 = 955.7$ gallons, nearly. **Ans.**

THE SPHERE.



FIG. 37.

120. A sphere is a solid bounded by a uniformly curved surface, every point of which is equally distant from a point within, called the center. (Fig. 37.)

The word *ball* is often used instead of sphere.

121. The diameter of a sphere is a straight line passing through its center, the ends of which terminate at the surface.

122. The radius of a sphere is a line drawn from the center to the surface.

123. Rule.—*To find the area of the surface of a sphere, square the diameter and multiply the result by 3.1416.*

Let A = area of surface of sphere;

V = volume of sphere;

D = diameter of sphere;

Then, $A = \pi D^2 = 3.1416 D^2$.

EXAMPLE.—What is the area of the surface of a sphere whose diameter is 14 in.?

SOLUTION.—Area = $3.1416 D^2 = 3.1416 \times 14^2 = 615.75$ sq. in. Ans.

From this it will be seen that the surface of a sphere equals the circumference of a great circle multiplied by the diameter, a rule often used.

124. Rule.—*To find the volume of a sphere, cube the diameter and multiply the result by .5236.*

Formula, $V = \frac{1}{6} \pi D^3 = .5236 D^3$.

EXAMPLE.—What is the weight of a lead ball 12 in. in diameter, a cubic inch of lead weighing .41 lb.?

SOLUTION.—Volume = $.5236 D^3 = .5236 \times 12^3 = 904.78$ cu. in.

Weight of ball = $904.78 \times .41 = 370.96$ lb. Ans.

EXAMPLES FOR PRACTICE.

125. Solve the following:

1. What is the volume of a ball 6 in. in diameter? (b) What is the surface of the ball?

Ans. $\begin{cases} (a) & 113.098 \text{ cu. in.} \\ (b) & 113.098 \text{ sq. in.} \end{cases}$

2. Find the volume of a cone whose altitude is 12 in., and the circumference of whose base is 31.416 in. Ans. 314.16 cu. in.

NOTE.—Find the diameter of the base, and then its area.

3. A conical vessel is 22 in. deep and 3 ft. in diameter. How many gallons are required to fill it? Ans. 32.31 gal.

4. Find the volume of a log of wood 18 ft. long and 32 in. in diameter. Ans. 100.53 cu. ft.

5. The diameter of the earth is 7,918 mi. Assuming it to be a true sphere, what is its volume in cubic miles?

Ans. 259,923,849,377.3152 cu. mi.

6. The frustum of a triangular pyramid has an altitude of 15 ft. The lower base is a right-angled triangle whose short sides are 8 ft. and 15 ft., the upper base is another right-angled triangle whose short sides are 6 ft. and 11 ft. 3 in. Find the volume of the frustum.

Ans. 693.75 cu. ft.

7. A cubic inch of cast iron weighs 26 of a pound. What is the weight of a cast-iron cannon ball 8 in. in diameter? Ans. 69.7 lb.

8. How many square feet of copper are required to cover a ball 8 ft. 4 in. in diameter? Ans. 34.9 sq. ft.

9. (a) How many cubic inches in a pan 10 in. across the top, 8 in. across the bottom, and 4 in. deep? (b) How many liquid quarts will the pan hold?

Ans. (a) 255 52 cu. in.
(b) 4 424 qt.

NOTE.—The pan is a frustum of a cone.

GAUGING OF CASKS.

126. A cask resembles two frustums of cones with their larger bases placed together.

The **bung diameter** of a cask is the diameter measured half way between the two ends; it is usually the greatest diameter.

The **mean diameter** of a cask is the mean between the bung diameter and head diameter. To find the mean diameter, add together the head diameter and bung diameter and divide the sum by 2.

127. Rule. To find the number of gallons in a cask, multiply the square of the mean diameter in inches by the length in inches, and that product by .0034.

To find the number of liters in the cask, multiply by .0129 instead of .0034. If the cask is partly filled, stand it on end, find the mean diameter of the part filled, multiply its square by the height, and that product by .0034.

EXAMPLE.—The diameter of a cask is 27 in. at the head, 33 in. at the bung, and the cask is 3 ft. long; how many gallons will it hold?

SOLUTION.—Mean diameter = $\frac{27 + 33}{2}$ = 30 in. Length = 3 ft.
= 36 in. Capacity = $30^2 \times 36 \times .0034$ = 110 16 gal. Ans.

EXAMPLES FOR PRACTICE.**128.** Solve the following:

1. (a) What is the capacity in gallons of a cask whose length is 32 in., bung diameter 25 in., and head diameter 19 in.? (b) What is the capacity in liters?

Ans. $\begin{cases} (a) & 52.66 \text{ gal.} \\ (b) & 199.8 \text{ liters.} \end{cases}$

2. The height of the liquid in a cask is 14 in.; the head diameter is 20 in., and the diameter at the level of the liquid is 24 in. How many gallons are in the cask?

Ans. 23.04 gal.

3. A cask has the following dimensions: head diameter, 16 in.; bung diameter, 21 in.; length, 34 in. If the cask contains 23 gal., how many more gallons may be put into it?

Ans. 16.564 gal.



ARITHMETIC.

RATIO.

1. Suppose that it is desired to compare two numbers, say 20 and 4. If we wish to know how many times larger 20 is than 4, we divide 20 by 4 and obtain 5 for the quotient; thus, $20 \div 4 = 5$. Hence, we say that 20 is 5 times as large as 4, i. e., 20 contains 5 times as many units as 4. Again, suppose we desire to know what part of 20 is 4. We then divide 4 by 20 and obtain $\frac{1}{5}$; thus, $4 \div 20 = \frac{1}{5}$, or .2. Hence, 4 is $\frac{1}{5}$, or .2, of 20. This operation of comparing two numbers is termed *finding the ratio* of the two numbers. Ratio, then, is a comparison. It is evident that the two numbers to be compared must be expressed in the same unit; in other words, the two numbers must both be abstract numbers or concrete numbers of the same kind. For example, it would be absurd to compare 20 horses with 4 birds, or 20 horses with 4. Hence, **ratio** may be defined as a comparison between two numbers of the same kind.

2. A ratio may be *expressed* in three ways; thus, if it is desired to compare 20 and 4, and express this comparison as a ratio, it may be done as follows: $20 \div 4$, $20 : 4$, or $\frac{20}{4}$. All three are read *the ratio of 20 to 4*. The ratio of 4 to 20 would be expressed thus: $4 \div 20$, $4 : 20$, or $\frac{4}{20}$. The first method of expressing a ratio, although correct, is seldom or never used; the second form is the one most often met with, while the third is rapidly growing in favor, and is likely to supersede

the second. The third form, called the fractional form, is preferred by modern mathematicians, and possesses great advantages to students of algebra and of higher mathematical subjects. The second form seems to be better adapted to arithmetical subjects, and is the one we shall ordinarily adopt. There is still another way of expressing a ratio, though seldom or never used in the case of a simple ratio like that given above. Instead of the colon, a straight vertical line is used; thus, $20 \mid 4$.

3. The terms of a ratio are the two numbers to be compared; thus, in the above ratio, 20 and 4 are the terms. When both terms are considered together, they are called a **couplet**; when considered separately, the first term is called the **antecedent**, and the second term, the **consequent**. Thus, in the ratio $20 : 4$, 20 and 4 form a couplet, and 20 is the antecedent and 4 the consequent.

When a ratio is expressed in the fractional form, the antecedent becomes the numerator and the consequent the denominator. Thus, the ratio of \$35 to \$7 is written \$35 : \$7, or $\frac{\$35}{\$7}$, \$35 and \$7 being in both cases the antecedent and consequent, respectively.

4. A ratio may be **direct** or **inverse**. The *direct ratio* of 20 to 4 is $20 : 4$, while the *inverse ratio* of 20 to 4 is $4 : 20$. The direct ratio of 4 to 20 is $4 : 20$, and the inverse ratio is $20 : 4$. An inverse ratio is sometimes called a **reciprocal ratio**. The **reciprocal** of a number is 1 divided by the number.

Thus, the reciprocal of 17 is $\frac{1}{17}$; of $\frac{3}{4}$ is $1 \div \frac{3}{4} = \frac{4}{3}$; i. e., the reciprocal of a fraction is the fraction inverted. The inverse ratio of 20 to 4 may be expressed as $4 : 20$, or as $\frac{1}{20} : \frac{1}{4}$. The two ratios have equal values; for,

$$4 \div 20 = \frac{1}{5}, \text{ and } \frac{1}{20} : \frac{1}{4} = \frac{1}{20} \times \frac{4}{1} = \frac{1}{5}.$$

5. The term **vary** implies a ratio. When we say that two numbers vary as some other two numbers, we mean that

the relation between the first two numbers is the same as the relation between the other two numbers.

6. The value of a ratio is the result obtained by performing the division indicated. Thus, the value of the ratio 20 : 4 is 5; it is the quotient obtained by dividing the antecedent by the consequent.

7. By expressing the ratio in the fractional form, for example, the ratio of 20 to 4 as $\frac{20}{4}$, it is easy to see, from the laws of fractions, that if both terms be multiplied or both divided by the same number, it will not alter the value of the ratio. Thus,

$$\frac{20}{4} = \frac{20 \times 5}{4 \times 5} = \frac{100}{20}; \text{ and } \frac{20}{4} = \frac{20 \div 4}{4 \div 4} = \frac{5}{1}.$$

8. It is also evident, from the laws of fractions, that multiplying the antecedent or dividing the consequent multiplies the ratio, and dividing the antecedent or multiplying the consequent divides the ratio.

9. When a ratio is expressed in words, as the ratio of 20 to 4, the first number named is always regarded as the antecedent and the second as the consequent, without regard to whether the ratio itself is direct or inverse. *When not otherwise specified, all ratios are understood to be direct.* To express an inverse ratio, the simplest way of doing it is to express it as if it were a direct ratio, with the first number named as the antecedent, and then transpose the antecedent to the place occupied by the consequent and the consequent to the place occupied by the antecedent; or, if the ratio is expressed in the fractional form, invert the fraction. Thus, to express the inverse ratio of 20 to 4, first write it 20 : 4, and then transpose the terms, as 4 : 20; or as $\frac{20}{4}$, and then invert, as $\frac{4}{20}$. Or, the reciprocals of the numbers may be taken, as explained above. To invert a ratio is to transpose its terms.

EXAMPLES FOR PRACTICE.

10. What is the value of the following?

- (a) The ratio of 98 to 49?
 (b) The ratio of \$45 to \$9?
 (c) The ratio of $6\frac{1}{2}$ to $\frac{1}{2}$?
 (d) The ratio of 35 to 45?
 (e) The inverse ratio of 76 to 19?
 (f) The inverse ratio of 49 to 98?
 (g) The inverse ratio of 18 to 24?
 (h) The inverse ratio of 9 to 15?
 (i) The ratio of 10 to 3, multiplied by 3?
 (j) The ratio of 35 to 49, multiplied by 7?
 (k) The ratio of 18 to 64, divided by 9?
 (l) The ratio of 14 to 28, divided by 5?
 (m) 111 gal. : 37 gal. = ?

Ans.	(a)	2.
	(b)	5.
	(c)	12 $\frac{1}{2}$.
	(d)	$\frac{7}{9}$.
	(e)	$\frac{1}{4}$.
	(f)	2.
	(g)	$\frac{1}{3}$.
	(h)	$\frac{1}{5}$.
	(i)	10.
	(j)	5.
	(k)	$\frac{2}{3}$.
	(l)	$\frac{1}{5}$.
	(m)	3.

11. Instead of expressing the value of a ratio by a single number as above, it is customary to express it by means of another ratio in which the consequent is 1. Thus, suppose that it is desired to find the ratio of the weights of two pieces of iron, one weighing 45 pounds and the other weighing 30 pounds. The ratio of the heavier to the lighter is then 45 : 30, an inconvenient expression. Using the fractional form, we have $\frac{45}{30}$. Dividing both terms by 30, the consequent, we obtain $\frac{1\frac{1}{2}}{1}$, or $1\frac{1}{2} : 1$. This is the same result as obtained above, for $1\frac{1}{2} \div 1 = 1\frac{1}{2}$, and $45 \div 30 = 1\frac{1}{2}$.

12. A ratio may be squared, cubed, or raised to any power, or any root of it may be taken. Thus, if the ratio of two numbers is 105 : 63, and it is desired to cube this ratio, the cube may be expressed as $105^3 : 63^3$. That this is correct is readily seen; for, expressing the ratio in the fractional form, it becomes $\frac{105}{63}$, and the cube is $\left(\frac{105}{63}\right)^3 = \frac{105^3}{63^3} = 105^3 : 63^3$. Also, if it is desired to extract the cube root of the ratio $105^3 : 63^3$ it may be done by simply dividing the exponents

by 3, obtaining $105 : 63$. This may be proved in the same way as in the case of cubing the ratio. Thus, $105^3 : 63^3 = \left(\frac{105}{63}\right)^3$,

$$\text{and } \sqrt[3]{\left(\frac{105}{63}\right)^3} = \frac{105}{63} = 105 : 63.$$

NOTE.—The root of a fraction may be found in two ways; either by reducing the fraction to a decimal and then extracting the root, or by extracting the root of both the numerator and the denominator. Thus,

$$\sqrt[4]{\frac{49}{64}} = \sqrt[4]{.765625} = .875; \text{ also, } \sqrt[4]{\frac{49}{64}} = \frac{\sqrt[4]{49}}{\sqrt[4]{64}} = \frac{7}{8} = .875.$$

13. Since $\left(\frac{105}{63}\right)^3 = \left(\frac{5}{3}\right)^3$, it follows that $105^3 : 63^3 = 5^3 : 3^3$ (this expression is read: the ratio of 105 cubed to 63 cubed equals the ratio of 5 cubed to 3 cubed), it follows that the antecedent and consequent may always be multiplied or divided by the same number, irrespective of any indicated powers or roots, without altering the value of the ratio. Thus, $24^2 : 18^2 = 4^2 : 3^2$. For, performing the operations indicated by the exponents, $24^2 = 576$ and $18^2 = 324$. Hence, $576 : 324 = 1\frac{7}{9}$, or $1\frac{7}{9} : 1$. Also, $4^2 = 16$ and $3^2 = 9$; hence, $16 : 9 = 1\frac{7}{9}$ or $1\frac{7}{9} : 1$, the same result as before. Also, $24^2 : 18^2 = \frac{24^2}{18^2} = \left(\frac{24}{18}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{4^2}{3^2} = 4^2 : 3^2$.

The case of equal roots of a ratio may be proved for roots in a similar manner. Thus, $\sqrt[3]{24^3} : \sqrt[3]{18^3} = \sqrt[3]{4^3} : \sqrt[3]{3^3}$. For the $\sqrt[3]{24^3} = 24$, and $\sqrt[3]{18^3} = 18$; and $24 : 18 = 1\frac{1}{3}$, or $1\frac{1}{3} : 1$. Also, $\sqrt[3]{4^3} = 4$, and $\sqrt[3]{3^3} = 3$; $4 : 3 = 1\frac{1}{3}$, or $1\frac{1}{3} : 1$.

NOTE.—If the numbers composing the antecedent and consequent have different exponents, or if different roots of those numbers are indicated, the operations described in Art. 13 cannot be performed. This is evident; for, consider the ratio $4^2 : 8^3$. When expressed in the fractional form it becomes $\frac{4^2}{8^3}$, which cannot be expressed either as $\left(\frac{4}{8}\right)^2$ or as $\left(\frac{4}{8}\right)^3$, and, hence, cannot be reduced as described above.

14. Since ratios are merely abstract numbers, they may be compared. For example, which is the greater, $7 : 5$ or $18 : 12$? Performing the divisions, $7 : 5 = 1.4$ and $18 : 12$

$= 1.5$; therefore, the latter is the greater. A convenient way of comparing ratios is to express them as fractions and reduce the fractions to a common denominator. For example, it is desired to find which is the greater, the ratio of 8 to 9 or the ratio of 9 to 10. Ratio $8 : 9 = \frac{8}{9}$ and $9 : 10 = \frac{9}{10}$.

Reducing to a common denominator, $\frac{8}{9} = \frac{80}{90}$ and $\frac{9}{10} = \frac{81}{90}$. Since 81 is greater than 80, the ratio of 9 to 10 is the greater.

EXAMPLES FOR PRACTICE.

15. Which is the greater :

(a) $10 : 7$ or $16 : 11$?

(b) $2 : 3$ or $7 : 10$?

(c) $22 : 7$ or $355 : 113$?

Ans. $\begin{cases} (a) & 16 : 11. \\ (b) & 7 : 10. \\ (c) & 22 : 7. \end{cases}$

1. What is the cube root of the ratio of 135 to 5 ? Ans. 3.

2. What is the square root of the ratio of 9 to 576 ? Ans. .125.

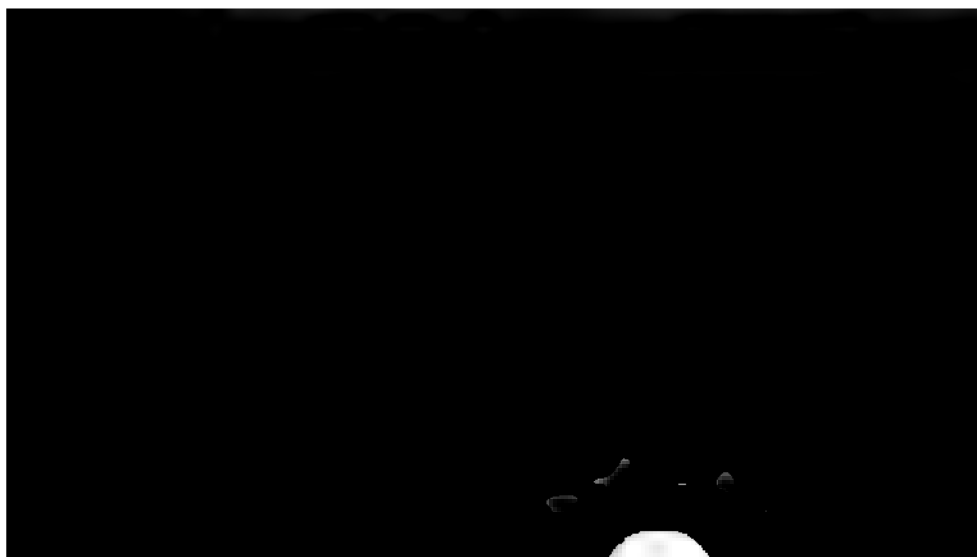
3. Which is the greater, the square of the ratio of 5 to 9, or the cube root of the ratio of 125 to 4,096 ? Ans. The latter.

4. If the antecedent is 5 hours and the consequent is 50 minutes, what is the ratio ? Ans. 6.

5. What is the ratio of $\frac{2}{3}$ to $1\frac{1}{4}$? Ans. $\frac{4}{15}$.

6. What is the square of the inverse ratio of $\frac{1}{2}$ to 3 ? Ans. 36.

7. If in the ratio $40 : 12$, the antecedent is multiplied by 3, what is the value of the new ratio ? Ans. 10.



17. The first form is the one most extensively used, by reason of its having been exclusively employed in all the older works on mathematics. The second and third forms are being adopted by all modern writers on mathematical subjects, and, in time, will probably supersede the first form. In this paper, we shall adopt the second form, unless some statement can be made clearer by using the third form.

18. A proportion may be *read* in two ways. The old way to read the above proportion is—*8 is to 4 as 6 is to 3*; the new way is—*the ratio of 8 to 4 equals the ratio of 6 to 3*. The student may read it either way, but we recommend the latter.

19. Each ratio of a proportion is termed a **couplet**. In the above proportion, 8 : 4 is a couplet; so, also, is 6 : 3.

20. The numbers forming a proportion are called **terms**; and they are numbered consecutively from left to right, thus:

$$\begin{array}{cccc} \textit{first} & \textit{second} & \textit{third} & \textit{fourth} \\ 8 & : & 4 & = & 6 & : & 3 \end{array}$$

Hence, in any proportion the ratio of the first term to the second term equals the ratio of the third term to the fourth term.

21. The first and fourth terms of a proportion are called the **extremes**, and the second and third terms, the **means**. Thus, in the foregoing proportion, 8 and 3 are the extremes and 4 and 6 are the means.

22. A **direct proportion** is one in which both couplets are direct ratios.

23. An **inverse proportion** is one which requires one of the couplets to be expressed as an inverse ratio. Thus, 8 is to 4 inversely as 3 is to 6, must be written $8 : 4 = 6 : 3$; i. e., the second ratio (couplet) must be inverted.

24. Proportion forms one of the most useful sections of arithmetic. In the arithmetics of our grandfathers it was called “The Rule of Three.”

25. The test of a proportion is the following principle:

In any proportion, the product of the extremes is equal to the product of the means

The truth of this principle may be shown by an example. The proportion $9 : 3 = 51 : 17$ is evidently true, since the ratio of each couplet is three. Expressed in the fractional form, the proportion is $\frac{9}{3} = \frac{51}{17}$. To reduce these fractions to a common denominator, multiply both terms of the first by 17 and both terms of the second by 3; thus, $\frac{9 \times 17}{3 \times 17} = \frac{51 \times 3}{17 \times 3}$. The fractions are still equal, because their values have not been changed by multiplying the numerator and denominator by the same number; and since the denominators are equal, each being the product 3×17 , the numerators 9×17 and 51×3 must be equal. But the numerator 9×17 is the product of the extremes, and the numerator 51×3 is the product of the means. Hence, in every case, these products are equal if the four numbers form a proportion.

EXAMPLE. Can the four numbers, 16, 125, 12, and 94 form a proportion?

SOLUTION.—Taking 16 and 94 for the extremes and 125 and 12 for the means the product $16 \times 94 = 1,504$ and $125 \times 12 = 1,500$ are not equal, and the numbers cannot form a proportion. Ans

26. The problem that most frequently occurs in proportion is to find one of the terms when the other three terms are given. Suppose it is required to find a number that will form a proportion with the numbers 6, 13, and 30; that is, $6 : 13 = 30 : \text{what number}$? Placing the product of the extremes equal to the product of the means,

$$6 \times \text{what number} = 13 \times 30$$

Since 6 times the required number is 13×30 , the number must be $\frac{1}{6}$ of 13×30 ; or,

$$\text{Number} = \frac{13 \times 30}{6} = 65$$

The unknown extreme is therefore equal to the product of the means divided by the known extreme.

27. Rule. I. *To find an unknown extreme, divide the product of the means by the given extreme.*

II. *To find an unknown mean, divide the product of the extremes by the given mean*

EXAMPLE.—What is the third term in the proportion $17 : 51 :: x : 42$?

SOLUTION.—In this case, a mean is unknown. The extremes are 17 and 42, and the other mean is 51. Hence,

$$\text{Third term} = \frac{17 \times 42}{51} = 14. \text{ Ans.}$$

EXAMPLE.—What is the first term in the proportion $x : 4 :: 21 : 6$?

SOLUTION.—Applying the rule,

$$\text{Unknown term} = 4 \times \frac{21}{6} = 14. \text{ Ans.}$$

28. In the statement of a problem, represent the unknown term by the letter x . Thus, in the last example, the proportion would be written $x : 4 :: 21 : 6$, and the solution would be written $x = \frac{4 \times 21}{6} = 14$.

29. Proportion is useful in solving a class of problems containing three quantities, of which two are of the same kind and have a certain ratio, and it is required to find a fourth quantity which shall bear the same ratio to the remaining quantity. The following is an example of a problem of this kind:

EXAMPLE.—If 4 horses can be bought for \$250 how much must be paid for 11 horses at the same price?

SOLUTION.—Here the 4 horses and the 11 horses are the two numbers of the same kind having a definite ratio, and we desire to find a fourth number to which \$250 shall bear the same relation that 4 horses do to 11 horses.

In solving a problem of this kind the student first asks himself "What is it I wish to find?" In the present case it is dollars. The 4 horses are bought for a known amount of money and the 11 horses, at the same price, cost an amount as yet unknown, but to be found. Since the horses are all bought at the same price, it is evident that the

cost of 11 horses bears the same relation to \$250, the cost of 4 horses, that 11 horses do to 4 horses. In other words, the ratio of the cost of 11 horses to the cost of 4 horses is equal to the ratio of 11 horses to 4 horses.

These two equal ratios may be put in the form of a proportion; thus,

Cost of 11 horses : \$250 = 11 horses : 4 horses.

The unknown term is an extreme, and is found by dividing the product of the means by the known extreme.

$$\text{Cost of 11 horses} = \frac{250 \cdot 11}{4} = \$687.50. \text{ Ans.}$$

EXAMPLES FOR PRACTICE.

30. Find the value of x in each of the following:

(a) \$16 : \$64 :: x : 4.

(b) x : 85 :: 10 : 17.

(c) 24 : x :: 15 : 40.

(d) 18 : 94 :: 2 : x .

(e) \$75 : \$100 :: x : 100.

(f) 15 pwt : x :: 21 : 10.

(g) x : 75 yd. :: \$15 : \$5.

$$\text{Ans. } \begin{cases} (a) & x = 1, \\ (b) & x = 50, \\ (c) & x = 64, \\ (d) & x = 10\frac{1}{2}, \\ (e) & x = 75, \\ (f) & x = 7\frac{1}{2} \text{ pwt.} \\ (g) & x = 225 \text{ yd.} \end{cases}$$

1. If 75 pounds of lead cost \$2.10, what would 125 pounds cost at the same rate? Ans. \$3.50.

2. If A does a piece of work in 4 days and B does it in 7 days, how long will it take A to do what B does in 63 days? Ans. 36 days.

3. The circumferences of any two circles are to each other as their diameters. If the circumference of a circle 7 inches in diameter is 23 inches, what is the circumference of a circle 31 inches in diameter?

Ans. 97½ inches.

INVERSE PROPORTION.

31. In the example given in Art. 29, the proportion was direct. The *greater* the number of horses bought, the *greater* must be the amount paid for them; that is, the cost of the horses varies *directly* as the number of horses. In some cases, the nature of the problem requires an inverse proportion. Such a case may be best shown by an example.

EXAMPLE.—If 6 men can do a piece of work in 8 days, in what time can 18 men do it working at the same rate?

SOLUTION.—It is clear that by employing 18 men instead of 6 the work will be done in *less* time; that is, the *greater* the number of men

the *less* the time required to do the work. The time varies *inversely* as the number of men, and the statement of the problem requires an inverse proportion. The ratio of the number of days required by 18 men to the 8 days required by 6 men is equal to the *inverse* ratio of 18 men to 6 men. Denoting the unknown number of days by x and writing the proportion as a direct proportion, the result is

$$6 : 18 :: 8 : x.$$

But since the proportion is an inverse one, one of the couplets must be inverted. If the first couplet is inverted, the result is $18 : 6 :: 8 : x$; if the second, $6 : 18 = x : 8$. In either case,

$$x = \frac{8 \times 6}{18} = 2\frac{2}{3} \text{ days. Ans.}$$

32. Sometimes the word *inverse* occurs in the statement of the example; in such cases the proportion can be written at once, merely inverting one of the couplets. But it frequently happens that only by carefully studying the conditions of the example, can it be ascertained whether the proportion is direct or inverse. When in doubt, the student can always satisfy himself as to whether the proportion is direct or inverse by first ascertaining what is required, and stating the proportion as a direct proportion. Then, in order that the proportion may be true, if the first term is smaller than the second term, the third term must be smaller than the fourth; or if the first term is larger than the second term, the third term must be larger than the fourth term. Keeping this in mind, the student can always tell whether the required term will be larger or smaller than the other term of the couplet to which the required term belongs. Having determined this, the student then refers to the example, and ascertains from its conditions whether the required term is to be larger or smaller than the other term of the same kind. If the two determinations agree, the proportion is direct, otherwise it is inverse, and one of the couplets must be inverted.

33. EXAMPLE.—A carriage wheel, 16 ft in circumference, makes 200 turns in passing over a certain distance. How many turns will a wheel 10 ft. in circumference make in going the same distance?

SOLUTION.—The quantities considered are the circumferences of the wheels and the number of turns made. Forming a direct proportion.

$$16 : 10 :: 200 : x.$$

An examination of this proportion shows that x (the number of turns of the smaller wheel) must be smaller than 200. But x must evidently be larger than 200, since a wheel 10 feet in diameter will make more turns in going a given distance than a wheel 16 feet in diameter. Hence, the proportion is an inverse one. Inverting the second couplet,

$$16 : 10 :: 1 : 200,$$

from which $x = \frac{16 \times 200}{10} = 320$ turns. Ans.

EXAMPLE.—If A's *rate* of doing work is to B's as 5 : 7, and A does a piece of work in 42 days, in what time will B do it?

SOLUTION.—The required term is the number of days it will take B to do the work. Hence, stating as a direct proportion,

$$5 : 7 :: 42 : x$$

Now, since 7 is greater than 5, x will be greater than 42. But, referring to the statement of the example, it is easy to see that B works faster than A, hence it will take B a less number of days to do the work than A. Therefore, the proportion is an inverse one, and should be stated,

$$5 : 7 :: x : 42,$$

from which $x = \frac{5 \times 42}{7} = 30$ days. Ans.

Had the example been stated thus: The time that A requires to do a piece of work is to the time that B requires as 5 : 7. A can do it in 42 days, in what time can B do it? it is evident that it would take B a longer time to do the work than it would A, hence, x would be greater than 42, and the proportion would be direct: the value of x being $\frac{7 \times 42}{5} = 58.8$ days.

34. The unknown term x may occupy the place of any term in a proportion without destroying the proportion. Thus, consider the example: If 4 men earn \$25 in one week, how much will 12 men earn in the same time? The proportion may be formed in any of the four following ways:

$$x : \$25 = 12 \text{ men} : 4 \text{ men};$$

$$\$25 : x = 4 \text{ men} : 12 \text{ men};$$

$$12 \text{ men} : 4 \text{ men} = x : \$25;$$

$$4 \text{ men} : 12 \text{ men} = \$25 : x.$$

From any one of the above proportions,

$$x = \frac{\$25 \times 12}{4} = \$75.$$

EXAMPLES FOR PRACTICE.

35. Solve the following:

1. If a pump which discharges 4 gal. of water per min. can fill a tank in 20 hr., how long will it take a pump discharging 12 gal. per min. to fill it? Ans. $6\frac{2}{3}$ hr.

2. If a pump discharges 90 gal. of water in 20 hr., in what time will it discharge 144 gal.? Ans. 32 hr.

3. The weight of any gas (the volume and pressure remaining the same) varies inversely as the absolute temperature. If a certain quantity of some gas weighs 2.927 lb. when the absolute temperature is 525° , what will the same volume of gas weigh when the absolute temperature is 600° , the pressure remaining the same? Ans. $2.561\frac{1}{2}$ lb.

4. If 50 cu. ft. of air weigh 4.2 pounds when the absolute temperature is 582° , what will be the absolute temperature when the same volume weighs 5.8 pounds, the pressure being the same in both cases? Ans. 407° , very nearly.

5. If a man can make a journey in 15 days traveling 10 hours per day, how long will it take him if he travels 8 hours per day? Ans. $18\frac{3}{4}$ da.

6. A merchant sells \$4,500 worth of goods and gains \$1,300. At the same rate, what would he gain if he sold \$8,000 worth? Ans. \$2,311 $\frac{1}{3}$.

7. If 14 men can dig a cellar in 12 days, how long will it take 8 men to dig it? Ans. 21 da.

PROPERTIES OF PROPORTION.

36. If the same operations (addition and subtraction excepted) be performed upon *all* the terms of a proportion, the proportion is not thereby destroyed. In other words, if all the terms of a proportion be (1) multiplied or (2) divided by the same number; (3) if all the terms be raised to the same power; if (4) the same root of all the terms be taken, or (5) if both couplets be inverted, the proportion still holds. We will prove these statements by a numerical example, and the student can satisfy himself by other similar ones. The fractional form will be used, as it is better suited to the purpose. Consider the proportion $8 : 4 = 6 : 3$. Expressing it in the fractional form, it becomes $\frac{8}{4} = \frac{6}{3}$. What we are to

prove is that if any of the five operations enumerated above be performed upon all the terms of this proportion, the first fraction will still equal the second fraction.

1. Multiplying all the terms by any number, say 7, $\frac{8 \times 7}{4 \times 7} = \frac{6 \times 7}{3 \times 7}$; or $\frac{56}{28} = \frac{42}{21}$. Now $\frac{56}{28}$ evidently equals $\frac{42}{21}$, since the value of either ratio is 2, and the same is true of the original proportion.

2. Dividing all the terms by any number, say 7, $\frac{8 \div 7}{4 \div 7} = \frac{6 \div 7}{3 \div 7}$; or $\frac{\frac{8}{7}}{\frac{4}{7}} = \frac{\frac{6}{7}}{\frac{3}{7}}$. But $\frac{8}{7} \div \frac{4}{7} = 2$, and $\frac{6}{7} \div \frac{3}{7} = 2$ also, the same as in the original proportion.

3. Raising all the terms to the same power, say the cube, $\frac{8^3}{4^3} = \frac{6^3}{3^3}$. This is evidently true, since $\frac{8^3}{4^3} = \left(\frac{8}{4}\right)^3 = 2^3 = 8$, and $\frac{6^3}{3^3} = \left(\frac{6}{3}\right)^3 = 2^3 = 8$ also.

4. Extracting the same root of all the terms, say the cube root, $\frac{\sqrt[3]{8}}{\sqrt[3]{4}} = \frac{\sqrt[3]{6}}{\sqrt[3]{3}}$. It is evident that this is likewise true, since $\frac{\sqrt[3]{8}}{\sqrt[3]{4}} = \sqrt[3]{\frac{8}{4}} = \sqrt[3]{2}$, and $\frac{\sqrt[3]{6}}{\sqrt[3]{3}} = \sqrt[3]{\frac{6}{3}} = \sqrt[3]{2}$ also.

5. Inverting both couplets, $\frac{4}{8} = \frac{3}{6}$, which is true, since each equals $\frac{1}{2}$.

POWERS AND ROOTS IN PROPORTION.

38. It was stated in Art. 12 that a ratio may be raised to any power, or any root of it may be taken. A proportion is frequently stated in such a manner that one of the couplets must be raised to some power or some root of it must be taken. In all such cases, both terms of the couplet so affected *must be raised to the same power, or the same root of both terms must be taken.*

39. EXAMPLE.—Knowing that the weight of a sphere varies as the cube of its diameter, what is the weight of a sphere 6 inches in diameter if a sphere 8 inches in diameter of the same material weighs 180 pounds?

SOLUTION.—This is evidently a direct proportion. Hence, we write

$$6^3 : 8^3 = x : 180.$$

Dividing both terms of the first couplet by 2 (See Art. 13),

$$8^3 : 4^3 = x : 180, \text{ or } 27 : 64 = x : 180;$$

whence, $x = \frac{27 \times 180}{64} = 75\frac{15}{8}$ pounds. Ans.

EXAMPLE.—A sphere 8 inches in diameter weighs 180 pounds; what is the diameter of another sphere of the same material which weighs $75\frac{15}{8}$ pounds?

SOLUTION.—Since the weights of any two spheres are to each other as the cubes of their diameters, we have the proportion

$$180 : 75\frac{15}{8} = 8^3 : x^3$$

x , the required term must be cubed, because the other term of the couplet is cubed (see Art. 38). But, $8^3 = 512$; hence,

$$180 : 75\frac{15}{8} = 512 : x^3, \text{ or } x^3 = \frac{75\frac{15}{8} \times 512}{180} = 216;$$

whence, $x = \sqrt[3]{216} = 6$ inches. Ans.

40. Since taking the same root of all the terms of a proportion does not change its value (Art. 36), the above example might have been solved by extracting the cube root of all of the numbers, thus obtaining $\sqrt[3]{180} : \sqrt[3]{75\frac{15}{8}} = 8 : x$; whence, $x = \frac{8 \times \sqrt[3]{75\frac{15}{8}}}{\sqrt[3]{180}} = 8 \times \sqrt[3]{\frac{75\frac{15}{8}}{180}} = 8 \sqrt[3]{\frac{1,215}{2,880}} = 8 \sqrt[3]{\frac{27}{64}} = 8 \times \frac{3}{4} = 6$ inches. The process, however, is longer and is not so direct, and the first method is to be preferred.

41. If two cylinders have *equal* volumes, but different diameters, the diameters are to each other inversely as the square roots of their lengths. Hence, if it is desired to find the diameter of a cylinder that is to be 15 inches long, and which shall have the same volume as one that is 9 inches in diameter and 12 inches long, we write the proportion

$$9 : x = \sqrt{15} : \sqrt{12}$$

Since neither 12 nor 15 is a perfect square, we square all of the terms (Arts. **40** and **36**) and obtain

$$81 : x^2 = 15 : 12; \text{ whence, } x^2 = \frac{81 \times 12}{15} = 64.8,$$

and $x = \sqrt{64.8} = 8.05$ inches = diameter of 15 in. cylinder.

EXAMPLES FOR PRACTICE.

42. Solve the following examples

1. The intensity of light varies inversely as the square of the distance from the source of light. If a gas jet illuminates an object 90 feet away with a certain distinctness, how much brighter will the object be at a distance of 20 feet? Ans. $2\frac{1}{4}$ times as bright.

2. In the last example, suppose that the object had been 40 feet from the gas jet, how bright would it have been compared with its brightness at 30 feet from the gas jet? Ans. $\frac{9}{16}$ as bright.

3. When comparing one light with another, the intensities of their illuminating powers vary as the squares of their distances from the objects they illuminate. If a man can just distinguish the time indicated by his watch, 50 feet from a certain light, at what distance could he just distinguish the time by a light 3 times as powerful. Ans. 86.6+ feet.

4. The quantity of air flowing through a mine varies directly as the square root of the pressure. If 60,000 cubic feet of air flow per minute when the pressure is 2.8 pounds per square foot, how much will flow when the pressure is 3.6 pounds per square foot? Ans. 68,034 cu. ft. per min., nearly.

5. In the last example, suppose that 70,000 cubic feet per minute had been required, what would be the pressure necessary for this quantity? Ans. 3.81+ lb. per sq. ft.

CAUSE AND EFFECT.

43. Many examples in proportion may be more easily solved by using the principle of *cause and effect*. That which may be regarded as producing a change or alteration in something, or as accomplishing something, may be called the **cause**, and the change or alteration, or thing accomplished, the **effect**.

44. *Like causes produce like effects.* Hence, when two causes of the same kind produce two effects of the same kind, the ratio of the causes equals the ratio of the effects; in other words, the ratio of the first cause to the second cause equals the ratio of the first effect to the second effect. Thus, in the question, if 3 men can lift 1,400 pounds, how many pounds can 7 men lift? we call 3 men and 7 men the *causes* (since they accomplish something, viz., the lifting of the weight), the number of pounds lifted, viz., 1,400 pounds and x pounds, are the effects. If we call 3 men the first cause, 1,400 pounds is the first effect; 7 men is the second cause and x pounds is the second effect. Hence, we may write

$$\begin{array}{ccccccc} 1st\ cause & 2d\ cause & & 1st\ effect & 2d\ effect & & \\ 3 & : & 7 & = & 1,400 & : & x \end{array}$$

whence $x = \frac{7 \times 1,400}{3} = 3,266\frac{2}{3}$ pounds.

45. The principle of cause and effect is extremely useful in the solution of examples in compound proportion, as we shall now show.

COMPOUND PROPORTION.

46. All the cases of proportion so far considered have been cases of **simple proportion**; i. e., each term has been composed of but one number. There are many cases, however, in which two or all of the terms have more than one number in them; all such cases belong to **compound proportion**. In all examples in compound proportion, both causes or both effects, or all four, consist of more than two numbers. We will illustrate this by an example.

EXAMPLE.—If 40 men earn \$1,200 in 16 days how much will 36 men earn in 31 days?

SOLUTION.—Since 40 men earn something, 40 men is a cause, and since they take 16 days in which to earn that something, 16 days is an element of the cause. For the same reason, 36 men working for 31 days is also a cause. The effects, that which is earned, are 1,200 dollars and x dollars. Then, 40 men and 16 days make up the first cause, and 36 men and 31 days make up the second cause. \$1,200 is the first effect and \$ x is the second effect. Hence, we write

$$\begin{array}{ccccccc} 1st\ cause & 2d\ cause & 1st\ effect & 2d\ effect \\ 40 & : & 36 & = & 1,200 & : & x \\ 16 & & 31 & & & & \end{array}$$

Now, instead of using the colon to express the ratio, we shall use the vertical line (see Art. 2), and the above becomes

$$\begin{array}{ccc} 40 & : & 36 \\ 16 & : & 31 \end{array} = 1,200 \mid x.$$

In the last expression, the product of all of the numbers included between the vertical lines must equal the product of all the numbers without them; i. e., $36 \times 31 \times 1,200 = 40 \times 16 \times x$.

$$\text{Or, } x = \frac{36 \times 31 \times 1,200}{40 \times 16} = \$2,232 \quad \text{Ans.}$$

47. The above might have been solved by canceling factors of the numbers in the original proportion. For, if any number within the lines has a factor common to any number without the lines, that factor may be canceled from both numbers. Thus, 16 is contained in

$$\begin{array}{ccc} 40 & : & 36 \\ 16 & : & 31 \end{array} = 1,200 \mid x.$$

the right of the second column, and on the right of this form a third column of the numbers composing the first effect, drawing a vertical line to the right; on the other side of this line, write for the fourth column the numbers composing the second effect. There must be as many numbers in the second cause as in the first cause, and in the second effect as in the first effect; hence, if any term is wanting, write x in its place. Multiply together all of the numbers within the vertical lines, and also all those without the lines (canceling previously, if possible), and divide the product of those numbers which do not contain x by the product of the others in which x occurs, and the result will be the value of x .

49. EXAMPLE.—If 40 men can dig a ditch 720 feet long, 5 feet wide, and 4 feet deep in a certain time, how long a ditch 6 feet deep and 3 feet wide can 24 men dig in the same time?

SOLUTION.—Here 40 men and 24 men are the causes and the two ditches are the effects. Hence,

$$40 \left| \begin{array}{c} 3 \\ 18 \\ 720 \\ 5 \\ 4 \end{array} \right| \begin{array}{c} x \\ 3 \\ 6 \\ 6 \end{array} \text{ whence, } x = 24 \times 5 \times 4 = 480 \text{ feet. Ans.}$$

50. EXAMPLE.—The volume of a cylinder varies directly as its length and directly as the square of its diameter. If the volume of a cylinder 10 inches in diameter and 20 inches long is 1,570.8 cubic inches, what is the volume of another cylinder 16 inches in diameter and 24 inches long?

SOLUTION. In this example, either the dimensions or the volumes may be considered the causes; say we take the dimensions for the causes. Then, squaring the diameters,

$$\begin{array}{c} 10^2 \\ 20 \end{array} \left| \begin{array}{c} 16^2 \\ 24 \end{array} \right| = 1,570.8 \quad x, \text{ or } \begin{array}{c} 100 \\ 20 \\ 5 \end{array} \left| \begin{array}{c} 256 \\ 24 \\ 6 \end{array} \right| = 1,570.8 \quad x;$$

$$\text{whence, } x = \frac{256 \times 6 \times 1,570.8}{5 \times 100} = 4,825.4976 \text{ cubic inches. Ans.}$$

51. EXAMPLE.—If a block of granite 8 ft. long, 5 ft. wide, and 3 ft. thick weighs 7,200 lb., what is the weight of a block of granite 12 ft. long, 8 ft. wide, and 5 ft. thick?

SOLUTION.—Taking the weights as the effects, we have

$$\begin{array}{c} 8 \\ 5 \\ 3 \end{array} \left| \begin{array}{c} 4 \\ 12 \\ 8 \end{array} \right| = 7,200 \quad x, \text{ or } x = 4 \times 7,200 = 28,800 \text{ pounds. Ans.}$$

52. EXAMPLE.—If 12 compositors in 30 days of 10 hours each set up 25 sheets of 16 pages each, 32 lines to the page, in how many days 8 hours long can 18 compositors set up, in the same type, 64 sheets of 12 pages each, 40 lines to the page?

SOLUTION.—Here compositors, days, and hours compose the causes, and sheets, pages, and lines, the effects. Hence,

$$\begin{array}{c|c|c|c}
 3 & 3 & 3 & 2 \\
 12 & 18 & 25 & 64 \\
 30 & x & 16 & 12 \\
 10 & 8 & 32 & 40
 \end{array}
 \quad
 \begin{array}{l}
 12, \text{ or } x = 3 \times 10 \times 2 = 60 \text{ days. Ans.}
 \end{array}$$

53. In examples stated like that in Art. 50, should an inverse proportion occur, write the various numbers as in the preceding examples, and then transpose those numbers which are said to vary inversely, from one side of the vertical line to the other side.

EXAMPLE.—The centrifugal force of a revolving body varies directly as its weight, as the square of its velocity, and inversely as the radius of the circle described by the center of the body. If the centrifugal force of a body weighing 15 pounds is 187 pounds when the body revolves in a circle having a radius of 12 inches, with a velocity of 20 feet per second, what will be the centrifugal force of the same body when the radius is increased to 18 inches and the speed is increased to 24 feet per second?

SOLUTION.—Calling the centrifugal force the effect, we have,

$$\begin{array}{c|c|c|c}
 15 & 15 & 187 & x \\
 20^2 & 24^2 & = & \\
 12 & 18 & &
 \end{array}$$

Transposing 12 and 18 (since the radii are to vary inversely) and squaring 20 and 24,

$$15 = 15$$

3. If 24 men can build a wall 72 rods long, 6 feet wide, and 5 feet high in 60 days of 10 hours each, how many days will it take 32 men to build a wall 96 rods long, 4 feet wide, and 8 feet high, working 8 hours a day ?
Ans. 80 days.

4. The horsepower of an engine varies as the mean effective pressure, as the piston speed, and as the square of the diameter of the cylinder. If an engine having a cylinder 14 inches in diameter develops 112 horsepower when the mean effective pressure is 48 pounds per square inch and the piston speed is 500 feet per minute, what horsepower will another engine develop, if the cylinder is 16 inches in diameter, piston speed is 600 feet per minute, and mean effective pressure is 56 pounds per square inch ?
Ans. 204.8 horsepower.

5. Referring to the example in Art. 50, what will be the volume of a cylinder 20 inches in diameter and 24 inches long ?
Ans. 7,539.84 cubic inches.

6. Knowing that the product of $3 \times 5 \times 7 \times 9$ is 945, what is the product of $6 \times 15 \times 14 \times 36$?
Ans. 45,360.

7. If 60 yards of carpet $\frac{3}{4}$ of a yard wide will cover a room 15 feet wide and 27 feet long, how many yards of carpet $\frac{7}{8}$ of a yard wide will cover a room 18 feet wide and 35 feet long ?
Ans. 80 yards.

8. If it costs \$72.50 to ship 9,000 pounds of freight a distance of 850 miles, how much will it cost to ship 22,000 pounds 675 miles at the same rate per mile ?
Ans. \$140.74, nearly.

PROPORTIONAL PARTS.

55. Proportion may be used to divide a number into two or more parts having a given ratio to each other. For example, it is required to divide the number 40 into two parts which shall be in the ratio of 3 to 5. Adding these proportional parts, $3 + 5 = 8$; then the ratio of the proportional part 3 to the sum 8 is the same as the ratio of the smaller of the two numbers whose sum is 40 to 40.

That is, $3 : 8 = \text{smaller number} : 40$;

similarly, $5 : 8 = \text{larger number} : 40$.

From these proportions,

$$\text{smaller number} = \frac{3 \times 40}{8} = 15;$$

$$\text{larger number} = \frac{5 \times 40}{8} = 25.$$

56. Rule.—*Form a proportion of which one of the proportional parts is the first term, the sum of the proportional parts is the second term, the unknown part is the third term, and the number to be divided into parts is the fourth term. Form as many of these proportions as there are proportional parts, and solve. The results will be the parts required.*

EXAMPLE.—If \$6,900 is divided among four men, A, B, C, and D, in the proportion of 3, 4, 7, and 9, how much does each receive?

SOLUTION.—Sum of proportional parts = $3+4+7+9 = 23$. The four proportions are

$$3 : 23 = \text{A's share} : \$6,900$$

$$4 : 23 = \text{B's share} : \$6,900$$

$$7 : 23 = \text{C's share} : \$6,900$$

$$9 : 23 = \text{D's share} : \$6,900$$

Hence,

$$\begin{aligned} \text{A's share} &= \frac{3 \times \$6,900}{23} = \$900 \\ \text{B's share} &= \frac{4 \times \$6,900}{23} = \$1,200 \\ \text{C's share} &= \frac{7 \times \$6,900}{23} = \$2,100 \\ \text{D's share} &= \frac{9 \times \$6,900}{23} = \$2,700. \quad \text{Ans} \end{aligned}$$

EXAMPLES FOR PRACTICE.

57. Solve the following:

1. Divide 1,350 into parts proportional to the numbers 4, 5, 6.

Ans. 360, 450, 540.

2. Divide 1,630 pounds into parts proportional to the fractions



ARITHMETIC.

PERCENTAGE.

DEFINITIONS AND PRINCIPLES.

1. In certain operations pertaining to business, it is very convenient to regard the quantity on which we are to operate as being divided into 100 equal parts; thus, instead of using the ordinary fractions $\frac{1}{4}$, $\frac{3}{5}$, $\frac{2}{7}$, we use the equivalent fractions

$\frac{25}{100}$, $\frac{60}{100}$, $\frac{28\frac{4}{7}}{100}$, or their equivalent decimals, .25, .60, .28 $\frac{4}{7}$.

This practice is a very convenient one in all computations involving United States money, because, since \$1 equals 100 cents, it is easier to comprehend what part of the whole $\frac{35}{100}$ is than some other equivalent fraction, as $\frac{49}{140}$; it is also much easier to compute with fractions whose denominators are 100 than it is to compute with fractions whose denominators are composed of other figures.

2. **Percentage** is a term applied to those arithmetical operations in which the number or quantity to be operated upon is supposed to be divided into 100 equal parts.

3. The term **per cent.** means *by the hundred*. Thus, 8 per cent. of a number means 8 hundredths, i. e., $\frac{8}{100}$, or .08, of that number; 8 per cent. of 250 is $250 \times \frac{8}{100}$, or $250 \times .08 = 20$; 47 per cent. of 75 bushels is $75 \times \frac{47}{100} = 75 \times .47 = 35.25$ bushels. The statement that the population of a city has increased 22 per cent. in a given time, say from 1880 to 1890, is equivalent to saying that the increase is 22 in

every hundred; that is, for every 100 in 1880, there are 22 more, or 122, in 1890.

4. The sign of per cent. is %, and is read *per cent.* Thus, 6% is read six per cent.; $12\frac{1}{2}\%$ is read twelve and one-half per cent., etc.

5. When expressing the per cent. of a number to use in calculations, it is customary to express it decimally instead of fractionally. Thus, instead of expressing 6%, 25%, and 43% as $\frac{6}{100}$, $\frac{25}{100}$, and $\frac{43}{100}$, it is usual to express them as .06, .25, and .43.

6. The following table will show how many per cent. can be expressed either as a decimal or as a fraction:

Per Cent.	Decimal	Fraction	Per Cent.	Decimal	Fraction.
1%01	$\frac{1}{100}$	$\frac{1}{4}\%$0025	$\frac{1}{400}$ or $\frac{1}{100}$
2%02	$\frac{2}{100}$ or $\frac{1}{50}$	$\frac{1}{2}\%$005	$\frac{1}{200}$ or $\frac{1}{100}$
5%05	$\frac{5}{100}$ or $\frac{1}{20}$	$1\frac{1}{2}\%$015	$\frac{3}{200}$ or $\frac{3}{100}$
10%10	$\frac{10}{100}$ or $\frac{1}{10}$	$6\frac{1}{2}\%$06 $\frac{1}{2}$	$\frac{13}{200}$ or $\frac{13}{100}$
25%25	$\frac{25}{100}$ or $\frac{1}{4}$	$8\frac{1}{2}\%$08 $\frac{1}{2}$	$\frac{17}{200}$ or $\frac{17}{100}$
50%50	$\frac{50}{100}$ or $\frac{1}{2}$	$12\frac{1}{2}\%$125	$\frac{25}{200}$ or $\frac{1}{4}$
75%75	$\frac{75}{100}$ or $\frac{3}{4}$	$16\frac{2}{3}\%$16 $\frac{2}{3}$	$\frac{161}{100}$ or $\frac{1}{6}$
100%	1.00	$\frac{100}{100}$ or 1	$33\frac{1}{3}\%$33 $\frac{1}{3}$	$\frac{331}{100}$ or $\frac{1}{3}$
125%	1.25	$\frac{125}{100}$ or $1\frac{1}{4}$	$37\frac{1}{2}\%$37 $\frac{1}{2}$	$\frac{371}{100}$ or $\frac{3}{8}$
150%	1.50	$\frac{150}{100}$ or $1\frac{1}{2}$	$62\frac{1}{2}\%$625	$\frac{625}{1000}$ or $\frac{5}{8}$
500%	5.00	$\frac{500}{100}$ or 5	$87\frac{1}{2}\%$875	$\frac{875}{1000}$ or $\frac{7}{8}$

cent. is always 100 times the *rate*. Thus, 7% of 125 and .07 of 125 amount in the end to the same thing; the former, 7, is the *rate per cent.*—the *number* of hundredths of 125 intended; the latter, .07, is the *rate*, the *part* of 125 that is to be found; 7% is used in *speech*, .07 is the form used in *computation*. So, also, $12\frac{1}{2}\% = .125$, $\frac{1}{2}\% = .005$, $1\frac{3}{4}\% = .0175$.

10. The **percentage** is the result obtained by multiplying the base by the rate. Thus, 7% of 125 = $125 \times .07 = 8.75$, the percentage.

11. The **amount** is the sum of the base and the percentage.

12. The **difference** is the remainder obtained when the percentage is subtracted from the base.

13. The terms amount and difference are ordinarily used when there is an increase or a decrease in the base. For example, suppose the population of a village is 1,500 and it increases 25 per cent. This means that for every 100 of the original 1,500 there is an increase of 25, or a total increase of $15 \times 25 = 375$. This increase added to the original population gives the *amount*, or the population after the increase. If the population had decreased 375, the final population would have been $1,500 - 375 = 1,125$, and this would be the *difference*. The original population, 1,500, is the base on which the percentage is computed; the 25 is the rate per cent., and the increase or decrease, 375, is the percentage. If the base increases, the final value is the amount, and if it decreases, its final value is the difference.

BASE, RATE, AND PERCENTAGE.

14. Rule.—*To find the percentage, multiply the base by the rate.*

EXAMPLE.—A farmer raised 650 bushels of wheat and sold 64% of it. How many bushels did he sell?

SOLUTION.—The base is 650 bushels. Out of every 100 bushels raised 64 were sold; that is, the number of bushels sold was $\frac{64}{100}$ or .64 of the number raised.

$$650 \times .64 = 416 \text{ bushels, the percentage. Ans.}$$

15. The rate per cent. also may be used in computation; it is then easy to see that it bears the same relation to 100 that the percentage does to the base. Thus, in the example just solved, the ratio 64% : 100% is equal to the ratio 416 : 650; that is,

$$64 : 100 = 416 : 650.$$

The rule of Art. 14 may be expressed by the proportion,

$$\text{rate per cent.} : 100 = \text{percentage} : \text{base.}$$

EXAMPLE.—A merchant gains 18% on goods that cost him \$3,500. How many dollars does he gain?

SOLUTION.—Base = \$3,500, rate per cent = 18.

Let x represent the percentage or gain. Then $18 : 100 = x : \$3,500$,

$$\text{or, } x = \frac{3,500 \times 18}{100} = \$630. \text{ Ans.}$$

16. Let both terms of the first couplet of the second proportion in Art. 15 be divided by 100 (see Art. 37, § 7); the resulting proportion is

$$\frac{\text{rate per cent.}}{100} : 1 = \text{percentage} : \text{base.}$$

Dividing the rate per cent. by 100 gives the rate. Thus, if the rate per cent. is 64, dividing by 100 gives $\frac{64}{100} = .64$, the rate. Hereafter the rate will generally be used instead of the rate per cent., in which case, the above proportion will be used in the form

$$\text{rate} : 1 = \text{percentage} : \text{base.}$$

This proportion is very important, since it is the foundation of all calculations in percentage.

17. Placing the product of the means equal to the product of the extremes,

$$\text{percentage} = \text{base} \times \text{rate.}$$

Each extreme is equal to the product of the means divided by the other extreme. Hence,

$$\text{rate} = \frac{\text{percentage}}{\text{base}}, \text{ or } R = \frac{P}{B}$$

$$\text{and } \text{base} = \frac{\text{percentage}}{\text{rate}}, \text{ or } B = \frac{P}{R}$$

18. Rule.—To find the rate, divide the percentage by the base

EXAMPLE.—Bought 300 bushels of apples and sold 228 bushels. What per cent. of the number of bushels bought was sold?

SOLUTION.—Here 300 is the base and 228 is the percentage; hence, applying rule,

$$\text{rate} = 228 \div 300 = .76 = 76\%. \quad \text{Ans.}$$

EXAMPLE.—What per cent. of 875 is 25?

SOLUTION.—Here 875 is the base, and 25 is the percentage; hence, applying rule,

$$25 \div 875 = .028 = 2\frac{8}{7}\%. \quad \text{Ans.}$$

PROOF.— $875 \times .028 = 25$.

19. Rule.—*To find the base when the percentage and rate are given, divide the percentage by the rate.*

EXAMPLE.—Bought a certain number of bushels of apples and sold 76% of them. If I sold 228 bushels, how many bushels did I buy?

SOLUTION.—Here 228 is the percentage, and .76 is the rate; hence, applying the rule,

$$228 \div .76 = 300 \text{ bushels.} \quad \text{Ans.}$$

Any problem coming under these three rules can be solved by the proportion given in Art. 16. Thus, in the last example,

$$\begin{aligned} .76 : 1 &= 228 : \text{base}; \\ \text{base} &= \frac{1 \times 228}{.76} = 300 \text{ bushels.} \quad \text{Ans.} \end{aligned}$$

EXAMPLES FOR PRACTICE.

20. What is

(a) 36% of 1,762?

(b) 19% of \$89?

(c) 47% of 2,400 bushels?

(d) 113% of \$1,640?

What per cent. of

(e) 360 is 90?

(f) \$900 is \$360?

(g) 125 is 25?

(h) 150 is 750?

(i) 280 horses is 112 horses?

(j) 400 is 200?

(k) 47 is 94?

(l) 500 days is 250 days?

(m) 42 is 6% of what number?

(n) 126 is 31½% of what number?

(o) 198 is 36% of what number?

Ans. $\left\{ \begin{array}{ll} (a) & 634.32. \\ (b) & \$16.91. \\ (c) & 1,128 \text{ bushels.} \\ (d) & \$1,853.20. \end{array} \right.$

Ans. $\left\{ \begin{array}{ll} (e) & 25\%. \\ (f) & 40\%. \\ (g) & 20\%. \\ (h) & 500\%. \\ (i) & 40\%. \\ (j) & 50\%. \\ (k) & 200\%. \\ (l) & 50\%. \end{array} \right.$

Ans. $\left\{ \begin{array}{ll} (m) & 700. \\ (n) & 400. \\ (o) & 550. \end{array} \right.$

ALIQOT PARTS IN PERCENTAGE.

21. When the rate per cent. is an aliquot part of 100 per cent., it is more convenient to use the equivalent fraction in computations. The principal aliquot parts are shown in the table, Art. 6.

EXAMPLE.—What is $16\frac{2}{3}\%$ of 684?

SOLUTION.— $16\frac{2}{3}\% = \frac{1}{3}$. $684 \times \frac{1}{3} = 228$. Ans.

EXAMPLE.—65 is 25% of what number?

SOLUTION.— $25\% = \frac{1}{4}$; 65 is $\frac{1}{4}$ of $65 \times 4 = 260$. Ans.

The student should become familiar with the most common aliquot parts of 100, and use the fractions wherever possible in percentage computations.

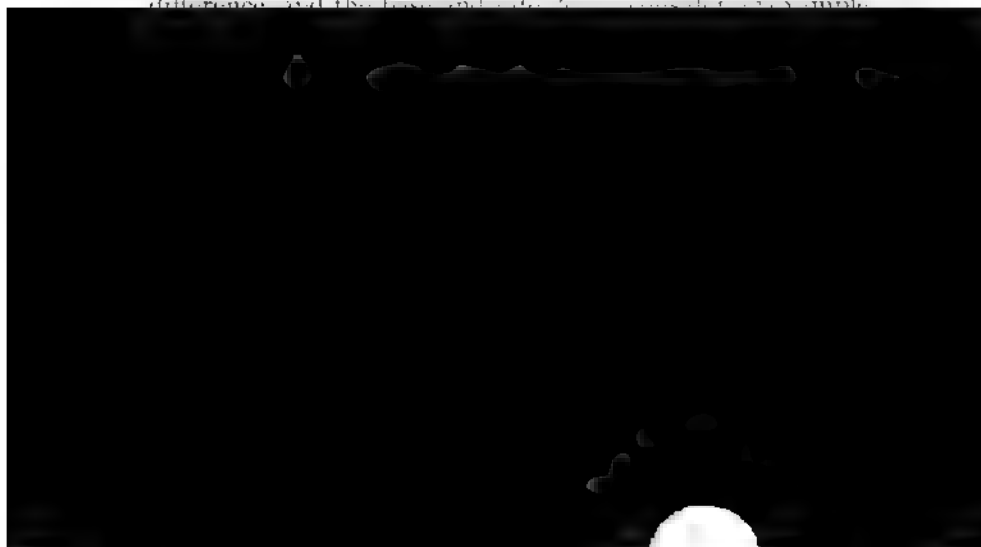
EXAMPLES FOR PRACTICE.

22. Solve the following by aliquot parts.

- | | | |
|---|--------|------------|
| (a) What is 50% of 1,964? | Ans. { | (a) 982. |
| (b) What is $33\frac{1}{3}\%$ of \$630? | | (b) \$210. |
| (c) What is $6\frac{1}{2}\%$ of 1,760? | | (c) 110. |
| (d) 85 is $12\frac{1}{2}\%$ of what number? | | (d) 680. |
| (e) 625 is $8\frac{1}{2}\%$ of what number? | | (e) 7,500. |

AMOUNT AND DIFFERENCE.

23. To find the relations existing between the amount or difference and the base and rate.



number of men increases from 2,100 to 2,268. Therefore, the ratio of 108 to 100 is the same as the ratio of 2,268 to 2,100, or,

$$108 : 100 = 2,268 : 2,100.$$

Dividing the terms of the first couplet by 100 (Art. 37, § 7),

$$1.08 : 1 = 2,268 : 2,100.$$

Since .08 is the rate, we have the proportion

$$1 + \text{rate} : 1 = \text{amount} : \text{base}.$$

24. If, in the above example, the force had been decreased 8%, the number of men thrown out of work would have been 168, and those left at work, $2,100 - 168 = 1,932$. As before, the base is 2,100, but the final number, 1,932, is the difference, since the base has decreased. Out of every 100 men formerly at work, 8 have been discharged, leaving $100 - 8 = 92$ still at work. Therefore,

$$92 : 100 = 1,932 : 2,100,$$

$$\text{or, } .92 : 1 = 1,932 : 2,100.$$

But $.92 = 1 - .08 = 1 - \text{rate}$; hence,

$$1 - \text{rate} : 1 = \text{difference} : \text{base}.$$

25. Placing the product of the means equal to the product of the extremes, the proportions of Arts. 23 and 24 give the following formulas:

$$\text{amount} = \text{base} \times (1 + \text{rate}), \text{ or } A = B \times (1 + R)$$

$$\text{difference} = \text{base} \times (1 - \text{rate}), \text{ or } D = B \times (1 - R).$$

26. Rule.—*To find the amount, the base and rate being given, multiply the base by 1 plus the rate.*

EXAMPLE.—The population of a city in 1880 was 43,000, and it increased 65% in the next ten years. What was its population in 1890?

SOLUTION.—In this case, 43,000 is the base, .65 is the rate, and the required population in 1890 is the amount. Applying the rule,

$$\text{amount} = 43,000 \times (1 + .65) = 43,000 \times 1.65 = 70,950. \text{ Ans.}$$

27. Rule.—*To find the difference, the base and rate being given, multiply the base by 1 minus the rate.*

EXAMPLE.—A speculator invested \$26,500 in a business enterprise, and lost 16% of his investment. How much had he left?

SOLUTION.—The original capital, \$26,500, is the base, and .16 is the

rate. Since the capital is diminished, the portion of it remaining is the difference. Applying the rule,

$$\text{difference} = \$26,500 \times (1 - .16) = \$26,500 \times .84 = \$22,260. \text{ Ans.}$$

28. In the proportions of Arts. 23 and 24, the fourth term is equal to the product of the means divided by the other extreme. That is,

$$\text{base} = \text{amount} \div (1 + \text{rate}), \text{ or } B = \frac{A}{1 + R}.$$

$$\text{base} = \text{difference} \div (1 - \text{rate}), \text{ or } B = \frac{D}{1 - R}.$$

29. Rule.—*To find the base, the amount and rate being given, divide the amount by 1 plus the rate.*

EXAMPLE. In 1895 the population of a village was 4,130, which was 18% more than the population in 1890. What was the population in 1890?

SOLUTION.—The unknown population in 1890 is the base upon which the 18% is computed. The final population in 1895 is the amount, since it is an increase. Applying the rule,

$$\text{base} = 4,130 \div (1 + .18) = 4,130 \div 1.18 = 3,500. \text{ Ans.}$$

30. Rule. *To find the base, the difference and rate being given, divide the difference by 1 minus the rate.*

EXAMPLE. A speculator lost 34% of an investment in stocks and had \$10,560 remaining. What was the original investment?

SOLUTION. The original investment on which the 34% is computed, is the base. Since there has been a loss, or decrease, the remaining \$10,560 is the difference. Applying the rule,

$$\text{base} = \$10,560 \div (1 - .34) = \$10,560 \div .66 = \$16,000. \text{ Ans.}$$

31. The difficulty that the student is most likely to experience in percentage is the identification of the terms or elements. When an example is given, he must first determine from it which is the base, the percentage, the amount, etc. The rate is always recognized by the words *per cent.* or by the sign %. The base is the most important element, and it can be identified by referring to the rate. The student asks himself: "Of what number do I wish to find the percentage?" The answer to this question is the base. The percentage is always a number of the same kind as the base. If, from the statement of the problem, it is seen that the base increases or decreases, the percentage is the increase or decrease, and the final value

obtained, when the base has been increased or decreased by the percentage, is the amount or the difference. To be able to recognize the elements immediately requires much practice. As an exercise, several examples are given.

32. EXAMPLE.— 15 tons of iron are obtained from 282 tons of ore. What per cent. of the ore is iron ?

SOLUTION.—Here the statement of the example shows that the rate is what is required. From the phrase, "what per cent. of the ore," we see that the ore is the thing of which a per cent. is taken. Therefore, the 282 tons must be the base. 15 tons is a number of the same kind as the base, and is the part of the base corresponding to the rate. It is, therefore, the percentage.

$$\text{Rate} = \text{percentage} \div \text{base} = 15 \div 282 = .05319 = 5.319\%. \quad \text{Ans.}$$

EXAMPLE.—Out of a cargo of oranges, 8% spoiled and 4,600 boxes remained. How many boxes in the cargo? How many boxes spoiled?

SOLUTION.—The rate is .08; .08 of what? The number of boxes in the cargo. Therefore, the base is the original number of boxes and the less number of boxes must be the difference. The number of boxes spoiled is the percentage.

$$\begin{aligned} \text{Base} &= \text{difference} \div (1 - \text{rate}) \\ &= 4,600 \div (1 - .08) = 4,600 \div .92 = 5,000 \text{ boxes.} \quad \text{Ans.} \end{aligned}$$

$$\text{Percentage or number of boxes spoiled} = 5,000 \times .08 = 400 \text{ boxes.} \quad \text{Ans.}$$

EXAMPLE.—A farmer lost 63 sheep, which was 18% of his flock. How many had he left?

SOLUTION.—The rate is .18. The number of sheep in the flock must be the base, since it is the number on which the 18% is computed. The decrease, or number of sheep lost, is the percentage, and the number remaining is the difference.

$$\begin{aligned} \text{Base} &= \text{percentage} \div \text{rate} \\ &= 63 \div .18 = 350 \text{ sheep} = \text{number originally in the flock.} \end{aligned}$$

$$\text{Difference} = \text{base} - \text{percentage} = 350 - 63 = 287 \text{ sheep left.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE.

33. Solve the following:

- | | |
|--|---|
| (a) What is $12\frac{1}{2}\%$ of \$900? | Ans. $\left\{ \begin{array}{l} (a) \quad \$112.50. \\ (b) \quad 5.016. \\ (c) \quad 18. \\ (d) \quad 146\frac{1}{11}. \\ (e) \quad 940.8. \\ (f) \quad 16\frac{2}{3}\%. \\ (g) \quad 12\frac{1}{2}\%. \\ (h) \quad 40\%. \end{array} \right.$ |
| (b) What is $\frac{1}{3}\%$ of 627? | |
| (c) What is $33\frac{1}{3}\%$ of 54? | |
| (d) 101 is $68\frac{2}{3}\%$ of what number? | |
| (e) 784 is $83\frac{1}{3}\%$ of what number? | |
| (f) What per cent. of 960 is 160? | |
| (g) What per cent. of \$3,606 is \$450 $\frac{3}{4}$? | |
| (h) What per cent. of 280 is 112? | |

1. A man's salary is \$1,800 per year and he saves \$225. (a) What per cent. of his salary does he save? (b) What per cent. of it does he spend?

Ans. $\begin{cases} (a) & 12\frac{1}{2}\% \\ (b) & 87\frac{1}{2}\% \end{cases}$

2. A man has 32% of his money invested in stocks, 18% in grain, and the remainder, which is \$7,620, in real estate. What is the total value of his property?

Ans. \$15,240.

3. If wool loses 32% of its weight in washing, how many pounds of unwashed wool are required to produce 35,360 pounds of washed wool?

Ans. 52,060 pounds.

4. In 1890 the population of a city was 85,000, which was 36% more than the population in 1880. What was the population in 1880?

Ans. 62,500.

5. If gunpowder contains 75% of saltpeter, 10% of sulphur, 15% of charcoal, how much of each is there in a ton of powder?

Ans. $\begin{cases} \text{Saltpeter,} & 1,500 \text{ lb.} \\ \text{Sulphur,} & 200 \text{ lb.} \\ \text{Charcoal,} & 300 \text{ lb.} \end{cases}$

6. A man bequeathed to a charity 32% of his estate. To another charity he gave \$23,100 which was 23% less than the amount given to the first charity. (a) What was the value of the estate? (b) What per cent. of the estate was given to the second charity?

Ans. $\begin{cases} (a) & \$93,750. \\ (b) & 24.04\% \end{cases}$

7. A man owning a ship worth \$225,000, sells $\frac{1}{3}$ of it to A, 20% of the remainder to B, and 35% of what then remains, to C. How much each do A, B, and C pay for their shares?

Ans. $\begin{cases} A, & \$56,250. \\ B, & \$33,750. \\ C, & \$47,250. \end{cases}$

PROFIT AND LOSS.

34. Profit and loss treats of the gains or losses arising in business transactions.

If the price for which merchandise is sold is greater than the cost of the merchandise, the difference is *profit* or *gain*. If the selling price is less than the cost, the difference is *loss*.

35. The *gross cost* of merchandise is its first cost plus the expenses of purchase, transportation, and storage. Such expenses are commission, freight, insurance, drayage, etc.

36. The **net selling price** is the gross selling price, less all discounts and expenses of sale.

37. Computations in profit and loss are made according to the rules of percentage. The gross cost of the merchandise is the *base*, upon which the *rate* of profit or loss is computed. The profit or loss is the *percentage*. If the merchandise is sold at a profit, the net selling price is the *amount*; if at a loss, the net selling price is the *difference*.

38. Rule.—*To find the profit or loss, multiply the gross cost by the rate of gain or loss. (Art. 14.)*

Formula.— $\text{Profit or loss} = \text{cost} \times \text{rate}.$

EXAMPLE.—A house costing \$3,000 is sold for 22% above cost. What is the profit?

SOLUTION.— $\text{Profit} = \text{cost} \times \text{rate} = \$3,000 \times .22 = \$660.$ Ans.

39. Rule.—*To find the rate of profit or loss, divide the difference between the selling price and gross cost by the gross cost; or divide the profit or loss by the gross cost. (Art. 18.)*

Formula.— $\text{Rate} = \text{profit or loss} \div \text{gross cost}.$

EXAMPLE.—A merchant sold for \$768 a lot of dry goods for which he paid \$900. What was the per cent. loss?

SOLUTION.— $\text{Loss} = \$900 - \$768 = \$132.$

$\text{Rate of loss} = \text{loss} \div \text{cost} = \$132 \div \$900 = .14\frac{2}{3}$ or $14\frac{2}{3}\%.$ Ans.

40. Rule.—*To find the selling price, the cost and rate of gain or loss being given, multiply the cost by 1 plus the rate of gain, or by 1 minus the rate of loss.*

Formulas.—

$$\text{Selling price} = \begin{cases} \text{Cost} \times (1 + \text{rate of gain}). & (\text{Art. 26.}) \\ \text{Cost} \times (1 - \text{rate of loss}). & (\text{Art. 27.}) \end{cases}$$

EXAMPLE.—If hay is bought for \$8 per ton, and if baling and shipping costs \$5.50 per ton additional, at what price must it be sold to yield a profit of 16%?

SOLUTION.— $\text{Gross cost} = \$8 + \$5.50 = \$13.50.$

$\text{Selling price} = \text{cost} \times (1 + \text{rate}) = \$13.50 \times 1.16 = \$15.66.$ Ans.

41. Rule.—*To find the cost, the selling price and rate of gain or loss being given, divide the selling price by 1 plus the rate of gain, or by 1 minus the rate of loss.*

Formulas.—

$$\text{Cost} = \begin{cases} \text{Selling price} \div (1 + \text{rate of gain}). & (\text{Art. 29.}) \\ \text{Selling price} \div (1 - \text{rate of loss}). & (\text{Art. 30.}) \end{cases}$$

EXAMPLE.—Sold drugs for \$112 and gained 75%. What was the cost of the drugs, and what was the profit?

SOLUTION.—

$$\text{Cost} = \text{Selling price} \div (1 + \text{rate}) = \$112 \div 1.75 = \$64. \quad \text{Ans.}$$

$$\text{Profit} = \$112 - \$64 = \$48. \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE.

42. What is the profit or loss

- (a) If the gross cost is \$85 and the rate of gain is 22%?
 (b) If the gross cost is \$87.50 and the rate of loss is 12%?
 (c) If the gross cost is \$240 and the rate of gain is 16%?

$$\text{Ans.} \begin{cases} (a) & \$27.20. \\ (b) & \$100.50. \\ (c) & \$40.00. \end{cases}$$

What is the rate of gain or loss

- (d) If the gross cost is \$6.50 and selling price is \$9.10?
 (e) If the gross cost is \$14.00 and selling price is \$12.50?
 (f) If the gross cost is \$3,500 and profit is \$500?

$$\text{Ans.} \begin{cases} (d) & 40\%. \\ (e) & 10\frac{1}{2}\%. \\ (f) & 14\frac{2}{3}\%. \end{cases}$$

What is the selling price

- (g) If the cost is \$945 and the rate of gain is 33%?
 (h) If the cost is \$3.50 and the rate of gain is 12%?
 (i) If the cost is \$125 and the rate of loss is 18%?

$$\text{Ans.} \begin{cases} (g) & \$1,260. \\ (h) & \$3.94. \end{cases}$$

2. What must be the selling price of a suit of clothes which cost \$18 in order that the profit may be $33\frac{1}{3}\%$? Ans. \$24.

3. A harvesting machine costs the hardware merchant \$90 net, and \$6 for freight and cartage. If sold for \$108, what is the gain per cent.? Ans. $12\frac{1}{2}\%$.

4. A carload of cattle is sold for \$875, which is at a loss of 16%. What was the cost of the cattle? Ans. \$1,041.67.

5. A sells a steam tug to B, gaining 14%, and B sells it to C for \$4,104 and gains 20%. How much did the tug cost A? Ans. \$3,000.

6. How much must hay sell for per ton, to gain 25%, if when sold for \$8.40 per ton, there is a gain of $16\frac{2}{3}\%$? Ans. \$9.

7. Six horses were sold at \$125 each; three of them at a profit of 25% and the others at a loss of 25%. What was the net gain or loss? Ans. \$50 loss.

TRADE DISCOUNTS.

43. Trade discounts are reductions made by manufacturers, jobbers, or merchants from their list or catalogue prices.

In many branches of business, manufacturers and dealers list their goods at a fixed price for each article, and allow a rate of discount on orders of a certain amount, a second discount on orders of larger amount, and perhaps a third discount on still larger orders. If it becomes necessary to raise or lower the price of the goods, the rate of discount is decreased or increased, the list price remaining the same. The system of discounts thus saves the expense of publishing a new price list every time prices change.

44. Merchandise is frequently sold at *time prices*; that is, payment is to be made in 30, 60, or 90 days after date of sale, and a certain rate of discount is allowed if payment is made at an earlier date. Business houses usually make announcements such as the following upon their bill heads: "Terms: 4 mo., or 5% 60 days;" "Terms: 60 days net; 30 days, 3% off; 10 days, 5% off." Even when no discount is stated in the terms, sellers will usually deduct the legal interest for the time remaining, if the payment is made

before it becomes due. Thus, if a payment due in 3 months is made 1 month after the due, the seller should deduct the interest for the remaining 2 months.

45. Trade discounts are computed by the rules of percentage, the list price of the goods being the base. When several discounts are allowed, the first discount is computed on the list price, the second is computed on the remainder after deducting the first discount, and so on, each remainder being regarded as a base for the computation of the next discount. The several discounts, if there are more than one, form a discount series.

46. Rule.—To find the selling price, multiply the list price by the rate, and subtract the discount thus derived from the list price. If there is a discount series, compute the second discount, using the first remainder as a base, and subtract the discount from the remainder. Repeat the process, using each successive remainder as a base for computing the next discount. The last remainder is the selling price.

EXAMPLE.—The list price of an article is \$62.50 and a discount of 40% is allowed. What is the selling price?

SOLUTION.—*First Method.*—

$$\text{Discount} = \$62.50 \times .40 = \$25.00.$$

$$\text{Selling price} = \$62.50 - \$25.00 = \$37.50 \quad \text{Ans.}$$

Second Method.—Since the list price is the base, and the selling price is the base less the percentage, i. e., the difference, the selling price may be found by applying the rule of Art. 27. Thus,

$$\text{Selling price} = \$62.50 \times (1 - .40) = \$37.50 \quad \text{Ans.}$$

EXAMPLE.—On a bill of goods amounting to \$720, discounts of 30%, 10%, and 5% are allowed. What is the selling price?

SOLUTION.—*First Method.*—

$$\text{First discount} = \$720 \times .30 = \$216.$$

$$\text{Remainder} = \$720 - \$216 = \$504.$$

$$\text{Second discount} = \$504 \times .10 = \$50.40.$$

$$\text{Remainder} = \$504 - \$50.40 = \$453.60.$$

$$\text{Third discount} = \$453.60 \times .05 = \$22.68.$$

$$\text{Selling price} = \$453.60 - \$22.68 = \$430.92 \quad \text{Ans.}$$

Second Method.—Regarding \$720 as divided into 100 parts, the first discount of 30% leaves $100 - 30 = 70$ parts. The second discount of 10% is computed on the remainder after the first discount has been deducted

that is, the second discount is 10% of 70 parts. Hence, the remainder after the second discount has been deducted is $70 \times (1 - .10) = 63$ parts. Similarly, the remainder after the third discount has been deducted is $63 \times (1 - .05) = 59.85$ parts, or 59.85% of \$720. Therefore,

$$\text{Selling price} = \$720 \times .5985 = \$430.92. \text{ Ans.}$$

Ordinarily, when applying this method, the work would be as follows:

$$\begin{aligned} & \$720 \times (1 - .30) \times (1 - .10) \times (1 - .05) \\ & = \$720 \times .70 \times .90 \times .95 = \$430.92. \text{ Ans.} \end{aligned}$$

47. The discounts usually allowed are aliquot parts of 100%, and the labor of computation may be shortened by using the fractions corresponding to the rates of discount.

EXAMPLE.—The gross amount of a bill of hardware is \$640, and discounts of 25%, 10%, and 5% are allowed. What is the net amount of the bill?

SOLUTION.—

$$25\% = \frac{1}{4}, 10\% = \frac{1}{10}, 5\% = \frac{1}{20}.$$

The solution is arranged as shown. To multiply \$640 by 25% or $\frac{1}{4}$, we divide by 4, then the discount \$160 is subtracted and the remainder, \$480, is divided by 10.

The second discount, \$48, is subtracted, and the remainder is

divided by 20. The final remainder, \$410.40, is the net amount or selling price.

4) \$ 6 4 0	
\$ 1 6 0	1st discount.
1 0) \$ 4 8 0	1st remainder.
\$ 4 8	2d discount.
2 0) \$ 4 3 2	2d remainder.
\$ 2 1 6 0	3d discount.
\$ 4 1 0 4 0	net amount. Ans.

48. When a discount series is allowed, business men usually reduce the series to an equivalent single discount; if there is a large number of sales, much labor of computation is saved by using the equivalent discount rather than the series.

49. Rule.—*To reduce a discount series to an equivalent single discount, subtract each rate of discount from 1, and multiply the remainders together. Subtract the product from 1, and the remainder will be the single discount. (See second example, second method, Art. 46.)*

EXAMPLE.—What single discount on the gross price is equivalent to a discount series of 25%, 20%, and 10%?

$$\text{SOLUTION} \quad 1 - .25 = .75; 1 - .20 = .80, 1 - .10 = .90.$$

$$.75 \times .80 \times .90 = .54.$$

$$1 - .54 = .46, \text{ or } 46\%. \text{ Ans.}$$

50. EXAMPLE—The cost of a line of goods is \$350. What must they be marked to give a profit of 20% and allow a discount of 30% on the selling price?

SOLUTION *First Method.*—The profit is $\$350 \times 20 = \70 , therefore, the actual selling price is $\$350 + \$70 = \$420$. This is what remains after deducting 30% from the marked price. Since the 30% discount is computed on the marked price, that price must be the base, and the less price, \$420, obtained by subtracting the discount, is the difference. According to the rule, Art. 30, $\text{base} = \text{difference} \div (1 - \text{rate})$, hence,

$$\text{marked price} = \$420 \div (1 - .30) = \$420 \div .70 = \$600 \quad \text{Ans.}$$

Second Method—Regarding the \$350 as divided into 100 parts, 20 parts must be added to this in order to gain 20%, that is, the selling price must be 120 parts or 120% of \$350. Now, if a discount of 30% is to be allowed from the list price and leave a remainder of 120 parts, it is evident that 120 parts is the difference, and the list price is the base. According to rule, Art. 30,

$$\text{list price (base)} = \text{selling price (difference)} \div (1 - \text{rate}), \text{ or}$$

$$\text{list price} = 120 \div (1 - .30) = 171\frac{1}{3} \text{ parts, or } 171\frac{1}{3}\%.$$

$$\text{Hence, list price} = \$350 \times 171\frac{1}{3}\% = \$600 \quad \text{Ans.}$$

51. Rule.—To find the price at which goods must be marked to insure a given profit after allowing a discount, or a discount series, add to the cost the profit required, and divide the sum by 1 minus the discount or equivalent single discount.

EXAMPLE.—The cost of manufacturing hats is \$36 per dozen. At what price per dozen must they be marked that the manufacturer may realize 16 $\frac{2}{3}$ % profit after allowing the trade discounts of 20% and 12 $\frac{1}{2}$ %?

SOLUTION—*First Method*— $16\frac{2}{3}\% = \frac{1}{3}$. Profit = $\$36 \times \frac{1}{3} = \6 , selling price = $\$36 + \$6 = \$42$. $1 - .20 = .80$, $1 - .12\frac{1}{2} = .87\frac{1}{2}$, $.80 \times .87\frac{1}{2} = .70$. The equivalent single discount is $1 - .70 = .30$. Marked price $\$42 \div (1 - .30) = \60 per dozen. Ans.

Second Method—Selling price expressed as per cent. = $100 + 16\frac{2}{3} = 116\frac{2}{3}$. Marked price expressed as per cent = $116\frac{2}{3} \div (1 - 20) \times (1 - 12\frac{1}{2}) = 116\frac{2}{3} \div .70 = 166\frac{2}{3}\%$. Hence, marked price = $\$36 \times 1.66\frac{2}{3} = \60 per dozen. Ans.

EXAMPLES FOR PRACTICE.

52. Reduce the following discount series to equivalent single discounts

(a) 25% and 16%.

(b) 30%, 20%, and 5%.

(c) 60%, 10%, and 5%

(d) 40%, 20%, 12 $\frac{1}{2}$ %, and 4%.

Ans. $\left\{ \begin{array}{l} (a) \quad 37\% \\ (b) \quad 46.8\% \\ (c) \quad 65.8\% \\ (d) \quad 59.68\% \end{array} \right.$

1. A musical instrument is listed at \$122 and discounts of 60% and 5% are allowed. What is the selling price? Ans. \$46.36.
 2. A bill of hardware is sold at the following discounts: \$452.00 at 80% and 10%, \$216 at 33 $\frac{1}{3}$ % and 5%; \$137.50 at 20%, and \$83.75 net. What is the total net amount of the bill? Ans. \$615.69
 3. A bill of goods amounting to \$886.72 was bought May 5, 1890, on the following terms: 4 months, or 5% off 30 days. How much would pay the bill June 4, 1890? Ans. \$794.88.
 4. A wholesale dealer sells books at discounts of 20% and 5%. What must he mark a set of books that cost him \$24 in order to make 26 $\frac{1}{2}$ % profit, after allowing discounts? Ans. \$40.
 5. Plows are bought at a discount of 40% from the list price. What per cent. is gained by selling them at the list price? Ans. 60 $\frac{1}{2}$ %.
 6. A wholesale dealer offers silks at \$3.50 per yard, subject to a discount of 20%, 12 $\frac{1}{2}$ %, and 10%. How many yards can be bought for \$352.80? Ans. 160.
 7. A suit of clothing is marked 50% off. By selling at this price the clothier loses 12 $\frac{1}{2}$ % of the cost of the suit, which was \$12. What was the marked price of the suit? Ans. \$21.00.
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COMMISSION AND BROKERAGE.

53. Commission, or brokerage, is the sum paid an agent for transacting business for another person; as, for buying or selling merchandise or property, for collecting or investing money, etc.

54. The agent or party who transacts the business is called a **commission merchant**, or **broker**; the party for whom the business is transacted is called the **principal**. The term broker is applied to one who sells and buys stocks, bonds, bills of exchange, and money securities.

55. A **consignment** is a shipment of goods from one party to another; the party that ships the goods is called the **consignor**, or **shipper**, and the party to whom they are shipped is called the **consignee**.

56. When goods are sold on credit, the agent charges an additional amount for guaranteeing the payment of the sale. This extra charge is termed **guaranty**.

57. The **gross proceeds** of a sale or collection is the total amount realized by the agent before deducting his commission and other expenses connected with the transaction. The **net proceeds** is the amount due the principal after the commission and all other charges have been deducted.

58. An **account sales** is a detailed statement made by the agent to his principal, showing the goods sold and the prices obtained, giving a list of the charges and expenses, and the net proceeds due the principal. The charges include freight, cartage, storage, insurance, inspection, advertising, commission, and guaranty.

59. The **prime cost** of a purchase is the sum paid by the agent for the goods or property. The **gross cost** is the prime cost plus the commission and expenses incident to the purchase.

60. An **account purchase** is a detailed statement made by the agent to his principal, showing the cost of goods or property bought, the expenses attending the purchase, and the gross cost.

61. The commission or brokerage is usually computed at a certain per cent. of the gross proceeds of a sale or the prime cost of a purchase. In some cases, however, it is computed at a certain price per unit of weight or measure; as, so much per ton, per bushel, or per barrel. Examples in commission are solved by the rules of percentage. Either the gross proceeds or prime cost is the *base*; the net proceeds is the *difference*; the gross cost is the *amount*; the commission is the *percentage*, and the rate of commission is the *rate per cent.* The remittance from the principal to the purchasing agent, including both the investment and the commission, is an *amount*. The following rules are derived directly from the principles of percentage:

62. Rule.—*To find the commission, multiply the prime cost or gross selling price by the rate of commission.*

Formula. -

$Commission = cost\ or\ selling\ price \times rate\ of\ commission$

EXAMPLE.—A real estate agent sells a house and lot for \$4,375 and receives 2% commission. What is the commission and what are the net proceeds?

SOLUTION.—Commission = selling price \times rate = $\$4,375 \times .02$
= \$87.50. Ans.

Net proceeds = selling price — commission = $\$4,375 - \87.50
= \$4,287.50. Ans.

63. Rule. *To find the prime cost or gross proceeds, the commission being given, divide the commission by the rate of commission.*

Formula.

Prime cost or gross proceeds = commission \div rate of commission.

EXAMPLE.—An agent received \$319.50 commission for selling apples. If the rate of commission charged was 1½%, what was the selling price of the apples?

SOLUTION.—Selling price or gross proceeds = $\$319.50 \div .015$
= \$21,300. Ans.

64. Rule. *—To find the prime cost and commission, the remittance from the principal being given, subtract from the remittance the expenses of the purchase, if any, and divide the remainder by 1 plus the rate of commission. The quotient is the prime cost. Subtract the prime cost from the remainder and the difference is the commission.*

EXAMPLE.—A principal sends his agent \$21,611 with orders to buy cotton after deducting his commission and other charges. The agent paid \$124.30 for freight, \$51.70 for cartage, \$15.00 for insurance, and deducted his commission of 2%. (a) How much remained to invest in cotton? (b) What was his commission?

SOLUTION. The expenses are first deducted $\$21,611 - (\$124.30 + \$51.70 + \$15)$ = \$21,420, which is the sum of the prime cost and commission. According to the rule,

Prime cost = $\$21,420 \div (1 + .02)$ = $\$21,420 \div 1.02$ = \$21,000. Ans.

Commission = $\$21,420 - \$21,000$ = \$420. Ans.

NOTE.—When a charge is made for guaranty, add the per cent. of guaranty to 1 plus the rate of commission, and proceed as above.

EXAMPLES FOR PRACTICE.

65. What is the commission

- (a) If the gross proceeds are \$300 and the rate of commission is 3½%?
- (b) If the gross proceeds are \$9,375 and the rate of commission is 2%?

- (c) If the prime cost is \$891.75 and the rate of commission is $1\frac{1}{2}\%$?
 (d) If the prime cost is \$960 and the rate of commission is $\frac{1}{2}\%$?

$$\text{Ans. } \left\{ \begin{array}{ll} (a) & \$10.50. \\ (b) & \$187.50. \\ (c) & \$10.40. \\ (d) & \$0.00. \end{array} \right.$$

What are the net proceeds

- (e) If the gross proceeds are \$340, rate of commission 3%, and other expenses \$4.90 ?
 (f) If the gross proceeds are \$6,375, rate of commission 4%, and other expenses \$32.50 ?
 (g) If the gross proceeds are \$195.40, rate of commission $1\frac{1}{2}\%$, and other expenses \$7.45 ?

$$\text{Ans. } \left\{ \begin{array}{ll} (e) & \$325.50. \\ (f) & \$6,087.50. \\ (g) & \$185.02. \end{array} \right.$$

What is the prime cost if

- (h) The gross cost is \$520 and the rate of commission is 4% ?
 (i) The gross cost is \$1,606 and the rate of commission is $\frac{1}{2}\%$?
 (j) The gross cost is \$843, the rate of commission is 2%, and the expenses of buying are \$27 ?

$$\text{Ans. } \left\{ \begin{array}{ll} (h) & \$500. \\ (i) & \$1,600. \\ (j) & \$800. \end{array} \right.$$

1. An agent receives \$13.00 commission for selling \$650 worth of goods. What rate of commission does he charge ? Ans. 2%.

2. A commission merchant sold a quantity of wool for \$4,650. He charged $2\frac{1}{2}\%$ commission, 2% guaranty, and the transportation, storage, and other expenses amounted to \$184. How much should he send his principal ? Ans. \$4,256.75.

3. An agent received \$550.50 to buy potatoes after deducting all expenses. He paid \$26.50 for drayage, \$32 for barrels, and charged $2\frac{1}{2}\%$ commission for buying. How many bushels did he buy at 60 cents per bushel ? Ans. 800 bushels.

4. A commission merchant has consigned to him 400 barrels of flour which he sells at \$4.75 per barrel, and charges $2\frac{1}{2}\%$ commission. With the net proceeds he buys sugar at $6\frac{1}{2}$ cents a pound and charges $2\frac{1}{2}\%$ commission for buying. (a) How many pounds of sugar does he buy ? (b) What is the amount of his commissions ? Ans. $\left\{ \begin{array}{ll} (a) & 28,917\frac{1}{2} \text{ lb.} \\ (b) & \$92.68. \end{array} \right.$

5. A New York agent received \$1,134 with which to purchase hats. If he charges 3% commission and 2% additional for guaranty of quality, how many dozen hats can he buy at \$13.50 per dozen ? Ans. 80 dozen.

6. A commission merchant sold 2,500 bushels of wheat at 64 cents a bushel and a quantity of corn at 23 cents per bushel. The rate of commission on each sale was $1\frac{1}{4}\%$, and the total commission was \$35.084. How many bushels of corn were sold? Ans. 1,760 bu.

INSURANCE.

66. Insurance is a contract by which one party, the **underwriter**, or **insurer**, agrees, for a consideration, to make good a loss sustained by another party.

67. Insurance is of two kinds, **property insurance** and **personal insurance**. Property insurance includes *fire insurance* (indemnity for loss or damage by fire); *marine insurance* (indemnity for losses at sea); *transit insurance* (indemnity for loss of, or damage to, merchandise during transportation); *stock insurance* (indemnity for loss of live stock); and *accident insurance* (indemnity for breakage of fragile materials, as plate glass, etc.).

Personal insurance includes *life insurance*, which secures the payment of a certain amount to a specified person at the death of the party insured, or after the lapse of a specified time; *accident insurance*, which secures the payment of a certain sum in case of accident to the insured; *health insurance*, which secures the payment of a weekly sum during sickness; and insurance against the dishonesty of employees.

68. The **policy** is the written contract between the insurance company and the party insured; it contains a description of the property insured, the conditions upon which the insurance is taken, and the amount to be paid in case of loss.

69. The **premium** is the amount paid to the insurer for assuming the risk of loss or damage. The premium is a certain per cent. of the amount of insurance, as $\frac{5}{8}\%$, $\frac{3}{4}\%$. The rate of premium depends upon the nature of the risk and upon the length of time the insurance has to run. It is customary to speak of the rate of premium as the cost per \$100 of insurance; as 60 cents per \$100, \$1.20 per \$100, etc.

70. In property insurance, all computations are based on the rules of percentage. The amount of insurance is the *base*, the premium is the *percentage*, and the rate of premium is the *rate*.

71. Rule.—*To find the premium, multiply the amount of insurance by the rate of premium.* (Art. 14.)

Formula.—

$$\text{Premium} = \text{amount of insurance} \times \text{rate}.$$

EXAMPLE.—A house and furniture are insured against fire for \$3,250, the rate of premium being $\frac{3}{4}\%$, or 75 cents per \$100 per year. What is the yearly premium?

SOLUTION.— $\frac{3}{4}\% = .0075$. Using the rule,

$$\text{Premium} = \$3,250 \times .0075 = \$24.37\frac{1}{2}. \text{ Ans.}$$

72. Rule.—*To find the amount of insurance, divide the premium by the rate of premium. To find the rate of premium, divide the premium by the amount of insurance.*

Formulas.—

$$\text{Amount of insurance} = \text{premium} \div \text{rate of premium}.$$

$$\text{Rate of premium} = \text{premium} \div \text{amount of insurance}.$$

EXAMPLE.—What amount of insurance can be obtained for \$137.50, the rate of premium being 55 cents per \$100?

SOLUTION.—Rate = $55 \div 100.00 = .0055$.

$$\text{Amount of insurance} = \text{premium} \div \text{rate} = \$137.50 \div .0055 = \$25,000. \text{ Ans.}$$

EXAMPLE.—A warehouse and contents together valued at \$13,500 is insured for $\frac{2}{3}$ of its value. If the premium is \$78.75 what is the rate of premium?

SOLUTION.—Amount of insurance = $\$13,500 \times \frac{2}{3} = \$9,000$.

$$\text{Rate of premium} = \$78.75 \div \$9,000 = .00875 = 87\frac{1}{2} \text{ cents per } \$100, \text{ or } \frac{7}{8}\%. \text{ Ans.}$$

EXAMPLES FOR PRACTICE.

73. 1 A store and contents valued at \$10,400 is insured for $\frac{3}{4}$ of its value at $1\frac{1}{4}\%$ premium. What is the cost per year of insurance?

Ans. \$110 70.

2. A premium of \$162.50 is paid for the insurance of a steamer worth \$32,500, which is insured for $\frac{1}{2}$ of its value. What is the premium in cents per \$100? Ans. 62½ cents per \$100.

3. A stock of goods is insured for $\frac{1}{2}$ of its value. The premium paid is \$43.75 and is at a rate of 75 cents per \$100. What is the value of the stock of goods? Ans. \$8,750.

4. A boat load of 8,600 bushels of corn, worth 32 cents per bushel, is insured for $\frac{1}{2}$ of its value at 1½% premium. If the corn is totally destroyed, what will be the owner's loss? Ans. \$721.54.

5. A building worth \$9,600 is insured for $\frac{1}{2}$ of its value in three companies. The first company takes $\frac{1}{3}$ of the risk at ½% premium; the second $\frac{1}{3}$ of it at ¾%; and the third the remainder at 7%. What is the total premium? Ans. \$50.40.

6. If it cost \$324 to insure a property for \$21,600, what will it cost at the same rate to insure a similar property for \$35,000, if \$1.50 is charged for the policy in the latter case? Ans. \$526.50.

7. A fire insurance company took a risk of \$42,000 at ¾% premium, and reinsured $\frac{1}{3}$ of it in another company at ½%, and $\frac{1}{2}$ of it in a third company at ¾%. What did the company gain by reinsuring? Ans. \$61.25.

TAXES.

74. **Taxes** are sums of money levied on persons, properties, or incomes, for public purposes. Thus, taxes are levied to support the state, county, and city governments; to support schools and charities; and to make improvements, such as paved streets and sewers.

75. A **capitation**, or **poll tax**, is a tax levied on persons; a **property tax** is a tax levied on real estate or personal property; an **income tax** is levied on incomes or salaries. The poll tax is usually a fixed amount for each citizen over 21 years of age. The property tax is estimated at a certain per cent. of the assessed valuation of the property subject to taxation.

76. **Rule.**—*To find a property tax, multiply the assessed value of the property by the rate of taxation.*

EXAMPLE.—What property tax must a person pay who owns real estate assessed at \$34,000 and personal property assessed at \$12,500, the rate of taxation being 8 mills per \$1.00, or rate = .008?

SOLUTION.—Total assessed value = \$34,000 + \$12,500 = \$46,500.

Tax = \$46,500 \times .008 = \$372. Ans.

77. Rule.—To find the rate of taxation, subtract from the total tax to be raised the poll tax, if any; divide the remainder by the total assessed valuation of the property to be taxed.

This rule is derived directly from the rule, Art. 18, since the tax is the *percentage*, and the assessed valuation is the *base*.

EXAMPLE.—The assessed valuation is \$1,375,000 on real estate and \$575,000 on personal property. The town votes to raise \$12,000 for schools, \$2,100 for streets and highways, \$3,000 for salaries, \$1,500 for support of the poor and \$650 for contingent expenses. There is a poll tax of \$1.00 each on 3,650 persons. What is the rate of taxation and how much must A pay who has property assessed at \$26,400 and pays for 3 polls?

SOLUTION.—Total tax = \$12,000 + \$2,100 + \$3,000 + \$1,500 + \$650 = \$19,250. The poll tax is \$3,650, therefore, the property tax is \$19,250 - \$3,650 = \$15,600.

Total assessed value of property = \$1,375,000 + \$575,000 = \$1,950,000.

Rate of tax on \$15,600 \div \$1,950,000 = .008 or 8 mills per \$1.00. Ans.

A's property tax is \$26,400 \times .008 = \$211.20, his poll tax is \$3.00.

\$211.20 + \$3.00 = \$214.20, A's total tax. Ans.

EXAMPLES FOR PRACTICE.

78. What is the tax if the assessed value of the property is

(a) \$6,300 and the rate of taxation is $1\frac{1}{2}\%$?

(b) \$34,300 and the rate of taxation is 64 mills per \$1.00?

(c) \$9,430 and the rate of taxation is 85 cents per \$100?

Ans. $\left\{ \begin{array}{l} (a) \ \$70.87\frac{1}{2} \\ (b) \ \$222.95 \\ (c) \ \$80.15\frac{1}{2} \end{array} \right.$

What is the rate of taxation (mills per \$1.00) if the assessed valuation is

(d) \$825,000 and the gross tax is \$5,230, including 460 polls at \$.50 each?

(e) \$1,364,000 and the gross tax is \$18,130, including 1,440 polls at \$.75 each?

(✓) \$34,000 and the gross tax is \$469.20 ?

Ans. $\left\{ \begin{array}{l} (d) \text{ 8 mills.} \\ (e) \text{ 12}\frac{1}{2} \text{ mills.} \\ (✓) \text{ 13.8 mills.} \end{array} \right.$

1. I own real estate worth \$8,500 and personal property worth \$3,750; both are assessed at $\frac{3}{4}$ of their value. The rate of taxation is 1.2%, but I receive a discount of $2\frac{1}{2}$ per cent. of my taxes for prompt payment. How much do I pay the tax receiver ? Ans. \$86.

2. A has 5 lots worth \$1,300 each; B has 4 lots worth \$1,000 each; C has 2 lots worth \$1,500 each; D has 7 lots worth \$800 each, and E has 1 lot worth \$1,150. A tax of \$1,417.50 for a street improvement is to be divided among them. What should each pay ?

Ans. $\left\{ \begin{array}{l} \text{A, \$455.} \\ \text{B, \$280.} \\ \text{C, \$210.} \\ \text{D, \$392.} \\ \text{E, \$80.50.} \end{array} \right.$

3. The rate of taxation for a certain state is $3\frac{1}{2}$ mills per \$1.00. How much state tax must be raised by a county whose valuation is fixed by the State Board of Equalization at \$13,876,394 ? Ans. \$48,567.38.

DUTIES.

79. Duties, or customs, are taxes levied by governments on imported goods for the purpose of producing revenue and for the protection of home industries.

80. There are two kinds of duties: *ad valorem* and *specific*. An *ad valorem* duty is estimated at a certain per cent. of the market value of the goods in the country from which they are imported; as, silks 50%, musical instruments 15%, etc. The market value of the goods is the invoice value after deducting discounts and before extra charges, such as commission, freight, boxing, etc., are added.

81. A *specific* duty is a duty levied on imported goods according to the weight, measurement, or number of the articles, without reference to their value; as, wheat 15 cents per bushel, coal 75 cents per ton, etc. Some kinds of merchandise are subject to both *ad valorem* and *specific* duties. In computing *specific* duties, the long ton of 2,240 pounds and the hundredweight of 112 pounds are used.

82. An **invoice** is an itemized statement of the merchandise shipped. It contains the names of purchaser and seller, a description of the quality and quantity of the goods, the price and incidental charges. Invoices are made out in the weights and measures and the currency of the country from which the goods are imported. Thus, the price and cost of goods imported from Germany would be given in *marks*; from France, in *francs*; from England, in *£ s. d.*

83. The following table gives the monetary units of leading foreign nations and their equivalents in United States money. These rates are proclaimed each year by the Secretary of the Treasury, and are used in Custom House computations:

Country	Monetary Unit.	Value in U. S. Gold.
Canada	Dollar = 100 cents . . .	\$1.00
Great Britain	Pound = 20 shillings . .	4.86½
France ..	Franc = 100 centimes.	.193
Belgium		
Switzerland }		
Italy	Lira = 100 centesimi . .	.193
Spain	Peseta 100 centimes . .	.193
German Empire . . .	Mark = 100 pfennige . .	.238
Denmark }	Crown = 100 öre268
Norway }		
Sweden }		
Japan.	Yen = 100 sen997

84. Before computing duties the following allowances are made: **Tare**, a deduction for the weight of boxes or crates; **leakage**, an allowance for loss of liquids imported in barrels or casks; and **breakage**, an allowance for loss of liquids imported in bottles. The **net quantity** is what remains after deducting tare, leakage, or breakage.

85. Ad valorem duties are computed by the rules of percentage; the net invoice price is regarded as the *base*, the ad valorem duty as the *percentage*, and the rate of duty as the *rate*.

Duties are not computed on fractions of a dollar; if the

cents are less than 50 they are rejected; if more, they are counted as a dollar.

86. Rule.—*To find the ad valorem duty, reduce the net invoice price to U. S. money, if necessary, deduct allowances, and multiply the remainder (expressed in even dollars) by the rate of ad valorem duty.*

To find the specific duty, multiply the net quantity by the rate of specific duty per unit of quantity.

EXAMPLE.—What is the duty on an invoice of silks valued at 24,860 francs, the ad valorem rate being 60%?

SOLUTION.—Referring to Art. 83, $24,860 \text{ francs} = 24,860 \times .193 = \$4,701.48$. Duty $= \$4,701 \times .60 = \$2,820.60$. Ans.

The 48 cents is rejected, being less than 50 cents. (Art. 85.)

EXAMPLE.—What is the duty on 820 gallons of brandy at \$1.50 per gallon, leakage 3%?

SOLUTION.— Leakage $= 820 \times .03 = 24.6$ gallons.

Net quantity $= 820 - 24.6 = 795.4$ gallons.

Duty $= 795.4 \times 1.50 = \$1,193.10$. Ans.

EXAMPLES FOR PRACTICE.

87. What is the ad valorem duty on an importation invoiced at

(a) £430 12 s. 4 d., allowing 5% breakage, rate of duty 40%?

(b) 36,750 lira, allowing 2% for tare, rate of duty 24%?

(c) 9,264 marks, rate of duty 85%?

(d) 4,700 yen, rate of duty 14%?

Ans. $\left\{ \begin{array}{ll} (a) & \$796.40. \\ (b) & \$1,668.24. \\ (c) & \$1,874.25. \\ (d) & \$656.04. \end{array} \right.$

What is the specific duty on an importation of

(e) 3,200 bu. potatoes at 15 cents per bushel?

(f) 60 dozen bottles of wine at \$3.00 per dozen, breakage 10%?

(g) 125 gross of empty bottles, breakage 6%, duty 10 cents per dozen?

(h) 3 T. 6 cwt. of iron castings at $\frac{3}{4}$ cent per pound?

Ans. $\left\{ \begin{array}{ll} (e) & \$480. \\ (f) & \$162. \\ (g) & \$141. \\ (h) & \$55.44. \end{array} \right.$

1. What is the duty on 25,670 lb. of pig iron at \$5.00 per ton ?
Ans. \$57.30.
2. What is the duty on 5 blocks of marble each 12 ft. long, 4 ft. wide, and $2\frac{1}{2}$ ft. thick, at 65 cents per cu. ft. ?
Ans. \$390.
3. What is the duty on an importation of 2,650 yards of woolen goods weighing 620 pounds net and valued at 72 cents per yard, the rates of duty being 60 cents per pound and 30% ad valorem ?
Ans. \$944.40.
4. An importation of musical instruments from Germany is valued at 13,670 marks. What is the duty at $17\frac{1}{2}$ % ad valorem ?
Ans. \$569.28.
5. An importer buys French silks at \$1.80 per yard and pays a duty of 35% ad valorem, and \$.60 per yard specific. At what price per yard must the silk be sold to yield a profit of 25% ?
Ans. \$3.79.
6. What is the duty at 65%, upon a consignment of 1,350 dozen kid gloves invoiced at 115 francs per dozen ?
Ans. \$19,475.95.

ARITHMETIC.

INTEREST.

SIMPLE INTEREST.

1. Interest is money paid for the use of money belonging to another.

2. The principal is the sum for which interest is paid.

3. The rate per cent. is the per cent. of the principal that is paid for its use for a given time, usually a year.

4. The amount is the sum of the principal and interest.

5. The legal rate is the rate established by law.

6. Usury is a rate that exceeds the legal rate. The penalty for usury is, in some States, the forfeiture of all interest, in others the forfeiture of both principal and interest. In a number of States, no legal notice is taken of usury.

7. In computing interest, a year is usually regarded as consisting of 12 months of 30 days each. Interest so computed is greater than it should be, unless the time is an exact number of years.

8. The elements in interest correspond with those of ordinary percentage as follows:

The *principal* is the *base*.

The *interest* is the *percentage*.

The *product of the rate per year by the time in years* is the *rate*. Thus, if the rate per cent. per year is 4% and the time is 5 years, the rate per cent. is 20%. That is, 20% of the principal

equals the interest. It is, of course, understood that fractions of a year are included in the expression "time in years."

9. From the correspondence between percentage and interest, it is obvious that all methods of computing interest depend upon the principle expressed in the following

Formula: *Interest = principal \times rate for 1 year \times time in years.*

10. As in percentage, the rate is used in computation instead of rate per cent. When no other rate per cent. is specified, 6% is to be understood.

FIRST METHOD.

EXAMPLE 1.—Find the interest of \$400 for 2 years 6 months at 5%.

SOLUTION.—2 years 6 months = $2\frac{1}{2}$ years; $\$400 \times .05 \times 2\frac{1}{2} = \50 .
Ans.

EXAMPLE 2.—What is the interest of \$878.60 for 3 yr 5 mo. 15 da. at 6%?

SOLUTION.— $\$878.60 \times .06 \times \frac{11}{2} = \182.31 . Ans.

EXPLANATION.—The time must first be changed to years. To do this, we begin with the days. 15 da. = $\frac{1}{2}$ mo.; $5\frac{1}{2}$ mo. = $\frac{11}{2}$ mo., or $\frac{11}{2 \times 12}$ yr. = $\frac{11}{24}$ yr.; $3\frac{1}{2}$ yr = $\frac{8\frac{1}{2}}{1}$ yr.

EXAMPLE 3.—Find the interest of \$697.23 for 2 yr. 11 mo. 24 da. at $3\frac{1}{2}$ %.

SOLUTION.— $\$697.23 \times .035 \times \frac{179}{60} = \72.80 . Ans.

EXPLANATION.—24 da. = $\frac{2}{3}$ mo.; $11\frac{2}{3}$ mo. = $\frac{59}{5}$ mo.
 $= \frac{59}{5 \times 12}$ yr. = $\frac{59}{60}$ yr.; $2\frac{2}{3}$ yr. = $\frac{179}{60}$ yr. The rate per cent. being $3\frac{1}{2}$ %, the rate is $.03\frac{1}{2}$, or .035. Performing the multiplication as indicated, and pointing off the result, gives \$72.80.

11. Rule.—*Multiply the principal by the rate, and that product by the time in years. The result will be the interest required.*

SECOND METHOD.

12. A very neat method of computing interest is the following:

EXAMPLE.—What is the interest at 5% of \$248.60 for 5 yr. 9 mo. 29 da.?

SOLUTION.—

$$\begin{array}{r}
 \$248.80 \\
 .05 \\
 \hline
 \$12.4400 = \text{Interest for 1 yr.} \\
 622000 = \text{Interest for 5 yr.} = 1 \text{ yr.} \times 5. \\
 62200 = \text{Interest for 6 mo.} = 1 \text{ yr.} \div 2. \\
 31100 = \text{Interest for 3 mo.} = 6 \text{ mo.} \div 2. \\
 .5200 = \text{Interest for 15 da.} = 3 \text{ mo.} \div 6. \\
 .4150 = \text{Interest for 12 da.} = 6 \text{ mo.} \div 15. \\
 .0690 = \text{Interest for 2 da.} = 12 \text{ da.} \div 6. \\
 \hline
 \$72.5340 = \text{Interest for 5 yr. 9 mo. 29 da. Ans.}
 \end{array}$$

EXPLANATION.—We first find the interest for 1 year, and then for 5 years. Having the interest for 1 year, we take half of that, which is the interest for 6 of the 9 months. Half of the interest for 6 months is the interest for 3 months. The interest for 29 days is found by taking $\frac{1}{6}$ of 3 months' interest, which gives the interest for 15 ($\frac{1}{2}$ mo.) of the 29 days. Since 12 days = $\frac{1}{5}$ of 6 months, we divide the interest for 6 months by 15* and get 12 days' interest. Only 2 of the 29 days remain; but 2 days = $\frac{1}{6}$ of 12 days. Hence, dividing 12 days' interest by 6, we have the interest for 2 days. 5 years + 6 months + 3 months + 15 days + 12 days + 2 days = 5 yr. 9 mo. 29 da., so that the sum of these several interests = the interest required.

It will be noticed that in the foregoing, 30 days are considered equal to 1 month, or 360 days to a year.

EXAMPLE.—What is the interest of \$6,400 for 3 yr. 11 mo. 26 da., at 4½%?

SOLUTION.—

$$\begin{array}{r}
 \$6400 \\
 .045 \\
 \hline
 \$28800 = \text{Interest for 1 yr.} \\
 86400 = \text{Interest for 3 yr.} = 1 \text{ yr.} \times 3. \\
 14400 = \text{Interest for 6 mo.} = 1 \text{ yr.} \div 2. \\
 7200 = \text{Interest for 3 mo.} = 6 \text{ mo.} \div 2. \\
 4800 = \text{Interest for 2 mo.} = 6 \text{ mo.} \div 3. \\
 1600 = \text{Interest for 20 da.} = 2 \text{ mo.} \div 3. \\
 400 = \text{Interest for 5 da.} = 20 \text{ da.} \div 4. \\
 .80 = \text{Interest for 1 da.} = 5 \text{ da.} \div 5. \\
 \hline
 \$1148.80 = \text{Interest for 3 yr. 11 mo. 26 da. Ans.}
 \end{array}$$

*To divide by 15, divide by 5 and then by 3.

13. Rule.—*Find the interest at the given rate for one year, and multiply this by the number of years. Find the interest for the given months by taking suitable parts of one year's interest, and for the days, suitable parts of the interest for one or more months. The sum of these partial results will be the total interest for the given time.*

EXAMPLES FOR PRACTICE.

14. Find the interest by the first method, and prove the correctness of your work by the second method.

1. Of \$600 for 1 yr 4 mo. 15 da. at 6%.
2. Of \$2,400 for 2 yr. 2 mo. 18 da. at 5%.
3. Of \$1,800 for 3 yr. 7 mo. 20 da. at $4\frac{1}{2}\%$.
4. Of \$1,250 for 9 mo. 25 da. at $3\frac{1}{2}\%$.
5. Of \$87.50 for 5 yr. 10 mo. 27 da. at 8%.
6. Of \$075.60 for 2 yr 8 mo. 12 da. at $4\frac{1}{2}\%$.
7. Of \$1,388.84 for 3 yr 7 mo. 17 da. at 4%.
8. Of \$725.50 for 7 yr. 5 mo. 23 da. at $7\frac{1}{2}\%$.
9. Of \$6,496.40 for 5 yr. 11 mo. 21 da. at 3%.
10. Of \$847.23 for 9 yr. 6 mo. 14 da. at $3\frac{1}{4}\%$.

Answers (1) \$49.50; (2) \$266, (3) \$213.75; (4) \$35.85+; (5) \$41.36—; (6) \$77.53—; (7) \$201.69; (8) \$407.04—; (9) \$1,164.48—; (10) \$303.06.

SIX-PER-CENT. METHOD.

15. Assuming that a year is composed of 12 months, each consisting of 30 days, it is clear that at 6% the interest of \$1 for 1 year is 6 cents, or \$.06; that for 1 month it is $\frac{1}{12}$ of \$.06, which is equal to 5 mills, or \$.005; and that for $\frac{1}{5}$ of 30 days, or 6 days, it is 1 mill, or \$.001. Or, in tabular form,

$$\begin{aligned} \text{Interest of \$1 for 1 year} &= \$.06 \\ \text{Interest of \$1 for 1 month} &= \$.005, \\ \text{Interest of \$1 for 6 days} &= \$.001 \end{aligned}$$

Hence, the interest of \$1 for any other time will be \$.06 for each year, \$.005 for each month, and \$.000 $\frac{1}{5}$ for each day. The sum of these three results will be the interest of \$1 for the given time.

EXAMPLE. What is the interest of \$1 for 5 yr 7 mo. 21 da. at 6%?

SOLUTION —

$$\text{Interest of \$1 for 5 yr} = \$.06 \times 5 = \$.30$$

$$\text{Interest of \$1 for 7 mo} = .005 \times 7 = .035$$

$$\text{Interest of \$1 for 21 da.} = .000\frac{1}{4} \times 21 = .0035$$

$$\text{Interest of \$1 for 5 yr 7 mo. 21 da. at 6\%} = \$.3385. \text{ Ans.}$$

Now, if we know the interest of \$1, it is simply a matter of multiplication to find the interest of any other number of dollars, or of dollars and cents.

Again, if we know the interest at 6% we may obtain the interest at 1% by dividing the interest at 6% by 6; and having the interest at 1%, we may find it at 4% by multiplying the interest at 1% by 4; similarly, for any other per cent.

EXAMPLE — What is the interest of \$654 for 3 yr. 9 mo. 28 da. at 5%?

SOLUTION —

$$\text{Interest of \$1 at 6\% for 3 yr.} = .06 \times 3 = \$.18$$

$$\text{Interest of \$1 at 6\% for 9 mo.} = .005 \times 9 = .045$$

$$\text{Interest of \$1 at 6\% for 28 da.} = .000\frac{1}{4} \times 28 = .007$$

$$\text{Interest of \$1 at 6\% for 3 yr. 9 mo. 28 da.} = \$.229\frac{1}{2}$$

$$\$.229\frac{1}{2} \times 654 = \$150.202 = \text{interest at 6\%}$$

$$\$150.202 \div 6 = \$25.033\frac{1}{3} = \text{interest at 1\%}$$

$$\$25.033\frac{1}{3} \times 5 = \$125.166\frac{2}{3} = \text{interest at 5\%. Ans.}$$

16. Rule. Find the interest of \$1 at 6% for the given time. To do this, multiply \$.06 by the number of years, \$.005 by the number of months, and \$.000 $\frac{1}{4}$ by the number of days. Multiply the sum of these results by the number of dollars in the principal, and the result will be the interest at 6%. For any other rate, divide the interest at 6% by 6, and multiply the quotient by the given rate.

17. In practice, it is better to find the interest at other rates than 6% by adding to, or subtracting from, the result for 6% suitable parts of itself.

The following partial table will illustrate:

$$\text{Interest at 6\%} + \left\{ \begin{array}{l} \frac{1}{6} \text{ of itself} = \text{Interest at 7\%} \\ \frac{1}{3} \text{ of itself} = \text{Interest at 8\%} \\ \frac{1}{2} \text{ of itself} = \text{Interest at 9\%} \\ \frac{2}{3} \text{ of itself} = \text{Interest at 10\%} \\ \frac{5}{6} \text{ of itself} = \text{Interest at 11\%} \end{array} \right.$$

$$\text{Interest at } 6\% = \begin{cases} \frac{1}{2} \text{ of itself} = \text{Interest at } 5\% \\ \frac{1}{4} \text{ of itself} = \text{Interest at } 4\frac{1}{2}\% \\ \frac{1}{8} \text{ of itself} = \text{Interest at } 4\% \\ \frac{1}{16} \text{ of itself} = \text{Interest at } 3\frac{1}{2}\% \\ \frac{1}{32} \text{ of itself} = \text{Interest at } 3\% \end{cases}$$

In the case of unusual rates per cent, it may be necessary to add or subtract two or more quotients.

EXAMPLE — Suppose we have found the interest of a certain principal at 6% to be \$237.68. How shall we find the interest of the same principal at 4½%?

SOLUTION —

$$\begin{array}{rcl} \$237.68 & = & \text{Interest at } 6\% \\ 59.42 & = & \text{Interest at } 1\frac{1}{2}\% \quad 6\% \div 4. \\ \hline \$178.26 & = & \text{Interest at } 4\frac{1}{2}\% \quad 6\% - 1\frac{1}{2}\% \\ 99.0+ & = & \text{Interest at } \frac{1}{2}\% \quad 1\frac{1}{2}\% \div 6. \\ \$188.16 & = & \text{Interest at } 4\frac{1}{2}\% \quad 4\frac{1}{2}\% + \frac{1}{2}\% \quad \text{Ans.} \end{array}$$

18. When the time for which the interest is to be computed is less than a year, it is customary to use the following method for finding the interest at 6%:

Since the interest for 1 day at 6% is \$.000½ (see Art. 15), the interest for any number of days may be found by multiplying the principal by the number of days, moving the decimal point *three* places to the left, and dividing the result by 6. It is usually easier, however, to divide the number of days by 6, multiply the quotient by the principal, and move the decimal point three places to the left.

EXAMPLE — Find the interest of \$1,215 for 86 days at 6%.

SOLUTION — $86 \div 6 = 14\frac{1}{3}$; \$1,215 = the principal with the decimal point moved three places to the left, $\$1,215 \times 14\frac{1}{3} = \$17,415$, or \$17.42. Ans.

When using this method, retain any fraction that may arise from dividing by 6. Thus, for 83 days, multiply by $13\frac{1}{2}$, rather than reduce the fraction to a decimal.

EXAMPLES FOR PRACTICE.

19. By the foregoing method, find the interest

1. \$484 for 2 yr. 3 mo. 15 da. at 3%.
2. \$700 for 1 yr. 0 mo. 20 da. at 4%.
3. \$1,200 for 3 yr. 6 mo. 24 da. at 5%.

4. Of \$9,600 for 4 yr. 7 mo. 27 da. at $3\frac{1}{2}\%$.
5. Of \$168.75 for 2 yr. 11 mo. 23 da. at $4\frac{1}{2}\%$.
6. Of \$437.50 for 5 yr. 8 mo. 18 da. at $5\frac{1}{2}\%$.
7. Of \$627.40 for 4 yr. 10 mo. 14 da. at $7\frac{1}{2}\%$.
8. Of \$909.96 for 3 yr. 9 mo. 22 da. at 7% .
9. Of \$1,237.50 for 7 yr. 2 mo. 26 da. at $2\frac{3}{4}\%$.
10. Of \$1,875.60 for 12 yr. 3 mo. 10 da. at $3\frac{1}{2}\%$.
11. Of \$784.15 for 57 da.
12. Of \$4,225 for 126 da.

Answers.—(1) \$35.005; (2) \$55.47—; (3) \$682.18—, (4) \$1,585.20
 (5) \$22.63+, (6) \$137.56—, (7) \$229.26+, (8) \$258.70+, (9) \$246.35—,
 (10) \$844.87—, (11) \$7.45—, (12) \$88.73—.

SIXTY-DAY METHOD.

20. For 1 year, at 6%, the interest of any principal is .06 of the principal itself, and for 2 months, or 60 days, the interest is .01 of the principal. Hence,

If the decimal point of any sum be moved two places to the left, it will give the interest of that sum for 60 days at 6%.

Thus, the interest of \$3,472.75 for 60 days at 6% is \$34.73—, and of \$692 it is \$6.92.

Having the interest for 60 days, it is easy, by operations that will suggest themselves, to find the interest for any other number of days.

EXAMPLE 1.—Find the interest of \$8,368 for 90 days at 6%.

SOLUTION.—

\$ 8 3 6 8 = Interest for 60 days.
 4 1 8 4 = Interest for 30 days = $\frac{1}{2}$ of 60 days.
 8 3 6 8 = Interest for 6 days = $\frac{1}{10}$ of 60 days.
 4 1 8 4 = Interest for 3 days = $\frac{1}{2}$ of 6 days.
 \$ 1 3 8 0 7 2 = Interest for 90 days. Ans.

EXAMPLE 2. What is the interest at 9% of \$1,204.76 for 49 days?

SOLUTION.—

\$ 1 2 6 4 7 6 = Interest for 60 days at 6%.
 6 3 2 3 8 = Interest for 30 days = $\frac{1}{2}$ of 60 days.
 3 1 6 1 9 = Interest for 15 days = $\frac{1}{4}$ of 60 days.
 6 3 2 4 = Interest for 3 days = $\frac{1}{20}$ of 60 days.
 2 1 0 8 = Interest for 1 day = $\frac{1}{60}$ of 60 days.
 \$ 1 0 3 2 8 9 = Interest for 49 days, at 6%.
 5 1 6 4 4 = Interest for 49 days, at 3%.
 \$ 1 5 4 9 3 3 = Interest for 49 days, at 9% Ans.

21. Rule.—Take .01 of the principal for the interest at 6% for 60 days, and then, by the method of aliquot parts, find the interest for the given time at the rate specified.

EXAMPLES FOR PRACTICE.

22. By the sixty-day method, find the interest

1. Of \$8,000 for 87 days at 6%.
2. Of \$6,050 for 96 days at 3%.
3. Of \$875.28 for 77 days at 3½%.
4. Of \$1,468.80 for 123 days at 4%.
5. Of \$23,750 for 108 days at 4½%.
6. Of \$42,000 for 176 days at 3½%.
7. Of \$7,200 for 225 days at 5%.
8. Of \$468.24 for 101 days at 5½%.
9. Of \$6,880 for 186 days at 7%.
10. Of \$7,000 for 143 days at 7½%.

Answers. —(1) \$116; (2) \$48.40; (3) \$6.55+; (4) \$20.07+; (5) \$820.625; (6) \$765.28—; (7) \$225; (8) \$7.23—; (9) \$248.83—; (10) \$226.42—.

EXACT INTEREST.

23. When interest is to be computed for one or more entire years at a specified rate per year, the fact that 12 months of 30 days each are usually regarded as a year does not affect the result—it is only when months and days, or days alone, become an element of the given time, that the interest is greater than it should be. The average length of a month in an ordinary year is $30\frac{5}{12}$ days, and in a leap year it is $30\frac{1}{2}$ days. A day is not $\frac{1}{365}$ of a year, but $\frac{1}{365}$ of a common year, and $\frac{1}{366}$ of a leap year. Hence, 360 days = $\frac{360}{365}$, or $\frac{72}{73}$, of a common year, and $\frac{360}{366}$, or $\frac{60}{61}$, of a leap year. By the ordinary method of finding interest, the result is either $\frac{1}{73}$ or $\frac{1}{61}$ greater than it should be.

Thus, the interest of \$7,300 for 60 days at 6%, as found by the usual method, is \$73. In equity it is $\$7,300 \times .06 \times \frac{60}{365} = \72 . That is, each \$73 interest should be \$72.

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sed by the government and by
the interest for the number of

entire years in a period, and then treat the remaining days as so many 365ths of a year.

EXAMPLE.—Find the exact interest of \$8,000 from Jan. 5, 1873, to July 23, 1880, at 6%.

SOLUTION.—

From Jan. 5, 1873, to Jan. 5, 1880 = 7 yr.

From Jan. 5, 1880, to July 23, 1880 =

Jan. Feb. Mar. Apr. May June July

26 + 29 + 31 + 30 + 31 + 30 + 23 = 200 days = $\frac{200}{365}$ = $\frac{40}{73}$ yr.

$\$8,000 \times .06 \times 7\frac{40}{73} = \$3,622.30$ — Ans.

25. The same result is obtained by adding to the interest for 7 years the interest for 200 days less $\frac{1}{61}$ of itself, found by the usual method. Thus,

$$\$8,000 \times .06 \times 7 = \$3,360.00$$

$$\$8,000 \times .06 \times \frac{200}{365} = \$266.67$$

$$\text{Deducting } \frac{1}{61} \text{ of } \$266.67 \quad 4.37 = \underline{262.30}$$

$$\text{Exact interest for 7 yr. 200 da.} = \$3,622.30 \quad \text{Ans.}$$

EXAMPLE.—What is the exact interest of \$4,800 for 198 days of an ordinary year, at 6%?

SOLUTION.—

\$ 4 8.0 0 = Interest for 60 days.

9 6.0 0 = Interest for 120 days.

1 2.0 0 = Interest for 15 days.

2.4 0 = Interest for 3 days.

73) \$ 1 5 8.4 0 = Interest for 198 days, counting 360
2.1 7 days of the year.

\$ 1 5 6.2 3 = Exact interest for 198 days. Ans.

EXPLANATION.—The interest is first found by the usual method, and $\frac{1}{61}$ of the result deducted.

26. Rule.—Find the interest by the ordinary method for the whole number of years included in the period. Count the number of days that remain, and by the same method find the interest for the days. If the days are part of an ordinary year, diminish the interest for the days by $\frac{1}{365}$ of itself; if they are part of a leap year, diminish the interest by $\frac{1}{366}$ of itself. Add the interest for the years to that for the days, and the result will be the exact interest.

EXAMPLES FOR PRACTICE.

27. Find the exact interest

1. Of \$10,000 for 123 days at 6%.
2. Of \$12,800 for 168 days at 8%.
3. Of \$6,400 for 213 days at 5%.
4. Of \$22,800 for 2 yr. 73 da. at 4%.
5. Of \$960 for 5 yr. 300 da. at $3\frac{1}{2}\%$.
6. Of \$484.80 for 6 yr. 202 da. at $5\frac{1}{2}\%$.
7. Of \$13,000 from Jan. 17, 1897, to Nov. 29, 1897, at $4\frac{1}{2}\%$.
8. Of \$968.40 from Apr. 19, 1865, to July 1, 1896, at 3%.
9. Of \$1,234.80 from Dec. 23, 1888, to Mar. 17, 1890, at 6%.
10. Of \$43,000 from May 29, 1891, to Nov. 3, 1895, at 7%.

Answers.—(1) \$202.10+; (2) \$353.48; (3) \$186.74—, (4) \$2,006.40; (5) \$195.02—; (6) \$174.74; (7) \$506.47—; (8) \$906.41—; (9) \$91.12+; (10) \$13,342.96—.

ANNUAL INTEREST.

28. Unless otherwise specified, interest upon debts is understood to be payable annually. In case it is not so paid, it is permitted in some States to charge interest upon overdue interest. In some other States this practice is illegal. Where it is intended to charge "annual interest," the written obligation should contain the words "interest payable annually."

EXAMPLE.—What is the interest at 6% of \$2,400 for 6 years 6 months, interest payable annually, if no interest is paid until the end of the time?

SOLUTION —

$$\text{Interest for 1 year} = \$2,400 \times .06 = \$144.00$$

$$\text{Interest for 6 yr. 6 mo.} = \$144 \times 6\frac{1}{2} = \$936.00$$

$$\text{Interest of \$144 for } 5\frac{1}{2} + 4\frac{1}{2} + 3\frac{1}{2} + 2\frac{1}{2} + 1\frac{1}{2} + \frac{1}{2}$$

$$= \text{Interest for 18 yr.} = \$144 \times .06 \times 18 = 155.52$$

$$\$1,091.52 \text{ Ans.}$$

EXPLANATION.—The first year's interest remains unpaid for $5\frac{1}{2}$ yr., the second for $4\frac{1}{2}$ yr., etc. The sum of these periods is 18 yr. One year's interest of the principal is \$144, and the interest of this for 18 yr is \$155.52. The sum of \$936 and \$155.52 is the entire interest due.

29. Rule.—*Find the interest of the principal for one year, and for the entire time the debt runs. Find the sum of the several periods that the annual interest remains unpaid, and for this time find the interest of one year's interest of the debt. The sum of the interest of the main debt and that of the unpaid annual interest will be the result required.*

EXAMPLES FOR PRACTICE.

30. In the following examples, assume that the annual interest is payable but unpaid, and find the entire interest due at the end of the given time

1. Of \$240 for 2 yr. 6 mo. at 6%.
2. Of \$380 for 3 yr. 8 mo. at 5%.
3. Of \$1,000 for 4 yr. 9 mo. at 7%.
4. Of \$1,200 for 3 yr. 3 mo. 15 da. at 6%.
5. Of \$387.50 for 5 yr. 6 mo. 20 da. at 4%.
6. Of \$7,625 for 8 yr. 7 mo. 18 da. at 5%.

Answers.—(1) \$37.73—; (2) \$74.42—; (3) \$376.60; (4) \$253.74; (5) \$94.03+, (6) \$3,921.79+.

PROBLEMS IN INTEREST.

31. Given the interest, rate, and time, to find the principal.

We know that $I = \text{Prin.} \times \text{rate} \times \text{time (in years)}$; or, more briefly,

$$\frac{Prt}{100} = I,$$

when r is the rate per cent. Hence, multiplying each side by 100, we have $Prt = 100I$. Now, if we divide each side by rt , we have $\frac{Prt}{rt} = \frac{100I}{rt}$. Canceling rt of the left member of this equation, we have

$$P = \frac{100I}{rt}.$$

That is, *the principal is equal to 100 times the interest divided by the product of the rate per cent. and the time.*

EXAMPLE.—What principal in 4 yr. 9 mo. at 5% will give \$152 interest ?

SOLUTION.—Applying the formula and noticing that 9 mo. = $\frac{3}{4}$ yr.,

$$P = \frac{152 \times 100}{5 \times 4\frac{3}{4}} = \frac{152 \times 100 \times 4}{5 \times 19} = \$640. \text{ Ans.}$$

EXPLANATION.— $4\frac{3}{4}$ being in the divisor, the fraction must be reduced to the form of an improper fraction and inverted. The rate also appears in the inverted form of $\frac{100}{5}$, since .05 cannot be inverted.

EXAMPLE.—What principal at $3\frac{1}{2}\%$ will, in 2 yr. 3 mo. 15 da., give \$374 interest ?

SOLUTION.—Changing the time to $\frac{17}{4}$ years, writing $3\frac{1}{2}\%$ as $\frac{17}{4}\%$, and applying the formula, we have

$$P = \frac{374 \times 100}{\frac{17}{4} \times \frac{17}{4}} = \frac{374 \times 100 \times 4 \times 4}{17 \times 17} = \$4,800. \text{ Ans.}$$

32. Rule.—*To find the principal when the interest, rate, and time are given, divide 100 times the interest by the product of the time in years and the rate per cent.*

EXAMPLES FOR PRACTICE.

33. Find the principal, when

1. Interest = \$96, time = 2 years, rate = 4%.
2. Interest = \$131.25, time = 2 yr 6 mo., rate = 6%.
3. Interest = \$12.50, time = 2 yr 3 mo., rate = 4%.

Resuming the equation,

$$Pr t = 100 I;$$

dividing each side by Pt , we have

$$\frac{Pr t}{Pt} = \frac{100 I}{Pt}, \text{ or } r = \frac{100 I}{Pt}.$$

That is, rate per cent. = (interest \times 100) \div (principal \times time).

EXAMPLE.—At what rate per cent. will \$480 in 3 years 10 months give \$92 interest?

SOLUTION.—Applying the formula,

$$r = \frac{100 \times 92}{480 \times 3\frac{5}{6}} = \frac{\overset{5}{100} \times \overset{4}{92} \times \cancel{6}}{\underset{20}{480} \times \underset{20}{23}} = 5\%. \text{ Ans.}$$

35. Rule.—*Change the time to years, and divide 100 times the interest by the product of the principal and the time.*

EXAMPLES FOR PRACTICE.

36. Find the rate per cent. when

1. Principal = \$2,875, time = 4 yr. 7 mo. 6 da., interest = \$529.
2. Principal = \$760, time = 3 yr. 9 mo. 18 da., interest = \$144.40.
3. Principal = \$1,260, time = 2 yr. 1 mo. 10 da., interest = \$119.70.
4. Principal = \$2,340, time = 2 yr. 6 mo. 20 da., interest = \$328.90.
5. Principal = \$4,870, time = 3 yr. 5 mo. 24 da., interest = \$1,017.83.
6. Principal = \$7,200, time = 123 days, interest = \$114.80.
7. Principal = \$1,500, time = 1 yr. 9 mo. 18 da., interest = \$99.
8. Principal = \$1,600, time = 5 yr. 7 mo. 6 da., interest = \$380.80.

Answers.—(1) 4%; (2) 5%; (3) 4½%; (4) 5½%; (5) 6%; (6) 4¼%; (7) 3¼%; (8) 4¼%.

37. Given the principal, interest, and rate per cent., to find the time.

If we divide the equation $Pr t = 100 I$ by Pr , we shall have

$$\frac{Pr t}{Pr} = \frac{100 I}{Pr}, \text{ or } t = \frac{100 I}{Pr}.$$

Otherwise expressed, this means that

$$\text{time (in years)} = (100 \times \text{interest}) \div (\text{principal} \times \text{rate per cent.})$$

EXAMPLE.—In what time will \$4,400 at 6% give \$871.36 interest?

SOLUTION.—Applying the formula,

$$\text{time} = \frac{100 \times \$871.36}{4,400 \times 6} = \frac{\overset{889}{\$7136}}{\underset{20}{4400} \times 6} = \frac{889}{120} \text{ yr.}$$

$$7\frac{11}{12} \text{ yr.} = 3\frac{11}{12} \text{ yr.}; \quad \frac{29}{120} \times 12 = \frac{29}{10} \text{ mo.} = 2\frac{9}{10} \text{ mo.}; \quad \frac{9}{10} \times 30 = 27 \text{ da.}$$

EXPLANATION.—The result is obtained in years, and must be reduced to years, months, and days. $7\frac{11}{12}$ years = $3\frac{11}{12}$ years.

Since there are 12 months in one year, $\frac{11}{12}$ of a year = $\frac{29 \times 12}{120}$

= $\frac{11}{5}$ months, or $2\frac{9}{10}$ months. Since there are 30 days in one

month, $\frac{9}{10}$ of a month = $\frac{9 \times 30}{10} = 27$ days. Hence, the

required time is 3 yr. 2 mo. 27 da. Ans.

38. Rule.—To find the time, when the principal, interest, and rate are given, divide 100 times the interest by the product of the principal and the rate per cent., and reduce the result, which is years, to years, months, and days.

EXAMPLES FOR PRACTICE.

39. Find the time, when

1. Principal = \$4,800, interest = \$652, rate = 6%.
2. Principal = \$680, interest = \$163.20, rate = 5%.
3. Principal = \$360, interest = \$26.325, rate = $4\frac{1}{2}\%$.
4. Principal = \$338.75, interest = \$35.23, rate = 4%.
5. Principal = \$1,080, interest = \$112.50, rate = 3%.
6. Principal = \$1,800, interest = \$63, rate = $3\frac{1}{4}\%$.
7. Principal = \$1,050, interest = \$45.50, rate = 5%.
8. Principal = \$1,000, interest = \$143, rate = $6\frac{1}{2}\%$.
9. Principal = \$2,400, interest = \$74.80, rate = $5\frac{1}{2}\%$.

Answers —(1) 2 yr 3 mo 5 da.; (2) 4 yr. 9 mo. 18 da.; (3) 1 yr. 7 mo. 15 da.; (4) 2 yr. 7 mo. 6 da.; (5) 3 yr. 5 mo. 20 da.; (6) 1 yr.; (7) 10 mo. 12 da.; (8) 2 yr. 2 mo. 12 da.; (9) 6 mo. 24 da.

40. Given the amount, time, and rate, to find the principal.

Since at 6% the interest of \$1 for, say, 2 years, is 12 cents, the amount is \$1 + 12 cents = \$1.12. Each \$1 of the principal will, in like manner, amount to \$1.12. Hence, if we divide the amount of any principal in a given time at a specified rate by the amount of \$1 for that time and rate, it will give the number of times \$1 is contained in the principal.

Or using the rate per cent. instead of the rate, we may, by means of the following formula, express the process of finding the principal:

$$P = \frac{100 A}{100 + r t}$$

That is, $(100 \times \text{amount}) \div (100 + \text{rate per cent.} \times \text{time}) = \text{principal}$.

EXAMPLE.—What principal will, in 5 yr. 6 mo., at 4%, amount to \$591.70?

SOLUTION.—Applying the formula,

$$P = \frac{100 \times 591.70}{100 + 4 \times 5\frac{1}{2}} = \frac{59,170}{123} = \$485. \text{ Ans.}$$

41. Rule.—*To find the principal, when the amount, rate, and time are given, divide 100 times the amount by 100 increased by the product of the rate per cent. and the time.*

NOTE.—In using this rule, and the formula given in Art. 40, care must be taken that the time be expressed in years and fractions of a year.

EXAMPLES FOR PRACTICE.

42. Find the principal that will amount

1. To \$1,005 in 5 yr. 8 mo. at 6%.
2. To \$3,459 in 3 yr. 4 mo. 24 da. at 4½%.
3. To \$2,985 in 2 yr. 1 mo. 10 da. at 5%.
4. To \$1,443.60 in 2 yr. 10 mo. 24 da. at 7%.
5. To \$2,353.50 in 1 yr. 6 mo. 12 da. at 3%.

Answers.—(1) \$750; (2) \$3,000; (3) \$2,700; (4) \$1,200; (5) \$2,250.

TRUE DISCOUNT.

43. The student has learned that any deduction made from a debt or other obligation is a *discount*. In making such deductions, the element of *time* may or may not be considered. When time is considered, we have one of the applications of interest. **True discount** is discount when time is considered and *no interest is allowed on the discount*. True discount corresponds exactly to the problems of Interest given in Arts. 40-42—the case in which the amount, rate, and time are given, to find the principal. The terms employed, however, are different.

The *principal* is called the **present worth**.

The *rate* is called the **rate of discount**.

The *interest* is called the **true discount**.

The *amount* is called the **debt, or obligation**.

True discount is so called to distinguish it from *bank discount*, which will be treated later.

44. The *present worth* of an obligation is a sum such that, if it be put at interest at a specified rate for a given time, it will amount to the obligation.

Thus, if the specified rate is 5%, a debt of \$105 due in one year is worth \$100 *now*, since \$100 placed at interest at 5% will in one year amount to \$105.

45. *True discount* is the difference between a debt due at a future time and its present worth.

Thus, \$5 in the illustration given above is the true discount of \$105 due in one year, when the rate of discount is 5%.

46. Given the debt, rate of discount, and time, to find the present worth and the discount.

The present worth may be found by means of the formula of Art. 40, or by the rule of Art. 41. Thus,

$$P = \frac{100 \cdot A}{100 + rt}, \text{ or present worth } = \frac{100 \times \text{debt}}{100 + rt}.$$

EXAMPLE.—A debt of \$773, which has 1 yr. 5 mo. 20 da. yet to run, is discounted at 5%. What is (a) the present worth, and (b) the discount?

SOLUTION.—1 yr. 5 mo. 20 da. = $1\frac{17}{12}$ years.

$$\begin{aligned} (a) \text{ Present worth} &= \frac{100 \times 773}{100 + 1\frac{17}{12} \times 5} \\ &= \frac{77,300 \times 36}{3,865} = \$720. \text{ Ans.} \end{aligned}$$

$$(b) \text{ Discount} = \$773 - \$720 = \$53. \text{ Ans.}$$

47. If the debt bears interest, its amount for the time it has to run must first be found, and that sum discounted at the specified rate.

EXAMPLE.—A debt of \$1,200 having 1 yr. 6 mo. to run bears interest at 6%. What is its present worth discounted at 5%?

SOLUTION.—Amount of \$1,200 in 1 yr. 6 mo. = \$1,308.

Applying the formula,

$$\text{Present worth} = \frac{100 \times 1,308}{100 + 1\frac{1}{2} \times 5} = \frac{130,800}{107.5} = \$1,216.74+. \text{ Ans.}$$

48. Rule.—I. *Divide 100 times the amount of the debt when it is due, by 100 increased by the product of the time and the rate of discount. The result will be the present worth.*

II. *Subtract the present worth from the debt, and the remainder will be the discount.*

EXAMPLES FOR PRACTICE.

49. What is the present worth and the discount

1. Of \$900 due in 9 months, discounted at 4%?
2. Of \$1,000 due in 1 yr. 6 mo., discounted at 6%?
3. Of \$800 due in 5 mo., discounted at 5%?
4. Of \$2,800 due in 1 yr. 3 mo. 12 da., discounted at 8%?
5. Of \$625 due in 2 yr. 5 mo. 15 da., discounted at 7%?
6. Of a note for \$600 bearing interest at 5%, having 2 yr. 10 mo. to run, and discounted at 6%?

Answers.—(1) \$873.79—, \$26.21+; (2) \$917.43+, \$82.57—; (3) \$783.67+, \$16.33—; (4) \$2,539.30—, \$260.70+; (5) \$533.24—, \$91.76+; (6) \$585.47, \$99.53.

COMPOUND INTEREST.

50. If the interest of a principal is added to the principal at regular intervals to form by each addition a new principal for the next interval, the resulting interest is called **compound interest**.

Thus, if \$100 be placed at compound interest at 6%, with the understanding that the interest is to be compounded annually, the principal will be \$100 for the first year, \$106 for the second year, \$112.36 for the third year, etc.

51. Most savings banks allow compound interest, although in most States its payment cannot be legally enforced, even though it be specified in a contract.

Unless otherwise stated, interest is understood to be compounded annually. If it be compounded semiannually, one-half the annual rate is taken as the rate; if quarterly, one-fourth the annual rate is taken; etc.

52. When the time is given in years, months, and days, the interest is compounded for the greatest number of entire periods included in the time, and the simple interest of the last principal is found for the remaining time.

EXAMPLE—Find the compound interest of \$800 for 1 yr. 9 mo. 20 da. at 6%, interest compounded semiannually.

SOLUTION—

$$\begin{array}{rcl}
 \$800 & = & \text{prin. 1st 6 mo.} \\
 \underline{24} & = & \text{int 1st 6 mo} = \$800 \times .03. \\
 \$824 & = & \text{prin. 2d 6 mo.} \\
 \underline{2472} & = & \text{int 2d 6 mo} = \$824 \times .03. \\
 \$84872 & = & \text{prin. 3d 6 mo.} \\
 \underline{2546} & = & \text{int 3d 6 mo} = \$84872 \times .03. \\
 \$87418 & = & \text{prin. for 3 mo. 20 da.} \\
 \underline{1603} & = & \text{int for 3 mo. 20 da.} = \$874.18 \times .06 \times \frac{1}{4}. \\
 \$890.21 & = & \text{amt for 1 yr. 9 mo. 20 da.} \\
 \underline{800} & = & \text{original prin} \\
 \$90.21 & = & \text{comp int for 1 yr. 9 mo. 20 da. Ans.}
 \end{array}$$

EXPLANATION.—In 1 yr. 9 mo. 20 da, there are three complete periods of 6 mo. each, and 3 mo. 20 da. besides. Since the

annual rate is 6%, for 6 mo. the rate per cent. is 3%. Finding the interest at 3%, and adding the principal for these three periods, gives \$874.18. The amount of this sum for the remaining 3 mo. 20 da. is \$890.21, from which we subtract the original principal. The remainder, \$90.21, is the required compound interest.

53. Compound interest is calculated in actual business by means of a table. The table shows the amount of \$1 at all the different rates, and for all the different times that are likely to occur. Having the amount of \$1 at any given rate and for any number of periods, we multiply it by the number of dollars in any given principal. The result will be the amount of that sum for the given time. If the original principal be subtracted from this amount, the remainder is the compound interest required.

The formula for compound interest is, therefore,

$$\text{Comp. Int.} = P(1+r)^n - P.$$

In this formula, r is the rate, and n the number of periods—it is an exponent expressing the power to which $1+r$ must be raised to give the amount of \$1 for that number of periods.

EXAMPLE.—What is the compound interest of \$600 for 4 years at 6%, interest compounded annually?

SOLUTION. —Applying the formula,

$$\begin{aligned} \text{Comp int} &= \$600 (1 + .06)^4 - \$600. \\ 1.06^4 &= 1.262477, \$600 \times 1.262477 = \$757.4862; \\ \$757.4862 - \$600 &= \$157.49. \text{ Ans.} \end{aligned}$$

54. In the foregoing formula, r is the rate for one of the equal intervals of time—it is not necessarily the rate per year. For instance, if the interest is compounded quarterly, r is one-fourth of the rate per year; if compounded semiannually, r is one-half the rate per year. In the following table, the numbers in the columns headed “2½ per cent.,” “3 per cent.,” etc., are the values obtained by raising $1+r$ to the power whose exponent is the number of complete periods during which the interest is compounded. Thus, in the column headed “4 per cent.,”

COMPOUND INTEREST TABLE.

Showing the amount of \$1. at various rates, compound interest, from 1 to 20 years, interest compounded annually.

Yr.	2½ per cent.	3 per cent.	3½ per cent.	4 per cent.	5 per cent.	6 per cent.
1	1.025000	1.030000	1.035000	1.040000	1.050000	1.060000
2	1.050625	1.060900	1.071225	1.081600	1.102500	1.123600
3	1.076891	1.092727	1.108718	1.124864	1.157625	1.191016
4	1.103813	1.125509	1.147523	1.169859	1.215506	1.262477
5	1.131408	1.159274	1.187686	1.216653	1.276282	1.338226
6	1.159693	1.194052	1.229255	1.265319	1.340096	1.418519
7	1.188686	1.229874	1.272279	1.315032	1.407100	1.503630
8	1.218403	1.266770	1.316809	1.368509	1.477455	1.593848
9	1.248863	1.304773	1.369297	1.423312	1.551328	1.689479
10	1.280095	1.343916	1.410599	1.480244	1.628895	1.790848
11	1.312087	1.384234	1.459970	1.539454	1.710339	1.898299
12	1.344889	1.425761	1.511069	1.610342	1.795856	2.012197
13	1.378511	1.468534	1.563956	1.665074	1.885649	2.132928
14	1.412974	1.512590	1.618695	1.731676	1.979982	2.260904
15	1.448298	1.557967	1.675349	1.800944	2.078928	2.396558
16	1.484500	1.604706	1.733986	1.872981	2.182875	2.540352
17	1.521618	1.652848	1.794676	1.947801	2.292018	2.692773
18	1.559659	1.702433	1.857186	2.025817	2.406619	2.854339
19	1.598650	1.753506	1.922501	2.106849	2.526950	3.025600
20	1.638616	1.806111	1.989789	2.191123	2.653298	3.207136

Yr.	7 per cent.	8 per cent.	9 per cent.	10 per cent.	11 per cent.	12 per cent.
1	1.070000	1.080000	1.090000	1.100000	1.110000	1.120000
2	1.144900	1.166400	1.188100	1.210000	1.232100	1.254400
3	1.225043	1.259712	1.295029	1.331000	1.367631	1.404908
4	1.310790	1.360489	1.411582	1.464100	1.518070	1.573519
5	1.402552	1.469928	1.538624	1.610510	1.685058	1.762342
6	1.500730	1.580874	1.677100	1.771561	1.870414	1.973822
7	1.605781	1.713824	1.828039	1.948717	2.076160	2.210081
8	1.718186	1.850930	1.992563	2.143589	2.304537	2.475963
9	1.838450	1.999005	2.171893	2.357048	2.558036	2.773078
10	1.967151	2.158925	2.367364	2.593742	2.810420	3.058548
11	2.104852	2.331639	2.580426	2.853117	3.151757	3.478549
12	2.252192	2.518170	2.812665	3.138428	3.498450	3.895975
13	2.409845	2.719624	3.065805	3.452271	3.883279	4.363492
14	2.578534	2.937194	3.341727	3.797498	4.310440	4.887111
15	2.759031	3.172169	3.642482	4.177248	4.784588	5.473565
16	2.952164	3.425043	3.970306	4.594973	5.310803	6.130392
17	3.158815	3.700018	4.327633	5.054470	5.895091	6.866040
18	3.379932	3.996019	4.717120	5.559917	6.543551	7.689964
19	3.616527	4.315701	5.141661	6.115909	7.263342	8.612760
20	3.869684	4.660957	5.604411	6.727500	8.062309	9.646291

1.872981 is the value of 1.04^{16} to 6 places of decimals; it is also the amount at compound interest of \$1 for 16 years at 4%.

compounded annually, of \$1 for 8 years at 8%, compounded semiannually, and of \$1 for 4 years at 16%, compounded quarterly, since in each case the number of periods is 16 and the rate is .04. Hence, if it was required to find the amount at compound interest of \$1 compounded annually, it may be obtained for any rate per cent. from 1 to 20 years from the table; if compounded semiannually, find the number of periods, divide the annual rate per cent. of interest by 2, and treat these results as "years" and "rate per cent.," respectively, when using the table; if compounded quarterly, divide the annual rate per cent. of interest by 4.

EXAMPLE. Find the amount of \$1 at compound interest for 3 yr. 6 mo. at 7% when compounded (a) annually; (b) semiannually; (c) quarterly.

SOLUTION.—(a) Referring to the table, the amount for 3 yr. at 7% is \$1.225043. The simple interest for 6 mo. on \$1.225043 at 7% is $\$1.225043 \times .035 = \0.42877 . The total amount is $\$1.225043 + \$0.42877 = \$1.653813$. Ans.

(b) When the interest is compounded semiannually, there are $3\frac{1}{2} \times 2 = 7$ periods in 3 yr. 6 mo., $7\% \div 2 = 3\frac{1}{2}\%$. Referring to the table, we find in the column headed "3½ per cent.," opposite 7, in column of years, 1.272279, hence, when the interest is compounded semiannually, the amount of \$1 for 3 yr. 6 mo. at 7% is \$1.272279. Ans.

(c) When the interest is compounded quarterly, there are $3\frac{1}{2} \times 4 = 14$ periods, $7\% \div 4 = 1\frac{3}{4}\%$. Since there is no column in the table headed "1¾ per cent.," the amount must be calculated directly. The amount of \$1 is, then, $\$1(1 + 0.0175)^{14} = \$1 \times 1.0175^{14} = \$1.274913$. Ans.

55. Rule.—Multiply the amount of \$1 for the given time and rate by the principal, and from the product subtract the principal. The remainder is the compound interest.

If there is a partial period, find the amount of the last principal for this partial period, and then subtract the original principal.

EXAMPLES FOR PRACTICE.

56. Find the compound interest of the following amounts for the times mentioned, and prove the correctness of your result by using the table. Unless otherwise stated the interest is to be compounded annually.

1. Of \$1,600 for 2 yr. at 8%.
2. Of \$980 for 3 yr. 8 mo. at 6%.
3. Of \$1,280 for 5 yr. 6 mo. at 4%.
4. Of \$2,400 for 3 yr. 10 mo. 20 da. at 5%.
5. Of \$360 for 4 yr. 4 mo. 12 da. at 6%.
6. Of \$450 for 3 yr. 9 mo. 24 da. at 8%, interest compounded semi-annually.
7. Of \$237.50 for 1 yr. 11 mo. 15 da. at 12%, interest compounded quarterly.
8. Of \$3,875 for 2 yr. 11 mo. 12 da. at 6%, interest compounded semi-annually.

Answers. (1) \$266.24, (2) \$229.11; (3) \$308.46, (4) \$501.78; (5) \$104.49; (6) \$167.65-, (7) \$61.00-, (8) \$738.48-.

PARTIAL PAYMENTS.

57. A debt or obligation may be discharged at one payment; or, from time to time, payments *in part* may be made, and finally at a time of *settlement* the remainder of the debt may be paid. Now, it is obvious that interest should be allowed upon such payments as are made, since interest is charged upon the obligation itself. But, if a payment should be less than the interest upon the debt since a previous payment had been made, to subtract such payment from the debt with accrued interest would result in increasing the principal. This would be a species of compound interest which, in many States, is illegal.

58. When a partial payment of a note is made, the date of payment and its amount are written upon the back of the note, and this record of it is called an **indorsement**.

The following rule for partial payments has been formulated by the Supreme Court of the United States, and has been adopted by most of the States:

59. United States Rule. 1. Find the amount of the principal to the time when the payment, or the sum of the

payments, is greater than the interest then due. From the amount subtract the payment or the sum of the payments, and treat the remainder as a new principal.

II. *Proceed in this manner to the date of settlement, and the last amount will be the sum still due.*

60. To compute partial payments by the United States rule.

EXAMPLE.—

\$1,200.

New York, Sept. 16, 1895.

On demand I promise to pay John Crawford, or order, Twelve Hundred Dollars, with interest at 6%, value received.

Edward G. Carson.

Indorsements: Jan. 1, 1896, \$120; May 7, 1896, \$300; Dec. 22, 1896, \$16; Sept. 19, 1897, \$400. What was due Jan. 1, 1898?

SOLUTION.—

Principal.....	\$ 1 2 0 0
Interest from Sept. 16, '95, to Jan. 1, '96 (3 mo. 15 da.) ...	2 1
Amount.....	1 2 2 1
First payment.....	1 2 0
New principal.....	1 1 0 1
Interest from Jan. 1, '96, to May 7, '96 (4 mo. 6 da.).....	2 3.1 2
Amount	1 1 2 4.1 2
Second payment.....	3 0 0
New principal.....	8 2 4.1 2
Interest from May 7, '96, to Sept. 19, '97 (1 yr. 4 mo. 12 da.)	6 7.5 8
Amount.....	8 9 1.7 0
Sum of third and fourth payments.....	4 1 6.0 0
New principal.....	4 7 5.7 0
Interest from Sept. 19, '97, to Jan. 1, '98 (3 mo. 12 da.)...	8.0 9
Amount due at time of settlement.....	\$ 4 8 8.7 9
	Ans.

In this example, 360 days are considered as 1 year. The third payment of \$16 is less than the interest due at the time it was made; hence, according to the rule, it is added to the next payment of \$400 and the interest is computed to the time of the fourth payment.

EXAMPLES FOR PRACTICE.

61. Find the amount due at the time of settlement on each of the following :

1. \$2,000.

Philadelphia, July 1, 1896.

One year after date, for value received, I promise to pay Wm. Gray, or order, Two Thousand Dollars, with interest at 6%.

Henry G. Brown.

Indorsements. Dec. 16, 1896, \$350; Mar. 1, 1897, \$25, Oct. 25, 1897, \$400, June 14, 1898, \$275. Time of settlement, July 1, 1899.

2. A note for \$3,000 at 6% dated Mar. 5, 1892, bore the following indorsements: Dec. 20, 1892, \$400, Mar. 14, 1893, \$60; Nov. 30, 1893, \$300, July 15, 1894, \$600. Time of settlement, Jan. 1, 1895.

3. A note for \$4,000 at 8% dated Sept. 1, 1886, is indorsed as follows Jan. 1, 1887, \$500; July 1, 1887, \$450. Jan. 1, 1888, \$300; Sept. 1, 1888, \$300. Time of settlement, Jan. 1, 1889.

4. Face of note, \$3,000, rate 8%, dated May 1, 1890. Indorsements: Dec. 1, 1890, \$600, Mar. 1, 1891, \$100; Dec. 1, 1891, \$800; July 1, 1892, \$1,000. Time of settlement, Dec. 1, 1892.

Answers. (1) \$1,216.80; (2) \$2,024.38, (3) \$2,625.50, (4) \$1,691.36.

When the time from the date of a note or other obligation is less than a year, settlement is usually made by a method called the *merchants' rule*.

62. The Merchants' Rule.—I. *By the method of exact interest, find the amount of each of the several payments from the time each is made to the date of settlement.*

II. *Subtract the sum of these amounts from the amount of the obligation from its date to the time of settlement. The remainder will be the amount still due.*

EXAMPLE.—Face of note, \$2,000; rate, 6%, date of note Dec. 31, 1888; time of settlement Nov. 15, 1889. Indorsements: Mar. 10, 1889, \$200, June 1, 1889, \$300, Aug. 20, 1889, \$400, Oct. 1, 1889, \$500. What was due at time of settlement?

SOLUTION.—

Principal	\$ 2000
Interest of \$2,000 for *319 da	104.88
Amount	\$ 2104.88

* Table XX, § 4, will be found very useful for finding the number of days between two dates.

Amount brought forward.....	\$ 2 1 0 4.8 8
Amount of \$200 for 250 da.....	\$ 2 0 8.2 2
Amount of \$300 for 167 da.....	\$ 3 0 8.2 4
Amount of \$400 for 87 da.....	\$ 4 0 5.7 2
Amount of \$500 for 45 da.....	\$ 5 0 3.7 0
Sum of payments, with interest.....	\$ 1 4 2 5.8 8
Amount due at time of settlement.....	\$ 6 7 9.0 0
	Ans.

EXAMPLES FOR PRACTICE.

63. Solve the following examples by the merchants' rule:

1. Debt, \$5,000; rate, 5%; date of note, July 1, 1894; date of settlement, June 15, 1895. Indorsements: Sept. 10, 1894, \$1,000; Dec. 12, 1894, \$800; Mar. 12, 1895, \$1,200; May 1, 1895, \$1,200. What is due at the time of settlement?

2. Face of note, \$8,400; rate, 4%; date of note, Jan. 1, 1897; time of settlement, Oct. 20, 1897. Indorsements: Mar. 30, 1897, \$1,600; June 1, 1897, \$800; Aug. 10, 1897, \$2,000; Sept. 15, 1897, \$3,000. What is due on the note at time of settlement?

3. Debt, \$5,000; rate, 6%; date of note, Nov. 1, 1897; time of settlement, May 20, 1898. Indorsements: Dec. 20, 1897, \$900; Jan. 1, 1898, \$1,200; Mar. 24, 1898, \$1,200; Apr. 20, 1898, \$400. What is due at time of settlement?

Answers.—(1) \$957.67; (2) \$1,193.60; (3) \$1,401.41.

PROMISSORY NOTES.

64. A **promissory note** is a written promise to pay a certain sum at a certain time.

65. The **maker** or **drawer** of a note is the person that promises to pay; the **payee** is the person to whom the note is payable; and the **holder** is the person that owns it.

66. The **face** of a note is the sum promised to be paid. This sum should be written both in figures and in words.

67. Notes are of two kinds—notes *bearing interest*, and notes *not bearing interest*. When no rate of interest is specified, the legal rate in the state or county where the note is made is to be understood. If a note not bearing

interest is not paid when due, it bears interest at the legal rate after that time until paid.

68. The following table is given for reference:

INTEREST LAWS.

Laws of each state and territory regarding rates of interest and penalties for usury, with the law or custom as to days of grace on notes and drafts. Compiled from the latest state and territorial statutes.

States and Territories.	Legal Rate of Interest, Per Cent.	Rate Allowed by Contract, Per Cent.	Penalties for Usury.	Grace or No Grace.
Alabama	8	8	Forfeiture of entire interest.	Grace
Arizona	10	Any rate	None	Grace
Arkansas	6	10	Forfeiture of principal and int.	No statute.
California	7	Any rate	None	No grace.
Colorado	10	Any rate	None	Grace
Connecticut	6	4	None	Grace
Dakota	7	12	Forfeiture of excess.	Grace
Delaware	6	4	Forfeiture of principal	Grace
District of Columbia	6	10	Forfeiture of entire interest.	Grace
Florida	8	Any rate.	None	No statute
Georgia	7	8	Forfeiture of excess.	Grace
Illinois	10	12	Forfeiture of 3 times excess of int.	No grace
Idaho	6	8	Forfeiture of entire interest	Grace
Indiana	6	8	Forfeiture of excess of interest	Grace
Iowa	6	10	Forfeiture of 10% per year on amt.	Grace
Kansas	7	12	Forfeiture of excess of interest	Grace
Kentucky	6	10	Forfeiture of excess over 10%	Grace
Louisiana	5	8	Forfeiture of entire interest.	Grace
Maine	6	Any rate.	None	Grace.
Maryland	6	6	Forfeiture of excess of interest	Grace.
Massachusetts	6	Any rate	None	Grace.
Michigan	7	10	Forfeiture of excess of interest	Grace.
Minnesota	7	10	Forfeiture of excess over 10%	Grace.
Mississippi	6	10	Forfeiture of excess of interest	Grace.
Missouri	6	10	Forfeiture of entire interest	Grace
Montana	10	Any rate	None	No grace.
Nebraska	7	10	Forfeiture of interest and cost.	Grace
Nevada	10	Any rate.	None	Grace
New Hampshire	6	6	Forfeiture of three times the excess.	Grace
New Jersey	6	6	Forfeiture of entire interest.	No grace.
New Mexico	6	12	None	No grace.
New York	6	6	Forfeiture of principal and int.	No grace.
North Carolina	6	8	Forfeiture of entire interest	Grace
Ohio	6	9	Forfeiture of excess over 10%	Grace
Oregon	8	10	Forfeiture of principal and int.	No grace
Pennsylvania	6	6	Forfeiture of excess of interest	No grace.
Rhode Island	6	Any rate	None	Grace.
South Carolina	7	Any rate	None	Grace
Tennessee	6	10	Forf. of exc. int. and \$100 fine.	Grace
Texas	8	12	Forfeiture of entire interest	Grace
Vermont	10	Any rate	None	No grace.
Virginia	6	6	Forfeiture of excess of interest.	No grace
Washington	6	8	Forfeiture of excess over 12%	Grace
West Virginia	10	Any rate.	None	Grace
Wisconsin	6	6	Forfeiture of excess of interest.	Grace
Wyoming	7	10	Forfeiture of entire interest	No grace.
Yukon	12	Any rate	None	Grace

Means of \$5,000 or upwards, on collateral security, any rate of inter-

69. A note should be so written as to show where it was made and when, the sum promised to be paid, whether it does or does not bear interest, and the words "value received." The law assumes that no one is to be compelled to pay unless he has received what he deems an equivalent. If "value received" is omitted, the holder may have to prove that the maker of the note did actually receive value for the money promised in it.

70. A note usually specifies where it is to be paid usually at a bank. If no place is designated, the holder's place of business is understood.

71. If a note contains the words "or order," it is a **negotiable note**, and may pass like a bank note from one person to another. If the holder of a negotiable note wishes to dispose of it, he is generally required to guarantee its payment by *indorsing it*—that is, by writing his name across the back of the note. There are several kinds of indorsements. Thus, if the holder is John Smith, he may, on the back of the note, write *John Smith*. This is an indorsement *in blank*, and makes John Smith responsible for the payment of the note.

He may write, *Pay to William Jones*. The note is then payable only to William Jones.

If he indorses it, *Pay to William Jones, or order*, it is payable to William Jones, or to any one to whom William Jones may order it to be paid.

He may indorse it, *Pay to William Jones, or bearer*, and it is payable to any person that presents it.

If it be indorsed, *Pay to bearer*, it is payable to the person that presents it for payment.

72. A **joint and several note** is a note signed by two or more persons, who become collectively and individually responsible for its payment.

A note is **protested** when the holder of it notifies the indorsers in legal form and within the time prescribed by law that the note is unpaid. Unless such protest is legally

made, the indorsers are not responsible for its payment. This protest must reach the indorser not later than the day when the note is payable.

73. Some forms of notes used in actual business are given below

\$250. *New York, Sept. 17, 1896.*

*On demand I promise to pay George Camp, or order,
Two Hundred and Fifty Dollars, value received.*

Howard Gray.

\$1,000. *Scranton, July 5, 1898.*

*Three months after date, for value received, I promise
to pay Stephen Girard, or order, One Thousand Dollars, with
interest at 5%.*

Charles Goldwin.

\$3,000 *Philadelphia, July 5, 1898.*

*Six months after date, we, or either of us, will pay to
George Owen, Three Thousand Dollars, value received, with
interest at 6%.*

George Kirwin

Henry Potter.

Erastus Kirby.

Payable at the First National Bank.

74. In most states, *three days of grace* are allowed before a note must be paid. If a bank discounts a note, interest is charged for days of grace in States where days of grace are legal.

75. If a note falls due on a Sunday, or on a legal holiday, it is usually payable during banking hours on the business day preceding its maturity. Interest, however, is charged for three days of grace in such cases. In some States, a note falling due on a Sunday or a legal holiday is payable on the first business day thereafter.

EXAMPLES FOR PRACTICE.

76. Solve the following examples

- 1 Write a negotiable demand note for \$500, with interest at 6%, and make Brown Jones & Co the payee

2. Write a non-negotiable note for \$4,000, with interest at 5%, payable in 30 days, yourself being the maker and Howard Crosby the payee.

3. How much will pay the following note when due without grace?
\$575 $\frac{1}{8}$ %. *New York, Sept. 19, 1898.*

Sixty days after date, for value received, I promise to pay Ralph Newton, or order, Five Hundred Seventy-Five and $\frac{1}{8}$ Dollars, with interest at 4 $\frac{1}{2}$ %.

Henry Miles.

Ans. \$579.82.

BANK DISCOUNT.

77. **Bank discount** is the charge made by a bank for paying a note or other obligation before it is due. This charge is the interest on the amount of the obligation from the time it is discounted until its maturity. This interest is subtracted from the face of the obligation, and its holder receives for it the remainder, which is called the **proceeds**. Hence, bank discount is inequitable, since interest is charged, not only upon the sum actually paid for the obligation, but also upon the discount.

78. In states where days of grace are allowed, bank discount is calculated for 3 days more than the time specified in the note.

Thus, if a 60-day note for \$1,000 is discounted at a bank, the interest of \$1,000 is found for 63 days, and is subtracted from \$1,000. If the rate of discount is 6%, the holder will receive as proceeds $\$1,000 - \$10.50 = \$989.50$.

79. It is evident that the owner of the note should receive for it the *true present worth* of \$1,000 payable in 63 days, or \$989.61. The bank gives him only \$989.50, or 11 cents less than he should get. When it is considered that the sums annually discounted by most banks mount up into millions, it will be seen how much of their gain is unearned.

80. The **maturity** of a note is on the last day of grace. The time of maturity is generally indorsed on the note, thus, Mar. 7/10, which means that it matures nominally on Mar. 7, and legally on Mar. 10.

The **term of discount** is the time from the discounting of the note to its maturity.

81. In the case of an interest-bearing note, the sum discounted is the amount of the note at maturity.

82. Banks usually require that a discounted note shall be payable at the bank that discounted it, and they rarely discount notes having more than 90 days to run.

83. To find the time when a note matures, the term of discount, the discount, and the proceeds.

EXAMPLE.—Find (a) the discount, and (b) the proceeds of the following note

\$484.60 Newark, N. J., Oct. 4, 1897.
Sixty days after date, for value received, I promise to pay William Hall, or order, Four Hundred Eighty-Four and $\frac{60}{100}$ Dollars, at the Ninth National Bank.
Henry Parshall.

Discounted, Oct. 20, 1897, at 6%.

SOLUTION —

(a) Maturity, Dec. 3/6, 1897.
 Term of discount, 47 days.
 Discount, \$3.80. Ans.

(b) Proceeds, \$484.60 — \$3.80 = \$480.80. Ans.

EXPLANATION — Sixty days after Oct. 4 is Dec. 3, and three days of grace make the date of legal maturity Dec. 6. From the time of discount, Oct. 20, to Dec. 6 is 47 days, for which the interest at 6% is \$3.80. Subtracting the discount, \$3.80, from the face of the note, \$484.60, gives \$480.80, the proceeds.

EXAMPLE.—Find (a) the discount and (b) the proceeds of the following note:

\$1,000 Chicago, Ill., August 6, 1898.
For value received, I promise to pay, three months after date, to Ernest Hazard, or order, One Thousand Sixty Dollars, with interest at 5%.
Emil Reeves.

Discounted, Sept. 1, 1898, at 6%.

SOLUTION —

(a) Maturity, Nov. 6/9, 1898.
 Amount of note at maturity..... \$1078.69
 Term of discount, 69 days.....
 Discount..... 12.85 Ans.
 (b) Proceeds..... \$1061.84 Ans.

84. Rule.—I. *If the note bears interest, find its amount at the time of maturity.*

II. *Find the interest on the face of the note, or, if it is an interest-bearing note, on the amount of the note at maturity at the given rate of discount for 3 days more than the time it has to run until its nominal maturity, and the result will be the bank discount.*

III. *Subtract the bank discount from the face of the note, or from its amount at maturity, and the remainder will be the proceeds.*

EXAMPLES FOR PRACTICE.

85. Solve the following examples:

1. Find (a) the bank discount and (b) the proceeds of a note for \$5,000, due in 60 days, discounted at 6%.

Ans. $\left\{ \begin{array}{l} (a) \text{ } \$52.50. \\ (b) \text{ } \$4,947.50. \end{array} \right.$

2. Find (a) the bank discount and (b) the proceeds of a note for \$4,000, due in 90 days, discounted at 5%.

Ans. $\left\{ \begin{array}{l} (a) \text{ } \$51.67. \\ (b) \text{ } \$3,948.33. \end{array} \right.$

3. Required, (a) the bank discount and (b) the proceeds of a note for \$7,600, due in 30 days discounted at 8%.

Ans. $\left\{ \begin{array}{l} (a) \text{ } \$55.73. \\ (b) \text{ } \$7,544.27. \end{array} \right.$

4. A note for \$8,000 dated July 5, 1898, is discounted at 5% on Sept. 7, 1898. If it is a 90-day note, what are the proceeds?

Ans. \$7,967.78.

5. A note for \$2,800 bearing interest at 6%, and due in 60 days, is discounted at 5%. What are the proceeds?

Ans. \$2,804.64.

6. \$8,476 $\frac{00}{100}$.

St. Louis, Mo., Jan. 8, 1898.

Six months after date I promise to pay to Charles Brown, or order, Eight Thousand Four Hundred Seventy-Six Dollars, value received, with interest at 6%.

Howard Bristow.

Find the proceeds of the foregoing note if discounted April 25, 1898, at 5%.

Ans. \$8,641.11.

7. A note for \$2,800, due in 3 months, is dated June 5, 1899, and bears interest at 7%. It is discounted July 20, at 6%. Find the proceeds.

Ans. \$2,826.87.

8. A note for \$96,000, with interest at 8%, is dated Nov. 16, 1897, and is due in 90 days. It is discounted Dec. 20, at 6%. Find how much the proceeds differ from the true present worth.

Ans. \$9.38.

86. To find the face of a note when the proceeds, time, and rate are given.

EXAMPLE.—The proceeds of a note discounted at a bank for 45 days at 6% were \$1,488. What was the face of the note?

SOLUTION.—

Proceeds of \$1 for 45 + 3 days = \$.993.

Face of the note = $\$1,488 \div \$.993 = \$1,500$. Ans.

87. Rule.—*Divide the proceeds by the proceeds of \$1 for 3 days more than the given time.*

EXAMPLES FOR PRACTICE.

88. Solve the following examples:

1. A 60-day note discounted at a bank at 6% yields \$1,246.77. What is its face? Ans. \$1,260.

2. The proceeds of a note for 2 mo. 12 da., discounted at a bank at 7%, are \$4,079.625. Find the face of the note. Ans. \$4,140.

3. Given, rate of discount, 8%; time, 90 days; proceeds, \$3,612.74. Find the face of the note. Ans. \$3,690.

4. Given, rate of discount, 7%; time, 30 days; proceeds, \$96,362.195. Find the face of the note. Ans. \$96,940.

5. Given, time, 51 days; rate, 4%; proceeds, \$484.575. What is the face of the note? Ans. \$487.50.

6. A note for 60 days discounted at $4\frac{1}{2}\%$ yields \$81,815.335. What is its face? Ans. \$82,464.75.

7. Write in proper form a 60-day note payable at the Chemical Bank of New York, which, when discounted when the note is made, will yield at 7% \$7,850 proceeds. Ans. Face, \$7,915.97

ARITHMETIC.

STOCKS AND BONDS.

1. If any work involving a large expenditure of money is to be undertaken, it is usual to organize a company, and procure a charter under the laws of some state. The chartered company then issues shares, which are sold to any persons having money to invest, and willing to incur the chances of loss. The advantage of having a charter for the company is that the shareholders, in case the business is unprofitable, are liable for the debts of the company only to the amount of their shares. Otherwise, any member of the company can be compelled to pay all of its debts. Moreover, a chartered company can do business just as an individual—that is, it may sue or be sued for debts, enter into contracts, etc.

2. The **par value** of shares is the amount—usually \$25, \$50, or \$100—specified in the certificates issued to its subscribers.

When the business of a company is profitable, its shares sell *above par*, or for more than their face value; if it is unprofitable, the shares sell at a *discount*, or *below par*.

3. When a company gains in its business, it pays its shareholders, from time to time, part of its profits, called **dividends**.

4. Dividends are *declared* quarterly, semiannually, or annually, and are usually paid at the general office of the company.

5. If the business is at a loss, the shareholders may be required to make good the loss. Such payments are called **assessments**.

6. If a man buys stocks of a company and makes partial payments on them, such payments are called **instalments**.

7. A **bond** is a written obligation under seal to pay a certain sum at a specified time.

8. Bonds to furnish money for the national or any state government, or for a city, county, town, village, or for an incorporated company, are prepared and sold in the open market. The money accruing from such sales may be used for current expenses, or for such improvements as may be desired. The bonds are secured by the property of those who issue them, and bear interest payable quarterly, semi-annually, or annually.

9. **Registered bonds** are numbered, and the names of their purchasers are recorded. To sell registered bonds, the transfer must be recorded on the books of the company that issued them. Sometimes bonds have **coupons** attached, stating the amount of interest due at certain times. These coupons may be detached, and exchanged for money at the general office of the company, or at a bank acting for the company.

10. Government or State bonds are usually designated by the interest they bear, or by the time when they are payable. Thus, "U. S. $3\frac{1}{2}$'s, 1907" are bonds of the United States Government bearing interest at $3\frac{1}{2}\%$, and payable in 1907.

11. A **stock broker** is a person whose business consists in buying and selling bonds or stocks for others. His compensation is a certain per cent. of the *par value* of the stocks bought or sold. The compensation of a broker is called **brokerage**.

12. The *par value* of stocks is to be understood as 100, unless some other value is given. Whatever may be the market price of stocks and bonds, brokerage is calculated on their *par value*.

EXAMPLE 1.—Find the cost of 480 shares of Canadian Pacific stock bought at $123\frac{1}{2}$, if the brokerage is $\frac{1}{8}\%$.

SOLUTION.— $(123\frac{1}{2} + \frac{1}{8}) \times 480 = \$59,340$. Ans.

EXAMPLE 2.—How many shares of bank stock selling at 112 can be bought for \$89,700, if the brokerage is $\frac{1}{8}\%$? The par value of the shares is \$50.00.

SOLUTION.—The cost of 1 share at the market price is $1.12 \times \$50 = \56 ; the brokerage per share is $.00\frac{1}{8} \times \$50 = \$0.06\frac{1}{4}$. Therefore, total cost of 1 share = $\$56 + \$0.06\frac{1}{4} = \$56.06\frac{1}{4}$, and the number of shares bought = $\$89,700 \div \$56.06\frac{1}{4} = 1,600$ shares. Ans.

13. NOTE.—Unless otherwise stated, the par value of each share will always be assumed to be \$100.00. Then, in example 1 above, the cost of 1 share is \$123.50. But the broker gets $\frac{1}{8}\%$, or $\frac{1}{8}$ of \$1.00 per share for selling them. Therefore, the total cost to the purchaser of 480 shares is $(123\frac{1}{2} + \frac{1}{8}) \times 480 = \$59,340$.

14. Rule.—I. *To find the cost of any number of shares of stock, multiply the sum of the market price per share and the brokerage by the number of shares, and the product will be the cost.*

II. *To find the number of shares that can be bought for a given sum, divide the given sum by the cost of one share, including the brokerage, and the quotient will be the number of shares.*

15. EXAMPLE.—How much must be invested in railroad stock that pays a quarterly dividend of $2\frac{1}{2}\%$, in order to have an income of \$4,000, if they are bought at $104\frac{1}{2}$, brokerage being $\frac{1}{8}\%$?

SOLUTION.—The expression, “a dividend of $2\frac{1}{2}\%$,” means $2\frac{1}{2}$ per cent. on the *par* value of the stock. Hence, since the *quarterly* dividend is $2\frac{1}{2}\%$, the *annual* income per share of \$100 will be \$10. Consequently, to obtain an annual income of \$4,000, there must be bought $4,000 \div 10 = 400$ shares; then each share will cost $\$104\frac{1}{2} + \$\frac{1}{8}$, and their total cost will be 400 times as much, or $\$104\frac{5}{8} \times 400 = \$41,850$. Ans.

EXAMPLE.—What per cent. is realized by buying 4% bonds at $89\frac{7}{8}$, brokerage being $\frac{1}{8}\%$?

SOLUTION.—Since each share costs $\$89\frac{7}{8} + \$\frac{1}{8} = \$90$, and each share yields \$4 annual income, the per cent. realized will be found by dividing the gain, \$4, by the entire cost of one share. Hence,

$$\$4 \div (\$89\frac{7}{8} + \$\frac{1}{8}) = .04\frac{1}{3} = 4\frac{1}{3}\%. \text{ Ans.}$$

16. Rule.—I. *To find the investment that will yield a given income, divide the income by the gain from one share,*

and the quotient will be the number of shares that must be bought, then multiply the cost of one share by the number of shares, and the product will be the investment.

11. To find the rate per cent of income from money invested in stocks or bonds, divide the gain yielded by one share by the cost of a share, and multiply the quotient by 100.

EXAMPLES FOR PRACTICE.

17. 1. What must be paid for 128 shares of Standard Oil stock at $128\frac{1}{2}$, brokerage $\frac{1}{2}\%$? Ans. \$16,432

2. How many shares of Union Gas Co. stock at $98\frac{1}{2}$ can be bought for \$30,550, the brokerage being $\frac{1}{2}\%$? Ans. 400 shares.

3. How much will 68 U. S. $4\frac{1}{2}\%$ bonds of 1907 cost at $116\frac{1}{2}$, brokerage being $\frac{1}{2}\%$? Ans. \$7,930.50.

4. The cost of some railroad stock was \$18,150, for which a man paid \$187 $\frac{1}{2}$ per share, and $\frac{1}{2}\%$ brokerage. How many shares did he buy? Ans. 132 shares.

5. Find the cost of 240 shares of mining stock at $98\frac{1}{2}$, brokerage being $\frac{1}{2}\%$. Ans. \$23,670.

6. How much must be paid for 5% city bonds to yield an annual income of \$1,250, if they cost $104\frac{1}{2}$, brokerage $\frac{1}{2}\%$? Ans. \$26,250.

7. What is the rate per cent received on the foregoing investment? Ans. $4\frac{1}{2}\%$.

8. Bank stock that pays an annual dividend of 10% is bought for $109\frac{1}{2}$, brokerage $\frac{1}{2}\%$. What per cent. is realized by investing in it? Ans. $9\frac{1}{2}\%$.

9. How much must be paid for the bank stock mentioned above, to endow a college professorship with an annual income of \$5,000? Ans. \$55,000.

10. How much more is the rate per cent. of income on 8% stock bought at $110\frac{1}{2}$ than on 6% stock bought at $107\frac{1}{2}$, brokerage in each case being $\frac{1}{2}\%$? Ans. $1\frac{1}{4}\%$.

11. How much per cent better is an investment in 8% securities bought at 80 than 7% stock at 84, not regarding brokerage? Ans. $1\frac{1}{4}\%$.

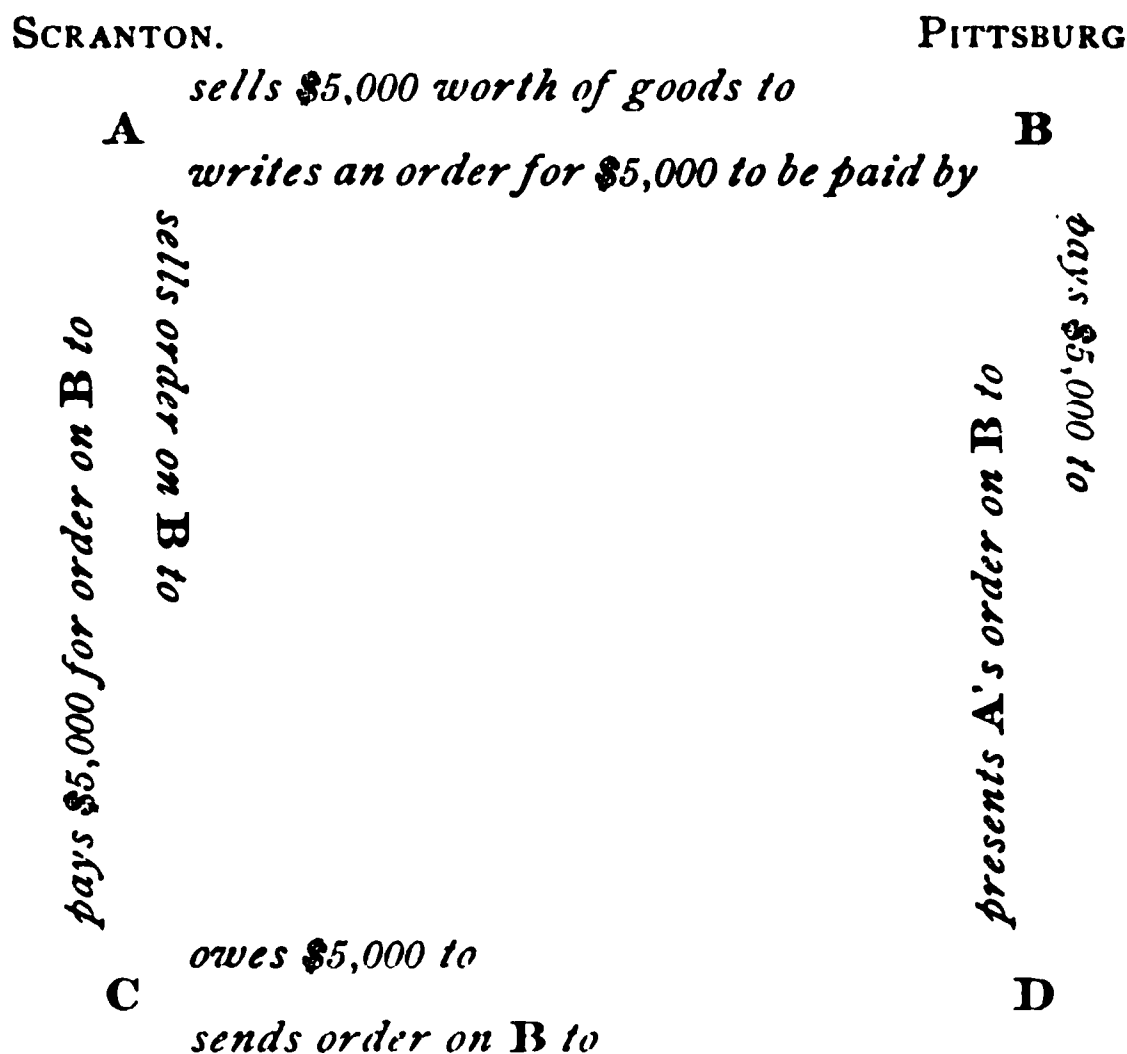
12. How much better is a gain of 20% on an investment at 80 than a gain of 18% on an investment at 90? Ans. 5%.

13. A man left his wife an annual income of \$8,000 by an investment in 6% bonds bought at $111\frac{1}{2}$, brokerage being $\frac{1}{2}\%$. What did they cost? Ans. \$178,200.

EXCHANGE.

18. **Exchange** is a method of paying debts in a distant place without transmitting money. It is done by means of *drafts*, or *bills of exchange*.

19. Thus, suppose that A, who does business in Scranton, *sells* \$5,000 worth of goods to B, who does business in Pittsburg; also, that C in Scranton *owes* \$5,000 to D in Pittsburg. Then, B in Pittsburg owes A in Scranton, and C in Scranton owes D in Pittsburg. A writes an order directing B to pay to C \$5,000, and sells this order to C, who pays A \$5,000 for it. C endorses the order, and makes it payable to D; he sends the order to D, who presents it to B, and B pays D \$5,000. A and D thus have the money that was due them, B and C have the goods, and no money had to be sent outside of Scranton or outside of Pittsburg. The diagram below will serve to make the operation clearer. The order directing payment to be made is called a **draft**, or a **bill of exchange**.



The foregoing explanation explains the purpose and convenience of a draft. Usually, however, the transaction is

conducted through a bank or broker. Thus, suppose that B in Pittsburg owes A in Scranton \$5,000, and that B wishes to pay A without incurring the danger and risk of actually sending the money. He goes to a bank in Pittsburg and buys a draft, say, on New York, paying \$5,000 and as much more as the bank charges him for its trouble. The Pittsburg bank then makes out a draft directing a New York bank to pay to A's order \$5,000. B sends draft to A, who endorses it and presents it either at the New York bank, or, more conveniently, at a Scranton bank, where it is cashed. The Scranton bank sends the draft to some other bank (preferably a New York bank) with which the Scranton bank has dealings, and this bank in turn presents it at the bank drawn on, which credits the bank presenting the draft, and debits the Pittsburg bank. At stated intervals the various banks balance their accounts, at which time the debtor banks forward to the creditor banks the amounts due them.

Or, again, suppose that B in Pittsburg owes A in Scranton \$5,000, and that A desires to collect this amount from B. A draws* a draft, or order, on B, directing him to pay the cashier of A's bank the \$5,000 due A. A then leaves this draft at his bank, and the bank sends it to some bank in Pittsburg, which bank presents the draft to B, who pays that bank the money. This bank then notifies the Scranton bank that it has collected the draft and has placed the money to the Scranton bank's credit. The Scranton bank then notifies A that the draft has been collected and that the money has been placed to A's credit.

20. Exchange between different parts of the *same* country is **domestic exchange**, and between *different* countries is **foreign exchange**.

21. A **sight draft** is payable when presented for payment; a **time draft** is payable after a specified time. Time drafts are usually made to run 30, 60, or 90 days, and in some places days of grace are allowed on them.

* To **draw** a draft is to request the payment of a certain sum, at a certain time, to a certain person, his order, or to bearer

When a time draft is presented to the drawee, if he desires to pay it, he writes across its face the word "Accepted," the date of acceptance, and his name. The draft then becomes due, the number of days stated in the draft, from the date of acceptance, instead of from the date of the draft. A time draft, thus accepted, in reality becomes a promissory note. Sight drafts do not need to be accepted, as they are usually paid when presented.

22. The **drawer** of a draft is the person that requests payment to be made; the **drawee** is the person requested to make such payment; and the **payee** is the person to whom the money is to be paid.

23. A draft is **accepted** by the drawee if he writes on its face the word "Accepted" and the date of acceptance, together with his name. He thus becomes liable for its payment.

24. **Discount** is allowed on time drafts, and is computed on the amount or on the face of the draft.

FORM OF A DRAFT.

*465 $\frac{25}{100}$.

Scranton, Pa., Jan. 3, 1898.

At sight, pay to the order of Henry Hudson, Four Hundred Sixty-five $\frac{25}{100}$ Dollars, value received, and charge to the account of

The Colliery Engineer Company.

*To Brown & Bird,
Chicago, Ill.*

EXPLANATION.—The Colliery Engineer Co. corresponds to A in Art. 19, Brown & Bird to B, and Henry Hudson to C. Henry Hudson endorses the draft, "Pay to the order of J. R. Robinson," and signs his name; he forwards the draft to J. R. Robinson, of Chicago (who corresponds to D), and he presents it to Brown & Bird for acceptance.

DOMESTIC EXCHANGE.

25. Exchange may be at *par*, *above par*, or *below par*. For example, consider two cities, as New York and New Orleans. If New Orleans owes more money in New York than New York owes in New Orleans, there will be more persons in New Orleans who wish to buy drafts on New York than there are sellers; hence, a buyer in New Orleans will be willing to pay a seller more than the face value for a draft on New York. Therefore, in New Orleans, exchange on New York will be above par, or at a *premium*. In New York, however, there will be more sellers than buyers, and the seller will be willing to take less than the face of a draft on New Orleans. Therefore, in New York, exchange on New Orleans will be below par, or at a *discount*. If there is an equality of debts between New York and New Orleans, exchange will be at par. The premium or discount is only so much as will suffice to cover the cost of safely transferring the money from the debtor city to the creditor city.

26. To find the cost of a sight draft.

EXAMPLE 1.—Find the cost of a sight draft on Baltimore for \$2,800, exchange being at $\frac{1}{2}\%$ premium.

SOLUTION.— $\$2,800 \times .005 = \14 ; $\$2,800 + \$14 = \$2,814$. Ans.

Or, $\$2,800 \times 1.005 = \$2,814$. Ans.

EXAMPLE 2.—What must be paid for a sight draft on New York for \$3,675, at $\frac{3}{4}\%$ discount?

SOLUTION.— $\$3,675 \times .0075 = \27.5625 ;

$\$3,675 - \$27.56 = \$3,647.44$. Ans.

Or, $\$3,675 \times (1 - .0075) = \$3,647.44$. Ans.

27. Rule.—Find the premium or the discount. The sum of the face of the draft and the premium, or the difference between the face of the draft and the discount, will be the cost of the draft.

EXAMPLES FOR PRACTICE.

28. Solve the following examples:

1. Find the cost of a sight draft for \$1,876 at (a) $1\frac{1}{2}\%$ premium, (b) $\frac{1}{2}\%$ discount.

Ans. } (a) \$1,899.45.
(b) \$1,866.62

2. The face of a sight draft is \$7,875.56, and the premium is $\frac{1}{4}\%$. Find its cost. Ans. \$7,938.56.

3. How much will it cost to pay, by a sight draft on San Francisco, a bill of \$7,528, when exchange is at $1\frac{1}{2}\%$ discount? Ans. \$7,415.08.

29. To find the cost of a time draft.

EXAMPLE.—Find the cost of the following draft at $\frac{3}{4}\%$ premium, when money is worth 5% interest.

\$4,800 $\frac{00}{100}$.

New York, July 1, 1898.

*Ninety days after sight, pay to William Wood, or order,
Four Thousand Eight Hundred Dollars, value received,
and charge to my account.*

John Steinway.

*To Henry Brothers,
New Orleans, La.*

SOLUTION.—In states where days of grace are allowed, 3 days must be added to the number of days specified in the draft. Hence, since the draft is *payable* in Louisiana, a state where days of grace are allowed (Art. 68, § 9), 90 days + 3 days of grace = 93 days.

$$\$4,800 \times .0129\frac{1}{2} = \$62 = \text{int. of } \$4,800 \text{ at } 5\% \text{ for } 93 \text{ da.}$$

$$\$4,800 - \$62 = \$4,738 = \text{proceeds of } \$4,800.$$

$$\$4,800 \times .0075 = \$36; \$4,738 + \$36 = \$4,774. \quad \text{Ans.}$$

EXPLANATION.—The bank discount at 5% for 93 days on \$4,800 is \$62; hence, the present worth, or proceeds, of \$4,800, payable in 93 days, is \$4,738. The premium on the face of a draft is \$36. This, with the proceeds, is the cost of the draft.

30. Rule.—Find the proceeds of the face of the draft for three days more than the time the draft has to run. Find, also, the premium or the discount, on the face of the draft. The sum of the proceeds and the premium, or the difference between the proceeds and the discount, will be the cost of the draft.

If days of grace are not allowed in the state where the draft is payable, find the proceeds for the time the draft has to run, and then proceed as before.

EXAMPLES FOR PRACTICE.

31. Solve the following examples.

1. The face of a draft is \$5,000, the discount is $\frac{1}{2}\%$, the time to elapse before it is payable is 60 days. Find the cost of the draft when money is worth 6% Ans. \$4,922.50.

2. How much will it cost to pay 30 days after sight a bill of \$3,250 in Charleston, S. C., at a premium of $1\frac{1}{2}\%$, money being worth 6%? Ans. \$3,280.87.

3. When exchange is at a discount of $\frac{1}{2}\%$, and money is worth 4%, what must be paid for a 60-day time draft for \$6,000? Ans. \$5,913.

32. To find the face of a sight or a time draft.

EXAMPLE — What is the face of a sight draft bought for \$3,000, exchange being $\frac{1}{2}\%$ discount?

SOLUTION — $\$1.00 \quad \$0.995 = \$0.995$
 $\$3,000 \div .995 \quad \$3,015.08 \quad \text{Ans}$

EXPLANATION. One dollar of the face will cost \$0.995; hence, there must be as many times one dollar of the face as .995 is contained times in the entire cost, or 3,015.08.

EXAMPLE. Find the face of a 60-day draft costing \$2,343.51 when money is worth 7% and exchange is at $1\frac{1}{2}\%$ premium.

SOLUTION — When the place where the draft is to be paid is not stated, days of grace should be allowed. Hence, the time is $60 + 3 = 63$ days.

Interest of \$1 for 63 days at 7% $= \$0.01225$
 Proceeds of \$1 $= \$1 - \$0.01225 = \$0.98775$
 Premium on \$1 $= \$0.01375$
 Proceeds + premium $= \$0.98775 + \$0.01375 = \$1.0015$
 Face of draft $= \$2,343.51 \div 1.0015 = \$2,340 \quad \text{Ans.}$

33. Rule. — *Divide the amount paid for the draft by the amount that will pay for \$1 of its face.*

EXAMPLES FOR PRACTICE.

34. Solve the following examples.

1. Find the face of a 90-day draft costing \$6,000, when discount is $1\frac{1}{2}\%$ and money is worth 5% Ans. \$6,156.48.

2. A man paid \$484.72 for a 60-day draft, premium being 1%, and money worth 6% interest. What was the face of the draft?

Ans. \$484.90

3. If a draft that is payable 90 days after sight costs \$2,800 when discount is $\frac{1}{2}\%$ and money worth 6%, what is its face? Ans. \$2,836.88.

4. Find the face of a sight draft costing \$1,200, when exchange is at $1\frac{1}{2}\%$ discount. Ans. \$1,216.73.

FOREIGN EXCHANGE.

35. Foreign bills of exchange are drawn in sets of two, called a **set of exchange**. These are numbered 1 and 2, and are sent by different mails; only the first presented for payment has any value. Formerly, foreign bills of exchange were drawn in sets of three, and are frequently so drawn now.

36. Exchange is at a premium or at a discount according to the balance of trade, or to the time that must elapse before payment is to be made.

Thus, let A and B denote two countries engaged with each other in commerce. Suppose that the balance of trade is in B's favor. By this is meant that A owes B more than B owes A. Now, it is clear that A must send money at some risk and expense to B to equalize matters. If one were to get in A a draft payable in B, it would put A more deeply in debt to B. This fact would put a premium upon the draft, and a discount upon a draft drawn in B upon A.

Again, a sight draft upon either country would cost less than a time draft.

37. The Secretary of the Treasury of the United States issues, on the first of January of each year, a statement showing, in terms of its own gold monetary unit, the value of the monetary unit of each other country, that is, in dollars and cents.

38. The daily papers of our commercial cities give quotations showing the rates of exchange from day to day. One of these follows:

Sterling exchange* was again weak and lower. Continental exchange was also lower. Rates are Long bills, \$4.82½ @ \$4.82½, sight drafts, \$4.84½ @ \$4.85, and cable transfers, \$4.85½ @ \$4.85½. Francs are quoted at 5.21½ for long and 5.20 for short, reichsmarks, 94½ @ 94½ for long and 95½ @ 95½ for short, guilders, 39½ @ 39½ for long and 40 @ 40½ for short.

39. A person going from New York to England carries with him, instead of money, a draft like the following:

New York, Oct. 1, 1897.

Exchange for £820-12-6 sterling.

At sight pay this First of Exchange, second of same tenor and date unpaid, to Edward Howe, or order, the sum of Eight Hundred Twenty Pounds £820-12-6 sterling. Value received, and charge to the account of

Smith, Jones & Co.,

To Baring Bros. & Co.,

London, England.

EXAMPLE 1—Find the cost of the foregoing draft in New York when exchange on London is 4.84½.

SOLUTION £820 12 6 £820 12s. 6d. £820.625.
 \$4.8475 × 820.625 \$3,977.98 Ans.

EXAMPLE 2 What must be paid for a draft on Paris of 8,000 francs, when \$1 is quoted at 5.21½?

SOLUTION 1 franc = $\frac{\$1}{5.21\frac{1}{2}}$;
 $\frac{\$1}{5.21\frac{1}{2}} \times 8,000 = \frac{\$8,000}{5.21\frac{1}{2}}$ \$1,534.04. Ans.

40. Rule.—*To find the cost of a draft upon a foreign country, multiply the quoted value of a foreign monetary unit by the given number of such units.*

* Sterling exchange is Bills of Exchange payable in English money called Pounds Sterling. Long bills are those payable 30, 60, 90 or more days after being received. Short bills are those payable from sight to 30 days after being received. Sight drafts are payable at sight, that is, as soon as received.

EXAMPLES FOR PRACTICE.

41. Solve the following examples

1. Find the cost of a draft on London, at 60 days' sight, for £987 16s., exchange being \$4 82½. Ans. \$4,768.60
 2. When exchange on Paris is quoted at 5.23, what must be paid for a sight draft for 2,800 francs? Ans. \$535.37.
 3. I bought a long draft on Berlin for 8,425 reichsmarks when exchange was quoted at 94½. What did it cost? Ans. \$7,982.69.
 4. What must be paid for a draft on Amsterdam for 8,000 guilders, exchange being 40½? Ans. \$3,205.
-

ARBITRATION OF EXCHANGE.

42. Arbitration of exchange is the process of finding the cost of a draft on one place through one or more intermediate places.

Thus, the quoted rates between New York and Vienna may be high, while those between New York and London, and between London and Vienna may be low. It may be cheaper for a man in New York to purchase a draft on Vienna through London than to purchase directly on Vienna.

In this roundabout or circuitous method, the intermediate brokers charge *brokerage* for their services, usually ½%.

43. **EXAMPLE 1.** When exchange between New York and Paris is 5.22, between New York and London is \$4.83, and between London and Paris 24.84 francs to the pound, which is cheaper—direct or circuitous exchange from New York upon Paris for 10,000 francs, London brokerage being ½%?

SOLUTION.—By direct exchange, the cost of a draft for 10,000 francs:

$$10\,000 \div 5.22 = \$1,915.71.$$

By circuitous exchange, the cost of the draft in pounds is

$$10,000 \div 24.84 = £402.5765.$$

$$\text{Cost of draft} + \text{brokerage} = £402.5765 \cdot 1.001 = £403.0797.$$

$$\text{Cost of draft in dollars} = \$4.83 \cdot 403.0797 = \$1,946.87.$$

Therefore, direct exchange is cheaper by

$$\$1,946.87 - \$1,915.71 = \$31.16. \quad \text{Ans.}$$

EXAMPLE 2.—A merchant sends 12,000 reichsmarks from New York to Berlin through London and Amsterdam. Exchange on London is \$4.85, between London and Amsterdam 11.86 guilders to the pound, and between Amsterdam and Berlin .43 reichsmark to the guilder. What is the cost of the draft in dollars, if brokerage at intermediate places is $\frac{1}{4}\%$?

SOLUTION 1.—We must first change the reichsmarks to guilders, for we do not know the relation between the reichsmark and the other monetary units. The broker gets his commission on the amount he actually exchanges, not on the amount given him to exchange. Hence $12,000 \div 1.00\frac{1}{4}$ is the number of reichsmarks that were exchanged for guilders, and $12,000 \times 1.00\frac{1}{4} \div .43 =$ the number of guilders received for 12,000 reichsmarks, less the brokerage of $\frac{1}{4}\%$. But $12,000 \times 1.00\frac{1}{4} \div .43 = (12,000 \times 1.00\frac{1}{4}) \div .43 = 27,941.86$ guilders. To change the guilders into pounds, taking out the brokerage of $\frac{1}{4}\%$, we have $27,941.86 \times 1.00\frac{1}{4} \div 11.86 = 2,358.92$. $\$4.85 \times 2,358.92 = \$11,440.76$.
Ans.

SOLUTION 2.—Another solution is the following, in which the vertical line is one of the signs of division, and indicates that the product of the numbers on the left of the line is to be divided by the product of the numbers on the right.

12,000 \times 1.00 $\frac{1}{4}$ reichsmarks	11.86 guilders.
1 \times 1.00 $\frac{1}{4}$ guilders	.43 reichsmark.
£1	£1.
\$4.85	

$$\frac{12,000 \times 1.00125 \times 1.00125 \times 4.85}{.43 \times 11.86} = \$11,440.76. \text{ Ans.}$$

To avoid the use of decimals having such an inconveniently large number of figures, it is better to use common fractions. Thus, $1.00\frac{1}{4}$

$$\frac{801}{800} \text{ whence, the last expression above becomes}$$

$$\frac{12,000 \times \frac{801}{800} \times \frac{801}{800} \times 4.85}{.43 \times 11.86} = \frac{12,000 \times 801 \times 801 \times 4.85}{.43 \times 11.86 \times 800 \times 800} = \$11,440.76.$$

By using common fractions instead of the decimals, the results will be more accurate and the principle of cancelation can be more readily employed.

44. It will be noticed that in the above arrangement the various monetary units appear once on each side of the vertical line of division. The same method may be employed with other than monetary units.

EXAMPLE —If 4 apples are worth 3 peaches, and 9 peaches are worth 5 oranges, how many apples must be given for 60 oranges?

SOLUTION.—

12		
60 oranges		x apples.
3		
9 peaches		5 oranges.
4 apples		3 peaches.

$$x = 12 \times 3 \times 4 = 144 \text{ apples. Ans.}$$

EXPLANATION.—The unknown quantity x should always be placed on the right-hand side of the vertical line of division, and the given quantity to which x is equivalent on the left-hand side. Then arrange the other quantities so that each quantity of the same kind appears on each side of the line. In the present example, 60 oranges are equivalent to a certain number of apples; hence, we place 60 oranges on the left and x on the right. Now, since 5 oranges are equivalent to 9 peaches, and we already have oranges on the left-hand side, we place the 5 oranges on the right-hand side and the 9 peaches on the left-hand side. For the same reason, we place 4 apples on the left-hand side and 3 peaches on the right-hand side. Canceling, we find that $x = 144$ apples.

EXAMPLES FOR PRACTICE.

45. Solve the following examples:

1. I sent from New York to Christiania, Norway, 4,740 crowns through London and Amsterdam. If 1 guilder = $.66\frac{7}{8}$ crowns, 11.85 guilders = £1, and \$4.85 = £1, what does the draft cost, brokerage at London on Amsterdam, and at Amsterdam on Christiania, being $\frac{1}{2}\%$?

Ans. \$2,908.19+.

2. If 5 cords of oak wood are worth 3 cords of hickory, and 4 cords of hickory are worth 10 cords of pine, what should be paid for oak when pine is \$1.25 per cord?

Ans. \$1.87 $\frac{1}{2}$.

3. If 9 bushels of wheat are worth 14 bushels of rye, 12 bushels of rye are worth 17 bushels of corn, and 3 bushels of corn are worth 5 bushels of oats, how many bushels of oats should be given for 324 bushels of wheat?

Ans. 1,190 bu.

AVERAGE OR EQUATION OF PAYMENTS.

46. Suppose that A owes B any sum, say \$100, due in 10 days, a second \$100, due in 20 days, and a third \$100, due in 30 days, there is evidently a time when he may pay B the entire \$300 without loss of interest to either party. Clearly, this time is 20 days after A incurs the debts. The conditions are that A may retain and use the first \$100 for 10 days, the second \$100 for 20 days, and the third \$100 for 30 days. But, so far as the interest is concerned, the use of \$100 for 10 days is equivalent to the use of $\$100 \times 10$, or \$1,000, for 1 day; \$100 for 20 days equals \$2,000 for 1 day; \$100 for 30 days equals \$3,000 for 1 day. A's privilege, therefore, equals $\$1,000 + \$2,000 + \$3,000$, or \$6,000 for 1 day. But \$6,000 for 1 day is the same as \$300 for 20 days. Expressing this argument more briefly, we have

$$\begin{array}{rcl}
 \$100 \text{ for 10 days} & = & \$1,000 \text{ for 1 day.} \\
 \$100 \text{ for 20 days} & = & \$2,000 \text{ for 1 day.} \\
 \$100 \text{ for 30 days} & = & \$3,000 \text{ for 1 day.} \\
 \hline
 \$300 & &) \$6,000 \\
 & & \underline{\hspace{1.5cm}} \\
 & & 20 \text{ days.}
 \end{array}$$

47. Average, or equation, of payments is the process of finding the equitable time when payment of several sums, due at different times, may be made in one payment.

48. The equated, or average, time of payment is the time when several debts with different *terms of credit* may be equitably made in one payment.

49. EXAMPLE 1.—At a certain time A agrees to pay \$1,000 as follows: \$300 in 30 days, \$200 in 60 days, and \$500 in 90 days. Find the equated time of payment, that is, the time at which the entire debt may be paid without interest and still be fair to both parties.

$$\begin{array}{rcl}
 \text{SOLUTION.}— & \$300 \times 30 = & \$9,000 \\
 & \$200 \times 60 = & \$12,000 \\
 & \$500 \times 90 = & \$45,000 \\
 \hline
 & \$1,000 &) \$66,000 \\
 & & \underline{\hspace{1.5cm}} \\
 & & 66 \text{ days.}
 \end{array}$$

The whole sum may equitably be paid 66 days after the obligation is incurred. Ans.

EXAMPLE 2.—A man owes \$250 due Mar. 1, \$300 due Apr. 20, \$450 due May 5, and \$500 due June 25. When is the equated time of payment?

$$\begin{array}{rcl}
 \text{SOLUTION.} & \$250 \times 0 = & 0 \\
 & \$300 \times 50 = & \$15000 \\
 & \$450 \times 65 = & \$29250 \\
 & \$500 \times 116 = & \$58000 \\
 \hline
 \$1500 &) \$102250 & \\
 & 68+ &
 \end{array}$$

68 days after Mar. 1, or May 8. Ans.

EXPLANATION.—Taking Mar. 1, the time when the first debt is due, as the *date of reference*, or the time from which to determine the terms of credit, the term of credit for \$250 is 0 days. The term of credit for \$300 is from Mar. 1 to Apr. 20, or 50 days; for \$450 the term of credit is from Mar. 1 to May 5, or 65 days; and for \$500 the term of credit is from Mar. 1 to June 25, or 116 days. We multiply each debt by its term of credit, and divide the sum of the products by the sum of the debts. The quotient, 68 days, is the number of days after Mar. 1 when one payment of the whole indebtedness may equitably be made, or May 8.

50. In the example just given, the length of time between the date when the equated time was computed and Mar. 1 was not stated. This does not matter, since the only effect that would be produced by introducing the number of days between this date and Mar. 1 would be to increase the numbers on the right of the signs of equality, and, since the total debt remains the same, the number of days obtained for the equated time will be increased by an amount just equal to the number of days between this date and Mar. 1, which, of course, does not change the date of settlement. For example, suppose that the equated time was computed 30 days preceding Mar. 1, that is on Jan. 29. Then, the first debt falls due in 30 days; the second, in $30 + 50 = 80$ days; the third, in $30 + 65 = 95$ days; and the fourth, in $30 + 116 = 146$ days.

In order, therefore, to find the equated time, we proceed as follows:

$$\begin{array}{r}
 \$250 \times 30 = \$7500 \\
 300 \times 80 = 24000 \\
 450 \times 95 = 42750 \\
 500 \times 146 = 73000 \\
 \hline
 \$1500 \quad) \$147250
 \end{array}$$

98.1† or 98 days after Jan. 29.

But 98 days after Jan. 29 is the same as 68 days after Mar. 1.

The number of days as computed above is always taken to the nearest integer; that is, if the fractional part is .5 or greater, 1 day is added. For instance, had the above result been 98.6, the number of days would have been taken as 99.

51. Rule.—*Taking as the date of reference the date when the first debt is due, find the term of credit for each debt.*

Multiply each debt by its term of credit, and divide the sum of the products by the sum of the debts. The quotient to the nearest integer will be the number of days from the date of reference to the equated time.

EXAMPLES FOR PRACTICE.

52. Solve the following examples:

1. A owes B \$500 due in 8 months, and \$900 due in 4 months. When may he equitably pay B both debts in one payment?

Ans. 5 mo. 13 da.

2. Find the equated time for paying \$400 due May 10, \$500 due June 20, \$900 due July 30, and \$1,000 due Aug. 15.

Ans. July 17.

3. Tefft, Weller & Co. sold to E. King & Co. goods as follows: on June 15, \$2,500 on 30 days' credit, and, on June 30, \$3,600 on 20 days' credit. Find the equated date of payment.

Ans. July 18.

4. What is the equated time for the payment of three notes: one for \$600, dated Aug. 9, 1897, for 3 months; the second for \$800 dated Oct. 1, 1897, for 2 months; the third for \$1,200, dated Dec. 21, 1897, for 6 months?

Ans. Feb. 27, 1898.

5. On Jan. 1, 1898, a merchant sold a bill of goods amounting to \$3,000, payable as follows: $\frac{1}{3}$ in 30 days, $\frac{1}{3}$ in 60 days, and the remainder in 90 days. Find the equated time of payment.

Ans. Feb. 20, 1898.

AVERAGE OR EQUATION OF ACCOUNTS.

53. Goods are usually sold on credit, the term of credit being commonly 30 days, 60 days, or 90 days. The prices of the goods are fixed for the time on which they are sold, interest being charged if payment is not made at the end of the time specified, and a discount being given if the debt is paid before the end of the term of credit. Now, if one or more payments are made on the bill before it is due, it is evident that a rebate ought to be given the purchaser, or else his term of credit on the remainder of the bill ought to be extended. For example, suppose that Wm. Marshall buys a bill of goods amounting to \$1,525.86 on Jan. 5, on 90 days' credit. The price was so fixed that the seller would not lose anything by selling the goods on 90 days' credit. Suppose that, on Jan. 27, Mr. Marshall pays \$425.40 on account, and on Feb. 24, \$506.62. There still remains unpaid $\$1,525.86 - (\$425.40 + \$506.62) = \593.84 . Mr. Marshall ought either to receive a rebate, or his term of credit on the \$593.84 still unpaid ought to be extended, in order to compensate him for having paid a part of the bill before it was due. To determine how long the term of credit should be extended, we reason as follows: The first payment was made 22 days, and the second payment 50 days, after the goods were bought. Hence, Mr. Marshall lost the use of \$425.40 for $90 - 22 = 68$ days, and of \$506.62 for $90 - 50 = 40$ days; or of $\$425.40 \times 68 = \$28,927.20$ for 1 day, and of $\$506.62 \times 40 = \$20,264.80$ for 1 day. Consequently, altogether, he lost the use of $\$28,927.20 + \$20,264.80 = \$49,192$ for 1 day. Therefore, to make things equal all around, the term of credit on the \$593.84 still unpaid should be extended $49,192 \div 593.84 = 83$ days, and the date of settlement should be $90 + 83 = 173$ days after Jan. 5, or June 27.

In equation of payments, only one side of the account is considered, the items being either all debits or all credits; but, when both sides of the account are considered, the process of finding the equated time is called **equation of accounts**. In averaging accounts, the method of finding

the equated time is nearly the same as in equation of payments; the method is shown in the following examples:

54. EXAMPLE.—Find the equated time for the settlement of the following account:

HENRY WARDELL.

1890.				1890.			
Jan	20	Mdse, 30 days,	800	Mar	1	Cash,	400
Feb.	18	" 30 "	600	"	20	"	600
Mar	14	" 60 "	1,000	Apr	1	"	1,000
Apr.	10	" 30 "	1,200	"	20	"	500

SOLUTION. The date of reference is Feb. 19, since 30 days after Jan. 20 is Feb. 19.

Feb. 19,	$800 \times 0 =$	0	Mar. 1,	$400 \times 10 =$	4 000
May 19,	$600 \times 89 =$	53 400	Mar. 20,	$600 \times 29 =$	17 400
May 13,	$1000 \times 83 =$	83 000	Apr. 1,	$1000 \times 41 =$	41 000
May 10,	$1200 \times 80 =$	96 000	Apr. 20,	$500 \times 60 =$	30 000
	3600	232 400		2500	92 400
	2500	92 400			
	1100	140 000			
		127			

Average term of credit, 127 days.

Equated time, 127 days after Feb. 19, or June 26. **Ans.**

EXPLANATION.—As in equation of payments, the debts are multiplied by the number of days from the date of reference to the dates when they respectively become due. It is most convenient to take as the date of reference (usually called the *focal date*) the date when the first debt becomes due; or, if a payment is made before the first debt becomes due, take the date of the first payment as the focal date. The same operation is performed on the payments, *using the same focal date*. The sum of the credits is then subtracted from the sum of the debits, the sum of the products on the credit side from the sum of the products on the debit side, and the second remainder is divided by the first remainder.

EXAMPLE.—In New York, what will be the cash balance of the following ledger account Jan. 10, 1898, interest at 6%?

WM. BONNER.

1897.				1897.			
Sept.	1	Mdse., 90 days,	800	Sept.	12	Cash,	600
	20	" 60 "	900	Oct.	20	Draft, 30 days,	500
Oct.	25	" 60 "	1,000	Nov.	15	Cash,	800
Nov.	1	" 30 "	2,000	Dec.	12	"	1,000

SOLUTION.—In this case it will be more convenient to take as the focal date Sept. 1. It will be noticed that one of the payments, that of \$500 on Oct. 20, is a 30-day draft. Since this draft can be cashed for its face value only after 30 days, i. e., on Nov. 19, this payment must be considered as having been made on Nov. 19; and, as days of grace are not allowed in New York, they are not added to the time the draft has to run. Proceeding as follows to find the equated time of settlement,

$800 \times 90 = 72000$	$600 \times 11 = 6600$
$900 \times 79 = 71100$	$500 \times 79 = 39500$
$1000 \times 114 = 114000$	$800 \times 75 = 60000$
$2000 \times 91 = 182000$	$1000 \times 102 = 102000$
<u>4700</u>	<u>2900</u>
439100	208100
<u>2900</u>	
1800	
<u>281000</u>	
128½ days.	

we find it to be 128 days after Sept. 1, or Jan. 7, 1898. If the bill is not settled until Jan. 10, it is clear that in equity 3 days' interest should be charged on the \$1,800 yet unpaid. The interest of \$1,800 for 3 days at 6% is \$.90; hence, the total amount that should be paid is \$1,800.90.

Ans.

EXAMPLE.—When should interest begin on the balance of the following account?

HENRY WELLINGTON.

1902.				1902.			
June	16	Mdse.,	1,200	July	21	Cash,	800
July	21	"	1,000	Aug.	11	"	1,200
Aug.	13	"	2,000		30	Draft, 30 days,	1,600
Sept.	15	"	3,600	Sept.	20	" 10 "	2,400

SOLUTION.—We take, as the focal date, June 1st. The reason for taking the first day of the month as the focal date is that it is easier to reckon the number of days between the first day of the month and some later date than it is to reckon the number of days between some other

day of the month and a later date. Days of grace are allowed on the two time drafts, because no state is mentioned in the example.

12000 × 15 =	180000	800 × 50 =	40000
10000 × 50	500000	12000 × 71	852000
20000 × 73	1460000	16000 × 123	1968000
36000 × 106 =	3816000	24000 × 124 =	2976000
78000	5956000	60000	6196000
00000			5956000
18000		18000	24000
			13 + days.

We find the products of the items and number of days as in the two previous examples. It will be noticed that the sum of the products on the credit side is greater than the sum of the products on the debit side, while the sum of the payments is less than the sum of the debts. We divide the difference between the sums of the products by the difference between the sum of the debts and the sum of the payments, obtaining for our result 13 days. Now, instead of *adding* this 13 days to the focal date, we *subtract*. The reason for subtracting will be evident when we consider that the last three payments were made a comparatively long time after the merchandise was bought. In other words, the merchant lost the use of the money due him in payment of the goods for a time equivalent to the difference between 13 days preceding June 1st (or May 19th) and the date of settlement, and he should receive interest on \$1,800 for this time. If, however, the sum of the products on the credit side had exceeded the sum of the products on the debit side, and the sum of the payments had exceeded the sum of the debts, the number of days obtained would have been *added* to the focal date, as in the two preceding examples. Hence, the interest on the balance of the above account should begin on May 19th. Ans.

55. Rule. Find the date on which each item of the account matures, and take the first day of the month in which the earliest of these dates occurs, as the focal date. Find the number of days between the focal date and the date of maturity of the different items, and multiply each item by the number of days so found. Divide the difference between the sums of the products by the difference between the sum of the debts and the sum of the payments, and the quotient will be the equated time. If the greater sum of the items and the greater sum of the products are both on the same side of the account, *add* the equated time to the focal date. But, if the greatest sum of the items and the greatest sum of the products

are on opposite sides of the account, **subtract** the equated time from the focal date. The result obtained by adding or subtracting the equated time from the focal date will be the date when the balance of the account is equitably due.

EXAMPLES FOR PRACTICE.

56. Solve the following examples, allowing 3 days of grace on time drafts.

1. Find the cash balance of the following account, July 1, 1903, interest at 6%; also, the equated time of settlement.

1903.				1903.			
GEORGE GRIFFIN.							
Jan	10	Mdse.,	2,500	Feb.	30	Cash,	4,000
Feb	10	"	4,900	Apr.	24	Mdse.,	1,900
Mar	10	"	2,000	May	7	Draft, 10 days,	2,900
May	20	"	6,900	June	12	Cash,	5,000

2. Find the equated time of settlement of the following account

1901				1901			
WALTER ROBERTS.							
Jan	1	Mdse., 20 days,	300	Feb	4	Cash,	800
Mar.	4	" 80 "	480	Apr.	6	Draft, 30 days,	900
May	13	" 60 "	640	May	7	" 60 "	600
June	12	" 80 "	960				

3. Find the equated time of settlement of the following account

1900.				1900			
WILLARD SMITH.							
Jan.	21	Mdse., 60 days,	4,000	Jan.	31	Cash,	2,500
Feb.	3	" 60 "	5,000	Feb.	14	Real Estate,	3,500
May	13	" 90 "	8,000	Apr.	15	Draft, 60 days,	6,000
June	15	" 80 "	7,800	May	1	Cash,	7,500
				July	1	"	2,000

Answers.—(1) \$2,628.67, Aug 31, 1902; (2) Aug. 16; (3) Apr. 27, 1897.

57. **NOTE**—The student will find the table given in Art. 68, § 4, of great assistance to him in working examples in equation of accounts.

PARTNERSHIP.

58. **Partnership** is an association of two or more persons for the transaction of business with joint capital.

59. The **capital** is the money or other property invested. The persons associated are **partners**, and these may be *active partners* or *silent partners*. An **active partner** is one who has a voice in the management of the enterprise. A **silent partner** is one having money invested in the business, but having no voice in its management.

60. The **assets** or **resources** of a firm are the property it owns and the debts owing to it.

The **liabilities** of a firm are the debts it owes.

WHEN THE PARTNERS INVEST FOR THE SAME PERIOD OF TIME.

61. The gains and the losses of a firm are apportioned among its members in proportion to each partner's investment, and the time during which it is invested. The operation is best performed by the method known as **distributive proportion**. When the shares of the several partners are invested for equal times, the apportionment of gains or losses is exactly similar to the division of a number into given proportional parts.

62. **EXAMPLE 1** — Divide the number 324 into three parts that are to one another as 5, 6, and 7.

1ST SOLUTION. The sum of the proportional parts is 18. It is required, therefore, to find 5, 6, and 7 of the 18 equal parts of 324.

$$\left. \begin{array}{l} 5+6+7:5=324:x; x=90 \\ 5+6+7:6=324:x; x=108 \\ 5+6+7:7=324:x; x=126 \end{array} \right\} \text{Ans.}$$

324.

2D SOLUTION. — It is evident that the required numbers must be multiples of 5, 6, and 7, hence, the sum of these multiples (which is 324

according to the conditions of the example) must also be a multiple of the sum of the given numbers 5, 6, and 7, i. e., a multiple of 18. Dividing 324 by 18, the quotient is 18, and 18 multiplied in succession by 5, 6, and 7 gives 90, 108, and 126, the required numbers. Ans.

EXAMPLE 2.—A, B, and C invest \$500, \$600, and \$700, respectively, in an enterprise by which they gain \$1,620. Apportion the gain.

1ST SOLUTION.—

$$\begin{aligned} \$500 + \$600 + \$700 &= \$1,800, \text{ the total amount invested.} \\ \$1,800 : \$500 &= \$1,620 : \text{A's share; A's share} = \$450. \\ \$1,800 : \$600 &= \$1,620 : \text{B's share; B's share} = \$540. \\ \$1,800 : \$700 &= \$1,620 : \text{C's share; C's share} = \$630. \end{aligned} \quad \left. \vphantom{\begin{aligned} & \\ & \\ & \end{aligned}} \right\} \text{Ans.}$$

2D SOLUTION.—Total amount invested is \$1,800. Since each partner shares the gain in proportion to the amount he invests, it is clear that A's share must be $\frac{500}{1,800}$ of \$1,620, or \$450; B's share $\frac{600}{1,800}$ of \$1,620, or \$540; and C's share $\frac{700}{1,800}$ of \$1,620, or \$630. Ans. This method is exactly the same as in the 2d solution of the last example; for, dividing 1,620 by 1,800, the result is .9, and .9 multiplied successively by \$500, \$600, and \$700, gives \$450, \$540, and \$630.

63. Rule.—*Divide the amount each partner invests by the total amount invested, and multiply the quotient by the gain or loss. The result will be each partner's gain or loss.*

EXAMPLES FOR PRACTICE.

64. Solve the following examples:

1. Divide \$1,440 into 3 parts that shall be to one another as 3, 5, and 8. Ans. \$270; \$450; \$720.

2. Divide the number 1,058 into 3 parts that shall be to one another as $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$. Ans. 276; 368; 414.

SUGGESTION.—Change the fractions to a common denominator, and divide the number into parts that are proportional to the numerators.

3. A and B engage in trade, and gain \$2,340. A's investment is to B's as 5 is to 8. Apportion the gain.

Ans. $\left\{ \begin{array}{l} \text{A's gain, \$900.} \\ \text{B's gain, \$1,440.} \end{array} \right.$

4. A puts \$8,000 into an enterprise, B \$1,100, and C \$1,300. They lose by it \$3,520. What does each lose?

$$\text{Ans. } \begin{cases} \text{A loses } \$2,707.69. \\ \text{B loses } \$372.81. \\ \text{C loses } \$440.00. \end{cases}$$

5. A man fails in business with assets amounting to \$7,080. He owes A \$7,500, B \$9,500, and C \$12,500. How much of the assets should each receive?

$$\text{Ans. } \begin{cases} \text{A, } \$1,800. \\ \text{B, } \$2,280. \\ \text{C, } \$3,000. \end{cases}$$

6. A, B, and C rent a pasture for \$63. A puts in 9 horses, B 12 horses, and C 15 horses. How much of the rent should each pay?

$$\text{Ans. } \begin{cases} \text{A, } \$15.75. \\ \text{B, } \$21.00. \\ \text{C, } \$26.25. \end{cases}$$

7. A's debts are to B's as $\frac{3}{4}$ to $\frac{7}{8}$, and together they owe \$4,003.50. Find the debt of each.

$$\text{Ans. } \begin{cases} \text{A, } \$1,884.00. \\ \text{B, } \$2,119.50. \end{cases}$$

WHEN PARTNERS INVEST FOR DIFFERENT PERIODS OF TIME.

65. EXAMPLE. Three men engage in business. A puts in \$8,000 for 8 months, B \$10,000 for 12 months, and C \$9,000 for 10 months. They lose \$10,960. What does each man lose?

$$\text{1ST SOLUTION.} - \$8,000 \times 8 + \$10,000 \times 12 + \$9,000 \times 10 = \$274,000.$$

$$\begin{aligned} \$274,000 : \$64,000 &= \$10,960 : \text{loss of A, or } \$2,560. \\ \$274,000 : \$120,000 &= \$10,960 : \text{loss of B, or } \$4,800. \\ \$274,000 : \$90,000 &= \$10,960 : \text{loss of C, or } \$3,600. \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{Ans.}$$

$$\$10,960$$

EXPLANATION.— \$8,000 for 8 months equals \$64,000 for 1 month; \$10,000 for 12 months equals \$120,000 for 1 month; \$9,000 for 10 months equals \$90,000 for 1 month. The remainder of the operation is as before explained.

$$\begin{aligned} \text{2D SOLUTION.} - \quad & \$8,000 \times 8 = \$64,000 \\ & \$10,000 \times 12 = \$120,000 \\ & \$9,000 \times 10 = \$90,000 \\ & \qquad \qquad \qquad \$274,000 \text{ for 1 month.} \end{aligned}$$

$$\begin{aligned} \text{A's loss} &= \frac{64,000}{274,000} \times \$10,960 = \$2,560. \\ \text{B's loss} &= \frac{120,000}{274,000} \times \$10,960 = \$4,800. \\ \text{C's loss} &= \frac{90,000}{274,000} \times \$10,960 = \$3,600. \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{Ans.}$$

EXAMPLE.—A and B engage in business for 1 year. During the first 5 months A's capital was \$8,000, at the end of which time he increased it by \$1,000. B's capital the first 8 months was \$10,000, he then drew out \$5,000 of it. At the end of the year they had gained \$8,120. Find each man's share.

1st SOLUTION.—Reducing the amounts invested to an equal interval of time, as 1 month, A had \$8,000 invested for 5 months, and \$8,000 + \$1,000 = \$9,000 for 7 months; B had \$10,000 invested for 8 months, and \$10,000 - \$5,000 = \$5,000 for 4 months.

$$\$8,000 \times 5 + \$9,000 \times 7 = \$103,000.$$

$$\$10,000 \times 8 + \$5,000 \times 4 = \$100,000.$$

$$203,000 : 103,000 = \$8,120 : A's \text{ share, or } \$4,120 \quad \text{Ans.}$$

$$203,000 : 100,000 = \$8,120 : B's \text{ share, or } \$4,000$$

$$\$8,120$$

2d SOLUTION.—Proceeding as before, the total amount invested for 1 month was \$203,000; hence,

$$\left. \begin{array}{l} A's \text{ gain was } \frac{103,000}{203,000} \times \$8,120 = \$4,120. \\ B's \text{ gain was } \frac{100,000}{203,000} \times \$8,120 = \$4,000. \end{array} \right\} \text{Ans.}$$

66. Rule.—Reduce the amounts invested to equivalent amounts for an equal interval of time, as 1 year, 1 month, or 1 day; then proceed as in Art. 63.

EXAMPLES FOR PRACTICE.

67. Solve the following examples:

1. A and B entered into partnership. A put in \$8,000 for 5 months, and B put in \$5,000 for 8 months. They gained \$3,500. What was each man's share of the gain?

$$\text{Ans. } \left\{ \begin{array}{l} A, \$1,500. \\ B, \$2,000. \end{array} \right.$$

2. A began business with a capital of \$12,000. At the end of 4 months he took in B with \$20,000, and 2 months later they took in C with \$30,000. At the end of the year, their gain was \$18,150. How much of the gain should each receive?

$$\text{Ans. } \left\{ \begin{array}{l} A, \$5,400. \\ B, \$6,000. \\ C, \$6,750. \end{array} \right.$$

3. A, B, and C with a combined capital of \$73,000 gained \$10,000. A's capital was in the business 4 months, B's 5 months, and C's 10 months. If A's capital was \$18,500, B's \$23,800, and C's \$30,700, how much did each gain?

$$\text{Ans. } \left\{ \begin{array}{l} A, \$1,480 \\ B, \$2,380 \\ C, \$6,140 \end{array} \right.$$

4. A contractor had 12 men on a piece of work for 10 days, when they went on a strike. He then employed 15 new men at the same daily wages, and they finished the work in 12 days. If he paid in wages for the entire work \$555, how much did each set of men receive, and how much did each man get per day?

Ans. $\left\{ \begin{array}{l} \text{1st set, \$222.} \\ \text{2d set, \$333.} \\ \text{\$1.85 a day.} \end{array} \right.$

5. Two men, A and B, hire a pasture for \$56. A puts in 12 horses for 15 days, and B puts in 24 oxen for 18 days. What should each pay, assuming that 4 oxen eat as much as 5 horses?

Ans. $\left\{ \begin{array}{l} \text{A, \$14.} \\ \text{B, \$42.} \end{array} \right.$

6. A and B are in partnership for 1 year. A begins the year with \$2,400, at the end of 3 months he increases his capital by \$1,000; and 4 months later he draws out \$600. B begins 3 months after A with \$3,600, and 5 months later he adds \$1,200 to his capital. At the end of the year they have gained \$6,120. How much of the gain should each receive?

Ans. $\left\{ \begin{array}{l} \text{A, \$2,958.} \\ \text{B, \$3,162.} \end{array} \right.$

ALLIGATION.

68. *Alligation* is a process of computation in which the *proportion* of the ingredients in a mixture, their respective *prices*, and the *price of a unit* of the mixture are the elements considered.

69. Alligation is of two kinds—*alligation medial* and *alligation alternate*.

ALLIGATION MEDIAL.

70. *Alligation medial* is the process of finding the value of a unit of a combination made up of several ingredients when the number of units and the price or quality per unit of each ingredient are given.

71. *EXAMPLE.*—A farmer sold 30 bushels of wheat at 75 cents a bushel, 72 bushels of rye at 45 cents a bushel, and 60 bushels of corn at 30 cents a bushel. Find the average price per bushel.

SOLUTION,—

$$\$.75 \times 30 = \$ 22.50$$

$$\$.45 \times 72 = \$ 32.40$$

$$\$.30 \times 60 = \$ 18.00$$

$$\begin{array}{r} 162 \overline{) \$ 72.90} \end{array}$$

\$.45 Ans.

EXPLANATION.—The operation is performed by the method of products, exactly as in equation of payments. The entire cost of the three kinds of grain is first found, and this sum is divided by the number of bushels sold.

72. Rule.—*Multiply the price or quality of a unit of each element of the combination by the number of such units, and divide the sum of the products by the entire number of units in the combination. The quotient will be the average cost or quality of a unit.*

EXAMPLES FOR PRACTICE.

73. Solve the following examples:

1. A grocer made a mixture of 15 lb. of 17-cent coffee, 15 lb. of 20-cent coffee, and 39 lb. of 30-cent coffee. For how much should it be sold per pound? Ans. \$.25 per lb.

2. A goldsmith melted together gold as follows, 48 oz. of 15 carats, 24 oz. of 17 carats, 12 oz. of 20 carats, and 36 oz. of 22 carats. How many carats fine was the mixture? Ans. 18 carats.

3. Teas worth \$.40, \$.50, \$.60, and \$.75 per pound are made into a mixture containing 15 lb. of the first kind, 60 lb. of the second, 45 lb. of the third, and 15 lb. of the fourth. Find the value of a pound of the mixture. Ans. \$.55 per lb.

4. A merchant sold silk in quantity and price as follows: 72 yd. at \$1.50 per yard, 24 yd. at \$1.87½, 12 yd. at \$2.25, and 48 yd. at \$2.75. What was the average price per yard? Ans. \$2 per yd.

5. A mixture of candy consists of 20 lb. worth 18 cents a pound, 40 lb. worth 25 cents a pound, 48 lb. worth 35 cents a pound, and 20 lb. worth 40 cents a pound. What is a pound of the mixture worth? Ans. \$.30 per lb.

ALLIGATION ALTERNATE.

74. Alligation alternate is the process of determining the proportional parts of the several ingredients in a combination or mixture having a given average unit value.

Thus, it may be required to find in what proportional parts we must mix coffee, tea, candies, etc. at different prices, so that the combination may be sold without loss at a given price per unit.

75. EXAMPLE.—Teas worth 38, 45, 52, and 60 cents per pound are mixed so as to be sold at 50 cents per pound. Find the proportional weights of the several kinds.

SOLUTION.

$$\text{I. } 50 \left\{ \begin{array}{l} 38 \\ 45 \\ 52 \\ 60 \end{array} \right\} \left| \begin{array}{l} \frac{1}{17} \\ \\ \\ \frac{1}{10} \end{array} \right| \left| \begin{array}{l} 5 \\ 2 \\ 5 \\ 6 \end{array} \right\} \text{Ans.} \quad \text{PROOF, } \left\{ \begin{array}{l} \$.38 \times 5 = \$ 1.90 \\ \$.45 \times 2 = \$.90 \\ \$.52 \times 5 = \$ 2.60 \\ \$.60 \times 6 = \$ 3.60 \\ 18 \quad) \$ 9.00 \\ \underline{50} \end{array} \right.$$

$$\text{II. } 50 \left\{ \begin{array}{l} 38 \\ 45 \\ 52 \\ 60 \end{array} \right\} \left| \begin{array}{l} \frac{1}{17} \\ \\ 1 \\ \frac{1}{10} \end{array} \right| \left| \begin{array}{l} 1 \\ 2 \\ 6 \\ 1 \end{array} \right\} \text{Ans.} \quad \text{PROOF, } \left\{ \begin{array}{l} \$.38 \times 1 = \$.38 \\ \$.45 \times 2 = \$.90 \\ \$.52 \times 6 = \$ 3.12 \\ \$.60 \times 1 = \$.60 \\ 10 \quad) \$ 5.00 \\ \underline{50} \\ \$.50 \end{array} \right.$$

EXPLANATION. The average price and the ingredient prices are written as shown. The ingredient prices are then linked in pairs, so that one of each pair may be greater, and one less, than the average price. In Solution I, 38 is linked with 60, and 45 with 52. Each different linking gives a different result. Considering Solution I, a pound worth \$0.38 sold at \$0.50 gains \$0.12, and to gain \$0.01 there must be sold $\frac{1}{17}$ of a pound. A pound worth \$0.60 sold at \$0.50 loses \$0.10, and to lose \$0.01 there must be sold $\frac{1}{10}$ of a pound. A gain of 1 and a loss of 1 is, on the whole, neither gain nor loss. It is clear that the sale of 60 times $\frac{1}{17}$ lb. and 60 times $\frac{1}{10}$ lb. will produce neither gain nor loss—that is, a sale of 5 and 6.

Applying the same reasoning to the other pair, we find that the sale of $\frac{1}{17}$ lb. and $\frac{1}{10}$ lb., or of 10 times these, will, in like manner, result neither in gain nor in loss. Hence, if 5, 2, 5, and 6 lb. of teas worth 38, 45, 52, and 60 cents per pound, respectively, be mixed, the mixture will be worth 50 cents per pound.

Any multiple of 5, 2, 5, and 6 will have the same average value per pound.

Solution II should be evident after considering the explanation of Solution I.

76. The fractions may be avoided by writing the difference between the average price and each ingredient price opposite the other ingredient price with which it is linked. This will be shown in the following example:

EXAMPLE. In what proportional parts must candies worth 25, 30, 36, and 50 cents a pound occur in a mixture worth 40 cents a pound?

SOLUTION -

40	25	10	10	Ans.
	30		10	
	36		10	
	50	15	10	
		4	29	

PROOF.

$\$.25 \times 10 = \$$	2.50
$\$.30 \times 10 = \$$	3.00
$\$.36 \times 10 = \$$	3.60
$\$.50 \times 29 = \$$	14.50
	59) \$ 23.60
	\$ 4.00

EXPLANATION.—Here, then, are three kinds of candy worth less than the average, 40 cents, and one kind worth more. Hence, we link 25 with 50, 30 with 50, and 36 with 50, as shown. We draw a vertical line and subtract 25 from 40, obtaining 15, which we write opposite 50; subtracting 40 from 50 we get 10, which we write opposite 25. Drawing another vertical line, we subtract 30 from 40, getting 10, which we write opposite 50; subtracting 40 from 50, we get 10, which we write opposite 30. Drawing another vertical line, we subtract 36 from 40, getting 4, which we write opposite 50; subtracting 40 from 50, we get 10, which we write opposite 36. Now, adding horizontally the numbers included between the vertical lines, the required proportion is 10 lb. of 25-cent, 10 lb. of 30-cent, 10 lb. of 36-cent, and 29 lb. of 50-cent candy.

77. The same method may be applied to examples in Art. 75, thus:

50	38	10	10	Ans.
	45		2	
	52		5	
	60	12	12	

78. There are usually many ways of linking the quantities in alligation alternate, but the answers will all be true, provided the two following conditions have been observed:

1. *That each pair must consist of one term below the average and one above.*
2. *That every quantity except that denoting the average must be included in the linking.*

79. When, however, only one quantity is greater or less than the average, it must be linked with each other quantity. In this case, there is only one method of linking, and only one set of numbers with its multiples will then meet the conditions of the example.

When there are two elements above the average and two below, there are several ways in which they may be linked. As the number of elements increases, the ways in which they may be linked become very numerous.

It is obvious that the conditions of an example will be met not only by any multiple of the several answers, but also by the sum of any two or more answers, provided that their elements are taken in the same order.

EXAMPLES FOR PRACTICE.

80. Solve the following examples

1. A miller mixes wheat worth \$.80, \$.95, \$1.02, and \$1.08 per bushel so as to sell the mixture at \$1.00 per bushel. Find the proportional number of bushels of each kind in the mixture.

Ans. $\left\{ \begin{array}{l} 8, 2, 5, 20. \\ 2, 2, 5, 5. \\ \text{etc.} \end{array} \right.$

2. A butcher bought lambs at \$4 each, sheep at \$7, hogs at \$9, and calves at \$12. In what proportion were they, if he paid for them an average of \$8½ each?

Ans. $\left\{ \begin{array}{l} 7, 1, 3, 9. \\ \text{etc.} \end{array} \right.$

3. A merchant bought different kinds of cloth at \$1.50, \$1.65, \$1.85, and \$2.00 per yard. If the average price was \$1.75 per yard, in what proportion were the different kinds?

Ans. $\left\{ \begin{array}{l} 2, 5, 5, 2. \\ \text{etc.} \end{array} \right.$

4. A mixture of 14, 19, 20, and 21 carat gold has an average fineness of 18 carats. Of what proportional parts is the mixture?

Ans. 3, 2, 2, 2.

5. The sum of certain multiples of 10, 23, 29, and 31 is exactly divisible by 24. What is the least possible sum of these numbers?

Ans. 240.

SUGGESTION.—First find the multipliers; then multiply the four given numbers by these multipliers, and add the products.

6. A pile of wood worth \$504 contains hickory worth \$8 a cord, oak worth \$5, maple worth \$4, and pine worth \$2. How many cords are there of each kind, if the average price per cord is \$4½?

Ans. $\left\{ \begin{array}{l} 40, 8, 8, 56. \\ \text{etc.} \end{array} \right.$

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SPELLING.

RULES FOR SPELLING.

1. The plural of nouns is generally formed by adding *s* to the singular; as, field, fields; friend, friends; temple, temples.
2. Nouns ending in *s*, *sh*, *ch* (soft), *x*, or *z*, add *es* for the plural; as, class, classes; brush, brushes; watch, watches; tax, taxes; buzz, buzzes.
3. Nouns ending in *y*, with a vowel* before the *y*, add *s* to form the plural; as, monkey, monkeys; day, days; valley, valleys.
4. Nouns ending in *y*, with a consonant before the *y*, change *y* to *i* and add *es* to form the plural; as, story, stories; army, armies; colony, colonies.
5. Nouns ending in *f* and *fe* generally change *f* or *fe* into *ves* to form the plural; as, calf, calves; knife, knives; thief, thieves.

FAMILIAR WORDS.

1.

clothes	cloak	clean	a'pron	words
shoe	your	slate	col'lar	spell
stock'ing	sharp	jack'et	rib'bon	les'son
coat	write	mit'ten	here	pen'cil

2.

peel	stem	mouth	tree	teeth
core	fore'head	chin	pulp	check
seeds	eye	flow'er	rind	ear
juice	nose	fruit	tongue	hair

* The *vowels* are *a*, *e*, *i*, *o*, *u*, and sometimes *w* and *y*. The other letters of the alphabet are called *consonants*.

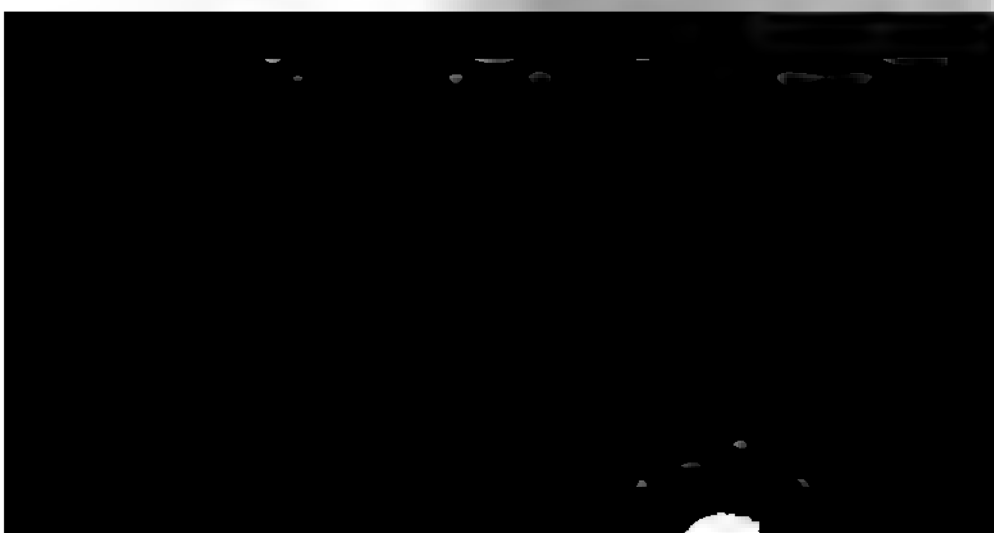
§ 11

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1. 1000
2. 1000
3. 1000
4. 1000

1. 1000
2. 1000
3. 1000
4. 1000

1. 1000
2. 1000
3. 1000
4. 1000



SPELLING.

3

9.—THE HOUSE.

chim'ney	man'tel	en'try
cup'board	fur'nace	pan'try
cel'lar	door'step	li'bra ry
hearth	at'tic	stair'case

10.—IN THE SCHOOLROOM.

wall	satch'el	pens
class	tran'som	globe
shelf	reg'is ter	pa'per
seats	pro'gram	shut'ters
chart	pen'-wi per	neat'ness
chairs	waste'-bas ket	at ten'tion

11.

switch	kit'ten	bring	click
skip	glimpse	width	wink
milk	still	fringe	cling
whist	miss	stiff	print
which	prim	knit	a bout'

12.

wedge	rem'e dy	dense	when
smell	wheth'er	meant	necks
shel'ter	cent	health	sweat
red'dish	deaf	next	depth

13.

cof'fin	con'cert	dodge	trot'ted	con vey'
for'est	shop	knock	hol'low	of fense'
bot'tle	knob	mock	pock'et	be yond'
gloss'y	notch	rock'et	fond	con verse'
bon'net	lodge	pop'py	bod'y	coun'tries

14.—BIRDS.

gull	grouse	swal'low	vul'ture	crane
swan	ra'ven	par'tridge	bob'o link	ea'gle
wren	par'rot	lark	night'in gale	cuck'oo

quall	os'trich	her'on	owl	spar'row
stork	pig'eon	o'ri ole	crow	con'dor
hawk	pea'cock	blue'jay	dove	lin'net

15.—WHAT BIRDS DO AND HAVE.

coo	car'ol	whis'tle	poise	tal'on
caw	hov'er	mi'grate	perch	wings
chirp	war'ble	soar	pin'ion	feath'ers
cheep	twit'ter	whir	beak	plu'mage

16.

crust	bunch	crutch	buzz	luck
ug'ly	jump	rub'ber	snug	mush
drum	brush	just	crumb	shrub
dumb	struck	dust	hut	pump

17.

live	space	moves	called	sur'face
ball	earth	through	breast	beau'ti ful
light	might	great	curled	won'der ful
gives	world	round	dressed	beau'ti ful ly

18.

po'ny	mus'tard	char'coal	tor'ment	hat'ter
med'al	par'ing	floun'der	scrib'ble	stick'y
cor'ner	mor'tar	hoe'ing	pres'ence	crip'ple
med'dle	pea'nuts	peel'ing	din'gy	grate'ful

PLATE OF PLATES

21.

lean	mid'dle	mis take'	rum'ple	south
lame	on'ly	naugh'ty	quar'rel	sor'ry
lamp	noise	ripe	pres'ent	sev'en
met'al	of'ten	or'der	soul	shad'ow
mer'ry	my self'	re fuse'	rust	shov'els

22.

taste	spo'ken	tum'ble	whale	worth
teach	thief	thought	up set'	wreck
stood	thirst	wake	up'per	whose
sprang	un less'	un til'	year	young
sto'ries	try'ing	waste	wool	bathe

23.

sure	talk'a tive	groan	shiv'er
quite	re la'tion	kite	whisk'ers
reach	mil'i ta ry	strength	gal'lon
speech	in ter rupt'	though	sal'low

KITCHEN.—24.—DINING ROOM.

tongs	sieve	urn	fork
ba'sin	stove	chi'na	spoon
pok'er	broom	plates	ta'ble
shov'el	ket'tle	sau'cer	cru'et
dip'per	scut'tle	gob'let	cast'ers
buck'et	dust'pan	tea'cup	nap'kin
coal'-hod	skim'mer	tu reen'	tum'bler
grid'i ron	sauce'pan	plat'ter	side'board

BEDROOM.—25.—PARLOR.

soap	sheet	vase	so'fa
bowl	lounge	car'pet	stool
tow'el	pil'low	mir'ror	scarf
bol'ster	bu'reau	cur'tain	mu'sic
pitch'er	blank'et	has'sock	pi an'o
scis'sors	mat'tress	book'case	por'trait
nee'dles	bed'stead	arm'chair	cab'i net
thim'ble	cov'er let	ot'to man	cush'ions

26.

balk	al'mond	tomb	dumb'ly
calk	salm'on	jamb	plumb'er
balm	be calm'	plumb	numb'ness
alms	em balm'	doubt	climb'er
salve	balm'y	debt'or	comb'ing
halves	psalm	crumbed	doubt'ful

27.

friend	like	sha'dy	sev'er al
glad	brook	cro quet'	ber'ries
birds	woods	ten'nis	ev'er y
fish	re ceive'	gained	moun'tains

28.

seal	er'mine	floe	moss'es
whale	rein'deer	Lapp	li'chens
sa'ble	po'lar bear	au ro'ra	red'snow
wal'rus	ei'der duck	snow hut	ice'bergs

29.—TREES.

pine	ash	al'der	birch
larch	elm	ma'ple	lo'cust
hol'ly	oak	eb'o ny	lin'den
ce'dar	beech	wil'low	wal'nut
spruce	ol'ive	hick'o ry	rose'wood
cy'press	pop'lar	chest'nut	pal met'to

30.—TOOLS.

file	gouge	lev'er	chis'el
adz	square	au'ger	lev'el
rake	lathe	gim'let	trow'el
spade	wrench	mal'let	hatch'et
plane	shears	pin'cers	cork'screw

31.

juice	loaf	liq'uid	sev'er al
cane	re fine'	hogs'head	plan ta'tion
mill	joints	mo las'ses	blos'som
scum	leaves	boiled	stom'ach

82.

know	va'por	er'rand	ques'tion
bruise	bal loon'	nei'ther	scare'crow
sneeze	no'tice	thir'teen	of'fi cer
rinse	e lev'en	be cause'	an oth'er
can'dy	e nough'	fair'y	nec'es sa ry

83.

On your way home you may see—

men	roads	gates	wom'en
stores	streets	parks	av'e nues
hous'es	gar'dens	hors'es	lamp'-posts
church'es	chil'dren	sta'bles	car'riag es

84.—CLOTHING.

hose	mack'in tosh	veil	muff
gloves	gai'ters	scarf	boots
cra vat'	col'lar	hood	tip'pet
skirt	trou'sers	ruf'fle	par'a sol
shawl	hand'ker chief	neck'tie	o'ver alls

85.—WORDS OF SIMILAR MEANING.

arch	curve	hank	skein
bluff	cliff	neat	ti'dy
bench	set tee'	quaint	strange
blithe	joy'ous	queer	droll
fleet	nim'ble	twain	coup'le
gruff	glum	tenth	tithe
gang	crew	van	front
giv'er	do'nor	wont	hab'it
pris'on	dun'geon	peo'ple	per'sons
late	tar'dy	soul	spir'it
snug	co'zy	rob	plun'der
rash	ha'sty	rich	fer'tile
grim	sur'ly	e rect'	up'right
hurt	in'jure	loose	un bound'
sole	sin'gle	com'fort	con sole'

36.—REVIEW.

beech	squash	sen'tence	pal'ace
sieve	fields	car'riage	lil'y
those	knife	pic'ture	dai'sy
wren	through	ea'gle	bu'reau
strait	tongs	chest'nut	tu reen'
pear	peach	on'ion	Eu'rope
sleeve	pen'cil	ba'sin	po'ny
fruit	tongue	re fuse'	os'trich
birch	Wednes'day	mi'grate	mo'ment

37.—REVIEW.

rob'in	let'tuce	meant	scis'sors
pig'eon	rai'sin	wrote	pump'kin
wal'nut	cur'rant	please	thim'ble
pop'lar	for'ward	ev'er y	pitch'er
for'est	ber'ries	qui'et	ba na'na
gob'let	rhu'barb	ache	a'pri cot
mat'tress	spin'ach	Christ'mas	beau'ti ful
cru'et	Eng'land	pil'low	Feb'ru a ry
min'ute	chis'el	mead'ow	po ta'toes

38.—IN THE COUNTRY.

farm'house	ditch	sheep	swing
vines	knoll	lambs	in'sects
gar'den	ar'bor	cat'tle	or'chard
flow'ers	hay-loft	fields	ber'ries
harp	mead'ow	wheat	but ter flies

hemmed
sleeves
learned

gath'ered
a'pron
eas'i ly

stitched
emp'ty
rap'id ly

seam'stress
rode
per'fect ly

41.

palms
cof'fee
cot'ton
sa'go
spi'ces

ca ca'o
cam'phor
in'di go
In'di a-rub'ber
sug'ar-cane

li'on
jag u ar'
gi raffe'
cam'el
mon'key

el'e phant
ser'pent
go ril'la
croc'o dile
leop'ard

42.

We should be—

good
frank
prompt
hon'est
no'ble
po'lite

cor'dial
sin cere'
lov'ing
truth'ful
care'ful
hope'ful

help'ful
thought'ful
ge'ni al
stu'di ous
pa'tient
court'e ous

civ'il
o blig'ing
gen'er ous
o be'di ent
tem'per ate
in dus'tri ous

43.

We should not be—

mean
curt
proud
la'zy
sulk'y
sau'cy

stin'gy
cru'el
self'ish
un kind'
fret'ful
sul'len

rude
sur'ly
care'less
haugh'ty
de ceit'ful
tat'tling

im po lite'
dis hon'est
cow'ard ly
quar'rel some
dis hon'or a ble
dis o be'di ent

44.—ABBREVIATIONS.

Jan'u a ry
Feb'ru a ry
March
A'pril
May
June

Jan.
Feb.
Mar.
Apr.
May
June

Ju ly'
Au'gust
Sep tem'ber
Oc to'ber
No vem'ber
De cem'ber

July
Aug.
Sept.
Oct.
Nov.
Dec.

WINDS.—45.—BOATS.

gale	cy'clone	barge	ca noe'
gust	si moom'	yawl	cut'ter
breeze	ty phoon'	sloop	schoon'er
squall	tor na'do	yacht	ves'sel
zeph'yr	whirl'wind	do'ry	frig'ate
tem pest	hur'ri cane	gon'do la	steam'er

46.

goat	ot'ter	oats	flax
deer	pan'ther	corn	hemp
wolf	squir'rel	maize	to bac'co
moose	buf'fa lo	bar'ley	tim'o thy
rab'bit	an'te lope	ce're al	mul'ber ry

47.

stealth	vague	whiff	ooze	wrought
trough	voice	trait	cruise	squeeze
thrift	vogue	grate	browse	grudge
twilled	vault	fierce	shrimp	sword

48.—FRUITS.

fig	peach	cher'ry	cran'ber ry
prune	grape	cit'ron	blue'ber ry
date	cur'rant	dam'son	straw'ber ry
plum	lem'on	pine'ap ple	rasp'ber ry
quince	mel'on	a'pri cot	goose'ber ry
pear	rai'sin	ba na'na	huck'le ber ry

49.—FLOWERS.

tu'lips	lark'spurs	pan'sies	sun'flow ers
as'ters	lil'ics	sweet peas	hol'ly hocks
li'lacs	dai'sies	hy'a cinths	vi'o lets
clo'vers	mar'i golds	dan'de li ons	ge ra'ni ums

50.

fell	cause	stum'bled	troub'le
lack	killed	horse'man	neg'li gence

steed
foe

caught
slight

in'ju ry
loos'ened

un fast'ened
at ten'tion

51.

co here'
se crete'
im pede'
con vene'
ex treme'
su preme'

e'qual
se vere'
de'cent
fe'male
pre'cept
de scribe'

need'y
speed'y
feed'ing
heed'ful
free'dom
cheer'ful

wear'y
drear'y
trea'son
cheap'ly
year'ling
mean'ing

52.

toil
moat
dirt
frown
filth
gape

gov'ern
heat
jest
joke
stray
pile

pawn
pledge
quoth
ruse
scheme
la'bor

scrub
scour
di rect'
sell
seize
scowl

trick
trench
yawn
wan'der
be fore'
col'umn

GEOGRAPHY.—53.—ARITHMETIC.

source
ax'is
or'bit
cir'cle
mo'tion
dai'ly
year'ly
trop'ic
pole

po'lar
cli'mate
sav'age
e qua'tor
ho ri'zon
ze'nith
par'al lel
hem'i sphere
ge og'ra phy

add
plus
sum
sub tract'
min'u end
sub'tra hend
re main'der
mul'ti ply
mul ti pli cand'

mul'ti pli er
prod'uct
di vide'
div'i dend
di vi'sor
quo'tient
proof
a rith'me tic
prob'lem

54.—USED IN COOKING.

rice
sage
mace
yeast
cloves

so'da
gin'ger
pars'ley
pep'per
nut'meg

all'spice
gel'a tine
choc'o late
sal e ra'tus
buck'wheat

va nil'la
vin'e gar
hom'i ny
tap i o'ca
cin'na mon

55.

Write the plurals of the words in this exercise:

skein	loaf	con'cert	man'sion
cit'y	dai'sy	sheaf	buf'fa lo
o'cean	tur'key	of'fi cer	jour'ney
wretch	suit'or	sur'face	os'trich
box	scratch	sand'wich	at tor'ney

56.

se rene'	rus'tic	waste'ful	si'lent
com'ic	tac'it	fear'ful	gloom'y
pu'trid	tur'bid	ru'ral	mud'dy
ea'ger	stur'dy	plac'id	flor'id
rud'dy	som'ber	ar'dent	rot'ten
lav'ish	tim'id	hard'y	mirth'ful

57.

gift	geese	gib'bous	gey'ser	guard
girt	ga'ble	gew'gaw	gal'lant	gid'dy
gild	gir'dle	tar'get	gaunt'let	gig'gle
gear	giz'zard	gal'lop	guest	guin'ea
gimp	gher'kin	gar'gle	gauze	gor'geous

58.—OPPOSITES.

right	wrong	dry	moist
find	lose	near	dis'tant
strong	weak	sweet	sour
gay	sad	quickly	slowly

bis'cuit
sal'ad
mut'ton

waf'fle
hon'ey
catch'up

muf'fin
por'ridge
chow'der

sauer'kraut
cus'tard
suc'co tash

60.

ag'ile
viv'id
doc'ile
hos'tile
mis'sile

rig'id
ac'tive
fes'tive
rep'tile
crev'ice

im'age
jus'tice
den'tist
ser'vice
prom'ise

fidg'et
res'pite
cor'nice
des'tine
doc'trine

61.

cowl
prowl
clown
crown
drown

crowd
bow'er
dow'er
cow'ard
drow'sy

rouse
shout
crouch
mound
flounce

de vour'
foun'dry
scoun'drel
com'pound
pro nounce'

62.

cleft
pyre
brief
copse
corpse
clench
cleanse

flare
dealt
freak
fraud
fledge
chasm
dredge

flaw
goad
farce
guile
gloat
gorge
gourd

liege
lapse
weird
grease
mold
mourn
league

rouge
rogue
shone
prism
pierce
plague
scourge

63.—OCCUPATIONS.

tai'lor
ba'ker
doc'tor
law'yer
cob'bler

flo'rist
join'er
gro'cer
bank'er
mer'chant

farm'er
weav'er
build'er
butch'er
drug'gist

mil'li ner
min'is ter
gar'den er
car'pen ter
black'smith

64.—OUTDOOR SPORTS.

fish'ing
sail'ing

ri'ding
ten'nis

dri'ving
play'ing

ska'ting
mar'bles

row'ing	leap'ing	bowl'ing	coast'ing
boat'ing	croquet'	jump'ing	base'ball
ba'thing	nut'ting	swing'ing	sleigh'ing

65.

so'lar	nec'tar	dro'ver	dif'fer	lar'der
lu'nar	lat'ter	can'ker	hin'der	blis'ter
tar'tar	an'ger	la'ter	gan'der	blub'ber
stel'lar	bet'ter	cof'fer	fil'ter	mem'ber

66.

loi'ter	coil	an noy'	in'voice
toi'let	broil	de coy'	re joice'
poi'son	spoil	boy'ish	pur loin'
coin'age	hoist	em ploy'	oint'ment
ap point'	choice	boy'cott	em broid'er

67.

cen'ser	cir'cus	ac'id	ceil'ing
cen'sus	ci'pher	cir'cuit	cym'bal
cen'tral	cin'der	cyl'in der	cen'tu ry
ce ment'	cer'tain	cel'e brate	ce les'tial
cen'taur	cen'sure	cem'e ter y	cen ten'ni al

68.—HOMONYMS.

1 { cent, a coin scent, an odor sent, did send	5 { to, as in "Give it to me" too, as in "too cold" two, a number
2 { plum, a fruit plumb, perpendicular	6 { threw, did throw through, as in "through the air"
3 { stare, to look earnestly stair, a step	7 { fir, a tree fur, fine, soft hair
4 { fore, in front four, a number	8 { earn, to get or merit by labor urn, a vase

Write the following sentences, putting the right word in the right place. Underline the words inserted.

It is not what we (8), but what we save that makes us rich.
—*Prov.* A bird in the hand is worth (5) in the bush. —*Prov.*

We (1) the silver (8) (5) them. I remember, the (7) trees, dark and high.—*Hood*. A (1) is one-hundredth part of a dollar. Prunes are dried (2)s. A dog has (4) feet, but the (4) feet are the two front feet. And all the world would (3).—*Cowper*. The (1) of the roses will hang round it still.—*Moore*. Who (6) the stone (6) the window? Russian sable is a costly (7). (5) many cooks spoil the broth. Masons test a wall with a (2)-line. White marble (3)s lead to the Capitol.

69.

nev'er	neth'er	fel'on	men'tal
tem'per	skep'tic	zeal'ot	plen'ty
per'ish	cher'ub	stead'y	pet'rel
cher'ish	her'ald	peas'ant	jeal'ous
wed'lock	blem'ish	threat'en	weath'er
ped'dler	thread'bare	pleas'ure	meas'ure

70.

com'pass	cac'tus	cha'os	chord
cul'prit	com'rade	chrome	chyle
cur'ry	cab'in	chro'mo	chyme
cur'few	ca'ble	chron'ic	chol'er a
com'et	com pute'	cho'ral	chron'i cle
com plete'	cu'pola	chem'ist	char'ac ter
col'umn	com plex'ion	Chris'tian	cha me'le on

71.—ARITHMETIC.

rate	cu'bic	in'te ger	a'mount
terms	fac'tor	dec'i mal	prin'ci pal
prime	frac'tion	mul'ti ple	di vis'i ble
dig'it	al'i quot	in'ter est	in sur'ance
ze'ro	dis'count	com pos'ite	bro'ker age
a'cre	ex am'ple	nu'mer ator	per cent'age
naught	hun'dredth	de nom'i na tor	av oir du pois'

72.—ANIMAL SOUNDS.

purr	yelp	bleat	snort	squeak
hum	howl	cluck	cack'le	roar

low	quack	neigh	whin'ny	scream
grunt	growl	croak	bel'low	buzz
squeal	mew	gob'ble	chir'rup	screech

73.—INSECTS.

bee	gnat	ant	drag'on-fly
wasp	moth	wee'vil	bum'ble bee
flea	roach	mos qui'to	but'ter fly
lo'cust	bee'tle	glow'worm	ka'ty did
hor'net	crick'et	silk'worm	grass'hop per

74.

a'li as	fa'cial	ef face'	fa'tal
a'gen cy	pa'tron	va'cant	ha'zy
ma'ni ac	sta'tion	en gage'	ba'bel
brace'let	an'cient	be came'	az'ure
fa'vor ite	pa rade'	pro fane'	ha'tred
va'por ize	dra'per y	dis place	man'ger

75.

gym'nast	gib'lets	hom'age	gen'u ine
gyp'sy	gen'ius	gib'bet	mag'is trate
en'gine	gen teel'	gen'e sis	gym nas'tics
mar'gin	herb'age	gest'ure	gym na'si um

76.—REVIEW.

ache	bruise	bu'reau	could
ac cept'	Arc'tic	care'ful	dai'sy
a fraid'	bis'cuit	car'riage	cur'tain
a gain'	au'tumn	cel'er y	col'ored
al'mond	ba na'na	chim'ney	con'ti nent
an'i mal	break'fast	Christ'mas	cran'ber ry

77.

dif'fer ent	friend	health'y	juice
dough'nut	gi raffe'	help'ful	knife
ear'nest	gru'el	hom'i ny	knuck'le

ei'ther
el'e phant
e nough'

e qua'tor
er'rand
Feb'ru a ry

ho ri'zon
hy'a cinth
in'ter est ing

laugh
learn
isth'mus

78.—REVIEW.

maize
li'chen
liq'uid
mat'tress
mi'grate
min'u end

mo las'ses
mos qui'to
moun'tain
nei'ther
neph'ew
niece

o'a sis
o blige'
om'e let
os'trich
oys'ter
par'al lel

rai'sin
pic'ture
pitch'er
pleas'ant
prai'rie
pump'kin

79.

rhu'barb
rough
salm'on
schol'ar
scis'sors
sen'tence

skein
sleigh
sev'er al
skel'e ton
spin'ach
squir'rel

tongue
tor'rid
un til'
tem'per ate
thous'and
vol ca'no

zone
whose
would
wrong
ze'nith
writ'ten

80.—SYNONYMS.

ef'fort
re past'
in close'
de cide'
re vere'
spring'y

en deav'or
col la'tion
en vel'op
de ter'mine
ven'cr ate
e las'tic

lack'ing
out'ward
down'cast
a tone'
con fuse'
de ride'

de fi'cient
ex ter'nal
de ject'ed
ex'pi ate
be wil'der
rid'i cule

81.

fool
lock
feast
hook

priest
del'ta
an'gel
se'cret

dra'ma
fel'low
mask
fos'sil

han'dle
hel'met
freck'le
liq'uor

lad'der
lob'ster
mar'ket
ma'tron

82.

rul'er
sto'ry

an'cient
ver'dant

tow'er ing
ex ten'sive

for'tress es
at tract'ive

roams	fruit'ful	sep'ul cher	ap pa ri'tion
stream	slum'bers	be stow'ing	pic tur esque'
stat'ed	en tombed'	re main'der	ben e dic'tion

83.—MASCULINE AND FEMININE.

he'ro	her'o ine	beau	belle
host	host'ess	wiz'ard	witch
act'or	ac'tress	sir	mad'am
god	god'dess	bach'e lor	maid, spin'ster
heir	heir'ess	wid'ow er	wid'ow
jan'i tor	jan i tress	man serv'ant	maid serv'ant
proph'et	proph'et ess	land'lord	land'la dy

84.—HOMONYMS.

1 { break, to part by force brake, for stopping wheels; a fern	5 { hail, frozen rain; to salute hale, healthy
2 { week, seven days weak, feeble	6 { wait, to stay weight, heaviness
3 { waist, part of the body waste, a desert; to squan- der	7 { heel, part of the foot heal, cure
4 { piece, a part; a composition peace, quiet	8 { peal, a loud sound peel, to strip off the skin

In the following sentences put the right word in the right place. Underline the words inserted.

Achilles was slain by being wounded in the (7). A (4) of banana (8) should not be thrown on the pavement. The (1)ing waves dashed high. *Hemans* (5), holy light — *Milton*. The engine whistled "Down (1)s." What is your (6)? I lay me down in (4) to sleep. *Willard* Physician, (7) thyself.—*Bible*. If you are (5), you cannot be (2). The deep thunder, (8) on (8), afar. *Byron*. There was a belt about her (3). Sunday is the first day of the (2). Learn to labor and to (6).—*Longfellow*. (3) not, want not.—*Prov*.

85.

jui'cy	tip'sy	po'sy	ma'zy
spi'cy	fuss'y	pal'sy	diz'zy

fan'cy
flee'cy
mer'cy

moss'y
mass'y
drop'sy

pro'sy
flim'sy
glass'y

cra'zy
breez'y
driz'zly

86.—SYNONYMS.

a base'
ab hor'
a bide'
ac quit'
ac cede'
a ban'don

de grade'
de test'
so'journ
ab solve'
com ply'
for sake'

ad'age
ac cost'
a dieu'
a dorn'
ad vice'
ac quaint'

max'im
sa lute'
good-by'
dec'o rate
coun'sel
in form'

87.—OPPOSITES.

ab'sent
be stow'
de stroy'
free
dis perse'
e merge'

pres'ent
re ceive'
con struct'
cap'tive
as sem'ble
im merge'

guilt'y
stub'born
in te'ri or
fail'ure
de crease'
em'i grate

in'no cent
yield'ing
ex te'ri or
suc cess'
aug ment'
im'mi grate

88.

sa li'va
en vi'ron
en ti'tle
vi'o late
ri'val ry
pi'e ty

i'ci cle
pli'a ble
di'a ry
li'a ble
si'phon
bi'ped

con fide'
com bine'
re cite'
sur mise'
re quire'
com prise'

pi'rate
bri'ny
mi'nus
li'bel
fi'nal
vi'per

89.

braid
beard
bloom
bleach

ex am'ine
ex cur'sion
de light'ful
in struc'tion

coax
skull
hoarse
a piece'

ra'zor
ven'ture
whith'er
smoth'er

90.—MINING.

zinc
lead

mi'ca
sul'phur

gran'ite
gyp'sum

lode
shaft

quartz
cop'per
ni'ter

car'bon
salt pe'ter
mer'cu ry

plat'i num
min'er al
me tal'lic

min'er
tun'nel
der'rick

91.—FARMING.

plow
reap'er
mow'er
scythe
sick'le

har'row
thresh'er
fal'low
swamp'y
fruit'ful

ster'ile
clay'ey
bar'ren
loam'y
marsh'y

fer'tile
bog'gy
ar'a ble
al lu'vi al
gua'no

92.

mu'ti ny
du'ti ful
pu'ri fy
cru'el ty
mu'tu al
lu'di crous

flu'id
mu'sic
glu'ten
tu'mor
fu'ture
cu'ri ous

dis pute'
pro cure'
in sure'
di lute'
en dure'
pre sume'

se clude'
de lude'
as sure'
a buse'
a muse'
al lure'

93.

doub'le
har'ness
mat'ting
val'u a ble

jew'el er
tomor'row
which ev'er
scen'ery

su pe'ri or
va ca'tion
twi'light
moun'tains

sleet
tacks
sphere
ce'dar

94.

blouse

cis'tern

knuckle

steppes

nymph	de sist'	a're a	cask'et
crea'ture	ad vance'	dis arm'	cit i zen
con sume'	di gest'	cha grin'	duc'tile

96.—HOMONYMS.

1 {	air, what we breathe e'er, ever ere, before heir, one who inherits	5 {	rain, water from clouds reign, rule rein, for a horse
2 {	sail, of a ship sale, a selling	6 {	coarse, rough course, way
3 {	ho'ly, sacred whol'ly, completely	7 {	col'lar, band for the neck chol'er, anger
4 {	plain, level ground; clear plane, flat surface; tree; tool	8 {	dy'ing, ceasing to live dye'ing, coloring

Write the following sentences, putting the right word in the right place:

Westward the (6) of empire takes its way.—*Berkeley*. O, there is sweetness in the morning (1).—*Byron*. (3) angels guard thy bed — *Watts*. How gladly would we buy time were it for (2). How beautiful is the (5) after the dust and heat.—*Longfellow*. The top of my desk is a (4) surface. What! drunk with (7).—*Shakespeare*. The (7) was made of cloth. We thought her (8) when she slept. The Prince of Wales is (1) to the English throne. Russia is almost (3) a vast (4). Write home (1) the ship (2)s. The (5) guides the horse. The cochineal insects furnish a red color for (8). Shakespeare lived during the (5) of Queen Elizabeth.

97.—CLOTH.

jean	sat'in	si le'sia	cal'i co
baize	vel'vet	me ri'no	al pac'a
serge	flan'nel	cam'bric	dam'ask
plush	mus'lin	bro cade'	cor'du roy
lin'en	mo'hair	chev'i ot	cas'si mere
tweed	de laine'	ging'ham	vel vet een'
chintz	broad'cloth	cash'mere	seer'suck er

98.

monk	piv'ot	pub'lic	tas'sel
mur'mur	part'ner	rum'mage	trel'lis
mys'ter y	phys'ic	skir'mish	tus'sle
par'a ble	pi'ous	sol'emn	weap'on
par'cel	pi'ra cy	spe'cie	wel'fare
mes'sage	pit'i ful	stam'mer	syr'inge

99.

tu'bu lar	bear'er	vic'tor	ju'ror
tab'u lar	lodg'er	val'or	fla'vor
pop'u lar	cor'o ner	tu'tor	ru'mor
cir'cu lar	mourn'er	tre'mor	or'a tor
cal'en dar	strag'gler	trai'tor	stu'por
sec'u lar	vend'er	tor'por	splen'dor
mus'cu lar	in tru'der	suit'or	sur vey'or

100.—ON THE WRITING DESK.

ream	ru'ler	let ter	pa'per-weight
quill	tab'let	e ra'ser	port fo'li o
quire	blot'ter	ink'stand	mu'ci lage
stamps	wa'fer	fools'cap	en'vel ope
pen'knife	cal'en dar	dic'tion a ry	seal'ing-wax

101. FISH.

eel	trout	dol'phin	her'ring
cod	pike	sar'dine	mack'er el
carp	shark	had'dock	pick'er el
perch	shad	suck'er	stur'geon
bass	smelt	min'now	hal'i but

102.

un just'	un just'ly	re al	re'al ly
lan'guid	lan'guid ly	meek	meek'ly
se cure'	se cure'ly	court	court'ly
se'ri ous	se'ri ous ly	an'nu al	an'nu al ly
spite'ful	spite'ful ly	in tent'	in tent'ly
un u'su al	un u'su al ly	for'mer	for'mer ly
dread'ful	dread'ful ly	fre'quent	fre'quent ly

103.

grief	griev'ous	pore	po'rous
joy	joy'ous	ri'ot	ri'ot ous
vice	vi'cious	glo'ry	glo'ri ous
stud'y	stu'di ous	dan'ger	dan'ger ous
la'bor	la bo'ri ous	mur'der	mur'der ous
in'dus try	in dus'tri ous	ma la'ri a	ma la'ri ous

104.—HOMONYMS.

1 { pair, two pare, to cut off pear, a fruit	5 { meat, animal food meet, proper; to come to- gether mete, to measure
2 { their, belonging to them there, in that place	6 { as cent', a rising as sent', agreement
3 { cap'i tal, chief town; stock cap'i tol, building	7 { pane, a plate of glass pain, an ache
4 { hear, to listen here, in this place	8 { rye, a grain wry, crooked

Write the following sentences, putting the right word in the right place:

(2) is nothing new under the sun. *Bible* (8) grows in cold countries. When shall we three (5) again?—*Shakespeare*. A (7) of glass. (4) rests his head upon the lap of earth. *Gray*. Can you (1) a (1) with a (1) of scissors? The (3) is a white marble building. The (6) of Mont Blanc is full of danger. Be silent that you may (4).—*Shakespeare*. Sweet is pleasure after (7).—*Dryden*. Birds in (2) little nests agree.—*Watts*. Washington is the (3) of the United States. Is not the life more than (5)?—*Bible*. He gave his (6) to the proposal. There is a bird called the (8) neck. With what measure ye (5), it shall be measured to you again.—*Bible*.

105.—KNOWN BY

<i>Seeing.</i>	<i>Smelling.</i>	<i>Tasting.</i>
squal'id	ran'cid	lus'cious
un couth'	fra'grant	pun'gent

col'ored
o paque'
ra'di ant

per'fumed
o'dor ous
ar o mat'ic

bit'ter
in sip'id
de li'cious

*Touching.**Hearing.*

warm
rough
sleek

smooth
tep'id
un e'ven

clear
loud
in dis tinct'

ech'o ing
muf'fled
noi'sy

106.

height
re sult'
con fess'
wrig'gle
fir'kin
anx i'e ty

count'er
awk'ward
bus'i ly
re mark'a ble
gen'tle man
u'su al ly

la'zi ly
hes' i tate
spe'cial
mus'cles
vil'lage
pro tect'

ach'ing
sur prise'
quan'ti ty
syl'la ble
scold'ed
gos'sa mer

107.—REVIEW.

e ra'ser
bar'y tone
or'ches tra
cam paign'
glac'i er
moun'tain

dic'tion a ry
mack'er el
jew'el ry
veg'e ta ble
cas'si mere
sar sa pa ril'la

scythe
feign
zeph'yr
liq'uor
giz'zard
gey'ser

com'mon
tac'it
gyp'sy
ges'ture
salm'on
wool'en

108.

al pac'a
sal'a ry
boy'cott
ox'y gen
em'er y
tur quoise'

par'lia ment
cyl'in der
cem'e ter y
am'e thyst
sep'a rate
hun'dredth

thief
toi'let
au'tumn
ac'id
pum'ice
ber'yl

dai'sy
doc'ile
rep'tile
im'age
cor'nice
ci'pher

109.—REVIEW.

skep'tic
squal'id
squir'rel

sol'emn
ver'ti cal
de li'cious

weird
myrrh
which

can'cel
con cede'
pleas'ure

schol'ar	syl'la ble	straight	ped'dler
myr'i ad	ker'o sene	weap'on	vil'lage
wiz'ard	med'i cine	syr'inge	griz'zly

110.

i'ci cle	ver mil'ion	a dieu'	tus'sle
rum'mage	Ni ag'a ra	sep'a rate	cer'tain
lunch'eon	nec'es sa ry	twelfth	cha grin'
sur'name	pen i ten'tia ry	be lieve'	cot'tage
car'a mel	re ceived'	the'a ter	sup pose'
span'iel	ex hi bi'tion	res er voir'	de vel'op

111.—HISTORY.

char'ter	squaw	scalped	wil'der ness
spear	pap poose'	ship'wreck	suf'fer ing
wig'wam	war'rior	ex plore'	set'tle ment
wam'pum	cal'u met	col'o ny	trea'ty
moc'ca sin	dis cov'er y	per'ma nent	per'se cute
tom'a hawk	pi o neers'	nav'i ga tor	mas'sa cre

112.—HOMONYMS.

1 {	ewe, a female sheep	5 {	knead, to work dough
	you, person addressed		need, want
	yew, an evergreen tree	6 {	stake, a post; a wager
2 {	cru'el, unkind		steak, a slice of meat
	crew'el, soft yarn	7 {	gait, manner of walking
3 {	choir, a band of singers		gate, a kind of door
	quire, 24 sheets of paper	8 {	main, chief
4 {	bough, a branch		mane, hair on horse's neck
	bow, to bend; front of ship		Maine, one of the U. S.

Write the following sentences, putting the right word in the right place:

I had most (5) of blessing. — *Shakespeare*. May I join the (3) invisible? — *George Eliot*. (1)s and bleating lambs. — *Milton*. Added woes may (4) me to the ground. — *Pope*. (2)s are used in embroidery. (1) must (5) the dough to make good bread. The man who walked through the (7) had a peculiar (7). Have a care for the (8) chance. — *Butler*. Twenty (3)s make a

ream. He bought a slice of sirloin (6). Me. is the abbreviation of (8). The Mistletoe (4) is a poem. Like a dewdrop from the lion's (8). Joan of Arc was burned at the (6). The wood of the (1) was used for making bows.

113.

cu'ti cle	lyr'ic al	cler'ic al	pay'a ble
mir'a cle	spher'ic al	crit'ic al	bla'ma ble
ob'sta cle	sur'gi cal	cyn'ic al	ta'ma ble
par'ti cle	trag'ic al	whim'si cal	teach'a ble
spec'ta cle	op'tic al	prac'ti cal	ten'a ble
'trea'cle	clas'sic al	phys'i cal	ca'pa ble
man'a cle	com'ic al	med'ic al	af'fa ble

114.

sing'er	preach'er	deaf'ness	soft'ness
wait'er	la'bor er	firm'ness	sweet'ness
start'er	lect'ur er	swift'ness	round'ness
catch'er	ex am'in er	fierce'ness	prompt'ness
think'er	com mand'er	hard'ness	wretch'ed ness

115.

ac crue'	a thwart'	con'quer	ath'lete
a chieve'	vict'uals	a'lien	fau'cet
a ghastr'	cro chet'	al'oes	cal'lous
greas'y	bou quet'	anx'ious	bur'glar
ca det'	bru nette'	brig'and	dul'cet
con geal'	car toon'	drug'get	daunt'less

Write the following sentences, putting the right word in the right place:

I see the (7), and I approve it too.—*Ovid*. It is a long (4) that has no turning.—*Prov*. We all do fade as a (1).—*Bible*. Do what you (6), come what may. Baptism is a religious (7). The (2) fishes eat up the little ones.—*Shakespeare*. I had as (1) sing (5). A ship (7) works in a shipyard. Have you (3) (6) against him? It is a (5) thing to betray a man who has trusted you.—*Prov*. A bright fire glowed in the (2). The lowing (3) winds slowly o'er the lea.—*Gray*. I never dare to (7) as funny as I can.—*Holmes*. The book has (4) on my table several days.

117.

ab'sence	fuzz	go'pher	la pel'
at'om	cleat	griz'zly	loi'ter
bar'gain	clev'er	haz'ard	lag'gard
but'ton	eclipse'	hu'mor	loz'enge
cat'kin	en cir'cle	jock'ey	ma chine'
glut'ton	fa tigue'	ker'o sene	med'i cine

118.

change	change'a ble	man'age	man'age a ble
charge	charge'a ble	mar'riage	mar'riage a ble
trace	trace'a ble	cour'age	cou ra'geous
no'tice	no'tice a ble	out'rage	out ra'geous
ser'vice	ser'vice a ble	ad vant'age	ad van ta'geous

119.

ey'ry	la'va	ma rine'	dis dain'
cyc'let	leis'ure	mar'tyr	glob'ule
frag'ile	jour'nal	naph'tha	hos'tage
frag'ment	lan'guage	nui'sance	cres'cent
ghast'ly	laun'dry	nos'trils	tor'toise
gor'geous	mal treat'	cha rade'	tor'ture

120.—HOMONYMS.

1 { rap, to knock wrap, to cover	4 { strait, narrow straight, direct
2 { peer, an equal; a nobleman pier, pillar of a bridge	5 { vain, useless; proud vein, a blood vessel vane, a weathercock
3 { colo'nel, an army officer ker'nel, central part of a nut	6 { cel'lar, an underground room sell'er, one who sells

Write the following sentences, putting the right word in the right place:

(5) is the help of man.—*Bible*. In song he never had his (2).—*Dryden*. Strive to enter in at the (4) gate.—*Bible*. He cut a (5) in his hand. (1) on the door. The (2)s were of granite. A (3) commands a regiment. The (6) of wines keeps his stock in a (6). A (5) was on the church steeple. (1) up warmly, for the road is bleak. The (3) of the nut serves the squirrel for food.

121.—CHRISTMAS DINNER.

soup	tur'key	on'ions	ice-cream'
rolls	squash	sauce	cake
pick'les	po ta'toes	mince pie	fruit
cel'er y	gra'vy	plum pud'ding	con'serve

123.

chas tise'	bap tize'	a pol'o gize	or'gan ize
crit'i cise	cap size'	har'mo nize	mag'net ize
cat'e chise	re'al ize	gal'va nize	sym'pa thize
ad'ver tise	i'dol ize	fer'til ize	sol'em nize
ex'er cise	civ'i lize	col'o nize	rec'og nize
mer'chan dise	cen'tral ize	tan'ta lize	pat'ron ize
en'ter prise	le'gal ize	dram'a tize	mem'o rize

124.—HOMONYMS.

Write the following sentences, using the right word:

Better alone than in (bad, bade) company. Don't give (to, two, too) much for the whistle. Everything comes in (thyme, time) to (hymn, him) who can (wait, weight). He who follows (two, too, to) (hares, hairs) is sure (two, too, to) catch neither. It never (reins, rains, reigns) but it (pores, pours). Men speak of the (fair, fare) as things went with them (there, their). Out of (site, sight, cite), out of (mind, mined). Time and (tied, tide) (wait, weight) for (know, no) man. Where (there's theirs) a will there's always a (weigh, way). (Faint, feint) (hart, heart) ne'er (won, one) (fare, fair) lady.

125.

val'leys	pur'chased	sal'a ry	ce'dar
cur'rent	a cross'	spruce	scen'er y
cap'i tal	glac'i ers	in hab'it ants	val'u a ble
cli'mate	va ri'e ty	re sem'ble	de spair'

126.

de clare'	can'cel	ap pease'	con sent'	af fair'
col lect'	a ver'	a bol'ish	re voke'	a gree'
al lay'	a mass'	as sert'	dis cuss'	an nul'
ap pall'	as suage'	con cede'	af firm'	ar'gue
at tach'	af fright'	zeal'ous	al low'	anx'ious
pac'i fy	ap pend'	con cern'	ar'dent	an'cient

127.

sluice	thyme	whoop	suite	spouse
shrift	type	wreathe	realm	manse
scribe	theme	writhe	myrrh	phrase

128.

nerve	un nerve'	ech'o	re ech'o
spell	mis spell'	e lect'	re e lect'
spend	mis spend'	le'gal	il le'gal
step	mis step'	sev'er	dis sev'er
en gage'	re en gage'	mor'tal	im mor'tal
sat'is fy	dis sat'is fy	sim'i lar	dis sim'i lar

129.—HOMONYMS.

Write the following sentences, selecting the right word:

The past is not (holy, wholly) (vane, vain, vein), if rising on its (wrecks, recks) to something nobler (we, wee) attain.—*Longfellow*. In the morning (sow, sew) thy (seed, cede).—*Bible*. Then he said, "Good (knight, night)," and with muffled (o'er, oar, ore), silently (road, rowed, rode) to the Charlestown shore.—*Longfellow*. The heaviest (dews, dues) fall on clear (nights, knights). The (pale, pail) light of the moon is the reflection of the (son's, sun's) light. "(There, Their) graves are green; they may be (scene, seen)," the little (made, maid) replied.—*Wordsworth*.

130.

leg'a cy	ur'gen cy	in'ti ma cy	em'bas sy
in'fan cy	cur'ren cy	ex'i gen cy	jeal'ous y
de'cen cy	con'stan cy	cel'i ba cy	min'stel sy
se'cre cy	fal'la cy	con spir'a cy	lep'ro sy
re'gen cy	clem'en cy	ac'cu ra cy	her'e sy
flu'en cy	bril'lian cy	e mer'gen cy	ec'sta sy
va'can cy	buoy'an cy	com'pe ten cy	hy poc'ri sy

131.

a bun'dance	tem'per ance	au'di ence	ig'no rant
venge'ance	ac quaint'ance	dil'i gence	as sist'ant

ig'no rance	an noy'ance	ab'sti nence	de pend'ent
sus'te nance	ve'he mence	rev'er ence	el'e gant
vig'i lance	el'o quence	dif'fi dence	dil'i gent

132.—DISEASES.

gout	ca tarrh'	jaun'dice	hem'or rhage
croup	asth'ma	quin'sy	rheu'ma tism
a'gue	mea'sles	scur'vy	neu ral'gi a
mumps	ul'cer	ty'phoid	pneu mo'ni a
fe'ver	ab'scess	scrof'u la	dys pep'si a
grippe	in flu en'za	nau se'a	pa ral'y sis
can'cer	chil'blain	pleu'ri sy	diph the'ri a
col'ic	ver'ti go	de lir'i um	hy dro pho'bi a

133.—HISTORY.

writs	bul'let	ef'fi gy	in de pend'ent
re sist'	pow'der	al li'ance	dec la ra'tion
mobbed	ram'part	as sem'bly	ca lam'i ty
war'rant	car'tridge	pro hib'it	vol un teer'
un'ion	pa'tri ot	tax a'tion	e vac'u ate
suf'frage	re pealed'	tyr'an ny	rev o lu'tion
of fi'cial	mi li'tia	op pres'sion	u ni ver'sal
re doubt'	lib'er ty	gov'ern ment	rep re sent'

134.—ARMY WORDS.

sol'dier	pla toon'	sen'try	siege
com'pa ny	bat tal'ion	pick'et	sut'ler
reg'i ment	sword	u'ni form	re cruit'
bri gade'	sa'ber	knap'sack	hos'pi tal
di vi'sion	bay'o net	can teen'	gar'ri son
corps	pis'tol	ep'au let	coun'ter sign
in'fan try	car'bine	bag'gage	court-mar'tial
cav'al ry	mus'ket	cais'son	am mu ni'tion
ar til'ler y	ri'fle	biv'ou ac	for ti fi ca'tion
en gi neers'	can'non	fur'lough	in trench'ment

135.—NEWSPAPER TERMS.

dai'ly	ed'i tor	ed i to'ri al
morn'ing	jour'nal ist	lead'er

even'ing
~~semi-weekly~~
 weekly
 bi-monthly
 monthly
 exchang'es
 edition
 proprie'tor
 pub'lish'er

con'trib'u'tor
 re'port'er
 cor're'spond'ent
 sub'scrib'er
 ad'vertis'er
 car'ri'er
 news'boy
 ex'tra
 col'umn

ar'ti'cle
 i'tem
 lo'cal
 tel'e'grams
 no'tic'es
 gos'sip
 fi'nan'cial
 a'muse'ments
 mis'cel'la'ne'ous

136.—REVIEW.

war'rior
 sci'ent'ist
 skil'ful
 pro'ceeds
 con'sign'ee
 Mis'sis'sip'pi

cyn'ic'al
 cler'ic'al
 proph'e'cy
 proph'e'sy
 mer'ri'ment
 prom'is'so'ry

fuch'si'a
 a'za'le'a
 wretch'ed
 an'te'ced'ent
 par'ti'ci'ple
 de'clar'a'tive

phlox
 cap'tain
 trea'cle
 pre'cede'
 pa'tience
 de'cease'

137.

pal'i'sade'
 cav'a'lier'
 nau'ti'lus
 or'i'fice
 se'cre'cy
 pes'ti'lence

aux'il'i'a'ry
 lin'e'a'ment
 Cin'cin'na'ti
 rheu'ma'tism
 dys'pep'si'a
 pneu'mo'ni'a

mal'ice
 vi'cious
 ec'sta'sy
 jeal'ous'y
 pen'sion
 mis'spell'

scal'lop
 su'ture
 si'phon
 fron'tier'
 mus'cle
 e'qual'ly

138.—REVIEW.

league
 gui'tar'
 cor'nice
 fa'cade'
 ac'crue'
 pres'sure

bat'ter'y
 sal'a'ble
 tab'er'na'cle
 syn'a'gogue
 pre'cen'tor
 cer'e'mo'ny

im'ag'ine
 fric'as'see'
 lat'i'tude
 mul'lein
 gor'geous
 tor'toise

niche
 sparse
 clique
 na'dir
 eye'let
 mi'rage'

139.

coup'le
 fro'zen
 vict'uals

pau'ci'ty
 venge'ance
 griev'ance

nui'sance
 mar'riage
 wretch'ed

sues
 route
 choked

bou quet'
bru nette'
drug'gist

ni'tro gen
cap'il la ry
es pe'cial ly

ap'pe tite
ha rangue'
phy sique'

ra vine'
rou tine'
pat'ent

140.—REVIEW.

nau'se a
mi li'tia
ver'ti cal
cal'en dar
ex'er cise
con'science

dis ap pear'
dis'si pate
glyc'er ine
ben'e fit ed
nec'es sa ry
re bel'lion

sep'a rate
re ceived'
sur'geon
sol'dier
bag'gage
fur'lough

sphinx
can'cer
ca tarrh'
cap size'
bea'con
bod'ice

141.

rec'i pe
def'i nite
sac'ri lege
right'eous
gra'cious
en gi neer'
chal'ice
chiv'al ry

in sur'er
ep'au let
ma neu'ver
ar'chi tect
guar an tee'
chrys'a lis
con'fis cate
côn va les'cent

fag'ot
for'feit
fer'rule
for bade'
sur'plice
hic'cough
ton'sil
per spire'

dis cern'
ex haust'
crip'ple
cau'tious
symp'tom
com'ment
con'crete
poul'tice

142.—REVIEW.

cha rade'
po'rous
plov'er
scep'ter
lar'ynx
tra'che a

bil'liards
am'a teur
per se vere'
beau'te ous
prej'u dice
ac com'plice

al lege'
ax'iom
pur'ple
ter'race
spig'ot
suc ceed'

whey
vig'or
vic'ar
sol'der
bil'ious
su'mac

143.

pit'y
a byss'
sol'ace
er'ror
busi'ness
re scind'
af'ghan
bi'cy cle

con'duit
col'umn
brick'kiln
ban dan'na
pam'phlet
cat'er pil lar
con ge'ni al
chi rop'o dist

bre vier'
wrin'kle
tres'tle
ten'ant
shud'der
res er voir'
wor'shiped
chan de lier'

siege
o'gre
trip'le
sher'iff
pal'ace
pi az'za
qui'nine
fla'grant

144.

per suade'	per'fi dy	pur'ga to ry	purge
per form'	per se vere'	pur vey'or	pur'port
per jure	per pet'u al	per'pe trate	pur'pose
per plex'	per son'i fy	per'ti nent	pur suit'

145.—RAILROAD.

mail	de'pot	lug'gage	tel'e graph
train	sig'nal	dis patch'	con duct'or
track	tick'et	sta'tion	post'al clerk
check	bun'dle	ex'press	pe ri od ic als
freight	pack'age	brake'man	lo co mo'tive

146.

at tain'a ble	el i gi ble	con tempt'i ble	beau'te ous
pit'i a ble	suit'a ble	ter'ri ble	be gin'ning
a gree'a ble	al'ma nac	vis'i ble	be ha'vior
see're ta ry	boun'da ry	cred'i ble	bil'ious
spec'i men	as cer tain'	cer tifi cate	cu ri os'i ty
ex pi'ri ation	ex treme'ly	dif'fer ent	treach'er y
grat'i tude	sum moned	co lo'ni al	pes'ti lence
observ'ance	re ceiv'a ble	ex cus'a ble	cer'e mony
av'er age	al real'y	al pha bet	ac knowl'edge
ex pla na'tion	an'thra cite	ap pren'tice	res er voir'

147.—WORDS LIABLE TO BE CONFOUNDED.

sculp'tor, <i>a carrier of stone</i>	light'en ing, <i>to make light</i>
sculp ture, <i>carved work</i>	light'ning, <i>electric flash</i>
proph'e sy, <i>to foretell</i>	pa'tients, <i>sick people</i>
proph'e cy, <i>a prediction</i>	pa'tience, <i>endurance</i>
pre cede', <i>to go before</i>	dis ease', <i>illness</i>
pro ceed', <i>to advance</i>	de cease', <i>death</i>

Write the following sentences, selecting the right word:

1. His (prophecy, prophesy) was fulfilled.
2. Let us run with (patience, patients) the race that is set before us. *Bible.*
3. (Lightening, lightning) must always (proceed, precede)

thunder. 4. Enjoy the kingdom after my (decease, disease).—*Shakespeare*. 5. Phidias was a famous (sculpture, sculptor) of ancient Greece. 6. The captain ordered the (lightning, lightening) of the vessel. 7. The remedy is worse than the (disease, decease).—*Shakespeare*. 8. The physician cured many (patience, patients). 9. He forth on his journey did (precede, proceed). 10. The Greeks ornamented their temples with (sculptor, sculpture).

148.—WORDS LIABLE TO BE CONFOUNDED.

1 {	ac cept', <i>to take</i>	4 {	for'mer ly, <i>time past</i>
	ex cept', <i>to leave out</i>		form'al ly, <i>in a formal way</i>
2 {	ad vice', <i>counsel</i>		
	ad vise', <i>to give counsel</i>	5 {	sta'tion a ry, <i>fixed</i>
	at tend'ance, <i>the persons</i>		sta'tion er y, <i>paper, pens,</i>
	<i>who attend any service, etc.</i>		<i>etc.</i>
3 {	at tend'ants, <i>those who at-</i>	6 {	pop'u lous, <i>full of people</i>
	<i>tend as servants, com-</i>		pop'u lace, <i>the people</i>
	<i>panions, etc.</i>		

Write the following sentences, using the right word from the list above:

China is a (6) country. Will you (1) my (2)? King Arthur had brave (3). The (6) swarmed in the streets. (4) buffaloes roamed over the Great Plains. A (5) engine drew a car to the top of the hill. The (3) was large at every lecture (1) the first. I shall no more (2) thee. A stationer sells (5). The meeting was (4) opened by the president.

149.—WORDS LIABLE TO BE CONFOUNDED.

ac cede', <i>to agree to</i>	bal'lot, <i>a vote</i>
ex ceed', <i>to go beyond</i>	close, <i>to shut</i>
af fect', <i>to act upon</i>	clothes, <i>articles of dress</i>
ef fect', <i>to accomplish</i>	cen'tu ry, <i>hundred years</i>
ad di'tion, <i>process of adding</i>	sen'try, <i>a sentinel</i>
e di'tion, <i>publication</i>	cel'e ry, <i>a vegetable</i>
as say', <i>to test metals</i>	sal'a ry, <i>wages</i>
es say', <i>to try</i>	de scent', <i>a going down</i>
bal'lad, <i>a song</i>	dis sent', <i>to disagree</i>

e lic'it, *to draw out*
 il lic'it, *unlawful*
 em'i nent, *distinguished*
 im'mi nent, *threatening*
 e lude', *to escape from*
 al lude', *to refer to*
 e rup'tion, *a bursting forth*

ir rup'tion, *an invasion*
 em i grate, *to leave*
 im'mi grate, *to move into*
 ex'er cise, *to use*
 ex'or cise, *to drive away*
 gla cier, *an ice field*
 gla'zier, *a glass setter*

150.—WORDS LIABLE TO BE CONFOUNDED.

in gen'ious, *skillful*
 in gen'u'ous, *honest*
 jest'er, *one who jests*
 ges'ture, *action*
 lin'i ment *liquid ointment*
 lin'e a ment, *a feature*
 lose, *to suffer loss*
 loose, *to untie*
 pas'tor, *a minister*
 pas'ture, *a field for cattle*
 pres'ence, *nearness*
 pres'ents, *gifts*
 pop'lar, *kind of tree*
 pop'u lar, *agreeable*
 pre'e dent, *an example*
 pre ced'ence, *superiority*
 pres'i dent, *chief magistrate*
 par ti'tion, *division*

pe ti'tion, *a request*
 rel'ic, *a memorial*
 rel'ict, *a widow*
 stat'ue, *an image*
 stat ure, *height*
 stat'ute, *a law*
 sur'plus, *the remainder*
 sur'plice, *clergyman's robe*
 se'ries, *a succession*
 se'ri'ous, *solemn*
 spe'cies, *a kind*
 spe'cious, *plausible*
 track, *a footstep*
 tract, *a region*
 ten'or, *part in music*
 ten'ure, *a holder of land*
 ve rac'i ty, *truthfulness*
 vo rac'i ty, *greediness*

STATES AND TERRITORIES WITH THEIR ABBREVIATIONS.

North Atlantic Division:

Maine	Me.
New Hamp'shire	N. H.
Ver mont'	Vt.
Mas sa chu'setts	Mass.
Rhode Is'land	R. I.
Con nect'i cut	Conn or Ct

South Atlantic Division:

Del'a ware	Del.
Ma'ry land	Md.
Dis'trict of Colum'bia	D. C.
Vir gin'i a	Va.
West Vir gin'i a	W. Va.
North Car o li'na	N. C.

New York	N. Y.	South Car o li'na	S. C.
New Jer'sey	N. J.	Geor'gi a	Ga.
Penn syl va'ni a	Pa.	Flor'i da	Fla.

*Northern Central Division:**Western Division:*

O hi'o	O.	Mon ta'na	Mont.
In di an'a	Ind.	Wy o'ming	Wyo.
Il li nois'	Ill.	Col o ra'do	Colo.
Mich'i gan	Mich.	New Mex'i co	N. Mex. Ty.
Wis con'sin	Wis.	Ar i zo'na	Ariz. Ty.
Min ne so'ta	Minn.	U'tah	U.
I'ow a	Ia.	Ne va'da	Nev.
Mis sou'ri	Mo.	I'da ho	Ida.
North Da ko'ta	N. Dak.	A las'ka	Alas. Ty.
South Da ko'ta	S. Dak.	Wash'ing ton	Wash.
Ne bras'ka	Neb.	Or'e gon	Or.
Kan'sas	Kan.	Cal i for'ni a	Cal.

Southern Central Division:

Ken tuck'y	Ky.	Lou i si a'na	La.
Ten nes sec'	Tenn.	Tex'as	Tex.
Al a ba'ma	Ala.	In'di an Ter'ri tory	Ind. Ty.
Mis sis sip'pi	Miss.	Ok la ho'ma	Okla. Ty.
	Ar'kan sas	Ark.	

The population of the United States in 1890 was 62,622,250.

TABLE OF COMMON ABBREVIATIONS.

A. B. Bachelor of Arts.	A. M. Master of Arts; before noon.
acct. or $\frac{a}{c}$ Account.	amt. Amount.
A. D. (<i>Anno Domini</i>) In the year of our Lord.	ans. Answer.
ad lib. (<i>ad libitum</i>) At pleasure.	Anon. Anonymous.
Admr. Administrator.	Atty. Attorney.
adj. Adjective.	Ave. or Av. Avenue.
adv. Adverb.	bal. Balance.
act. (<i>actate</i>) Aged.	bbl. Barrel.
	B. C. Before Christ.

Bro. Brother.	H. R. H. His (or Her) Royal Highness.
bu. Bushel.	ib. or ibid. (<i>ibidem</i>) In the same place.
Capt. Captain.	id. (<i>idem</i>) The same.
Cap. Capital.	i. e. (<i>id est</i>) That is.
C E. Civil engineer.	in Inch, inches.
Co. Company, county.	inst. (instant) Of the present month.
Col. Colonel.	Jr. or Jun. Junior.
Cr. Creditor, credit.	£, lb., lb, or lib. Pound.
cts. Cents.	LL. D. Doctor of Laws.
cwt. Hundredweight.	Lieut. Lieutenant.
D. D. Doctor of Divinity.	M. Monsieur, midday.
del. (<i>delineavit</i>) He drew it.	Maj. Major.
Dep. Deputy.	mas. Masculine.
Dept. Department.	M. C. Member of Congress.
do. (<i>ditto</i>) The same.	M. D. Doctor of Medicine.
doz. Dozen.	mem. (<i>memento</i>) Remember.
Dr. Doctor, debtor.	min. Minute, minutes.
D. V. (<i>Deo volente</i>) God willing.	Mlle. Mademoiselle.
ed. Edition, editor.	Mme. Madame
e. g. (<i>exempli gratia</i>) For example.	M. P. Member of Parliament.
Eng. England, English.	MS. Manuscript.
Esq. Esquire.	MSS. Manuscripts.
et al. (<i>et alii</i>) And others.	Mt. Mountain, mount.
etc. (<i>et cetera</i>) And the rest.	Mus. D. Doctor of Music.
Exr. Executor.	N. B. (<i>nota bene</i>) Mark well.
Fahr. or F. Fahrenheit.	neut. Neuter.
fem. Feminine.	N. G. National Guard.
Fr. France, French.	No. (<i>numero</i>) Number.
ft. Foot, feet.	ob. (<i>obiit</i>) Died.
gal. Gallon, gallons.	p. Page; pp., pages.
Gen. General.	per cent. or % (<i>per centum</i>) By the hundred.
Ger. Germany, German.	Ph. D. Doctor of Philosophy.
Gov. Governor.	P. M. (<i>post meridiem</i>) Afternoon, postmaster.
hhd. Hogshead.	
H. M. His (or Her) Majesty.	
Hon. Honorable.	

P. O. Post-Office.

pk. Peck.

pl. Plural.

pop. Population.

P. P. C. (*pour prendre congé*)

To take leave.

Pres. President.

Prof. Professor.

pro tem. (*tempore*) For the
time.

prox. (*proximo*) Of next
month.

P. S. Postscript.

Ps. Psalm or Psalms.

pwt. Pennyweight.

q. e. (*quod est*) Which is.

qt. Quart.

q. v. (*quod vide*) Which see.

R. A. Royal Academy.

Rec'd. Received.

Rev. Reverend.

R. R. Railroad.

R. S. V. P. (*répondes s'il
vous plaît*) Answer, if
you please.

Rt. Hon. Right Honorable.

Sec. Secretary.

sing. Singular.

St. Saint, street.

supp. Supplement.

Supt. Superintendent.

tr. Transpose, translator.

ult. (*ultimo*) Of last month.

U.S.A. United States Army.

U. S. M. United States Mail.

U.S.N. United States Navy.

viz. (*videlicet*) Namely.

vol. Volume, volumes.

vs. (*versus*) Against.

Xmas. Christmas.

yd. Yard, yards.

&c. And so forth.

PENMANSHIP.

PRELIMINARY INSTRUCTIONS.

GENERAL REMARKS.

1. Acquirement.—The acquirement of a handsome, yet plain and practical, style of penmanship is the duty of every one. It is not necessary that all of us become professional penmen, but *it is necessary* that we all have a style of penmanship easily and rapidly written, and plain to read. It is thought by some that a handsome style of penmanship is a gift. This is not so; *it is an acquirement*, and any one who will *intelligently* follow the course of study given in this Paper, with that effort necessary to master any other study, will acquire a style of penmanship that will be valuable both as an accomplishment and as a business qualification.

The course of instruction given in this Paper is designed to produce *practical results*, and includes everything needful for the acquirement of a plain and rapid handwriting.

2. Specimen.—Before reading any further, we desire you to write two pages of specimens, each containing one set of capitals, one set of small letters, one set of figures, and the following short letter:

§ 12

I _____ of the _____ copyright _____ in _____ dated _____ with the _____

..... Your address and the current date.

*The International Correspondence Schools,
Scranton, Pa.*

Gentlemen:

*This is a fair specimen of my writing
before beginning your course of penmanship lessons by your
correspondence method. Yours truly.*

..... Your signature.

One set of these specimens you keep; the other send immediately to us.

METHOD OF PRACTICE.

3. Routine.—There are sixteen lessons in this subject, and that you may get the most benefit from them it is desirable that you take them up in their order and closely follow the directions given.

Beginning on page 31 of this section is given the copies for this course of lessons and the instructions for studying and practicing them. Lesson 1 is composed of copies 1, 2, and 3. Study these copies carefully and practice on them as directed until you are fairly well satisfied with the results; then mail us a sample of your work, showing at least one-third of a page of each copy.

While waiting for return of these specimens, you may work on Lesson 2, copies 4, 5, and 6. Practice on these copies until you have obtained good results; then send us specimens of your work as before. While waiting for return of Lesson 2, take up Lesson 3, copies 7, 8, and 9. Unless otherwise directed, this will be the order of work throughout the Course. NEVER SEND US BUT ONE LESSON AT A TIME.

MATERIALS.

4. Penholders.—Before beginning the study and practice of these lessons, you must obtain suitable materials with which to work. For practical business writing, a common

straight holder with a taper stem, like the one shown in the figures, made entirely of wood, or of wood with a cork or a rubber tip, is generally preferred, and is certainly the best. Avoid all holders with polished metal piece at the bottom. The smooth, polished metal is very difficult to hold, and the student will acquire the habit of pinching the holder, which will contract the muscles and thus make the whole arm and hand rigid, and entirely prevent the acquirement of that free and easy movement which is absolutely necessary for good writing.

Many "professional" penmen use the "oblique" holder. For their class of work it has many advantages, but for business writing it is not to be recommended.

5. Pens.—For practice and study it is best to use moderately fine steel pens, even if their use is not to be continued. No difficulty will be experienced in changing from a fine to a coarse pen later on, if one is preferred. With a fine pen you will make the lines more accurately and be able to locate your errors with more certainty.

Its use will also cultivate that lightness of touch which is actually necessary to good writing. Gillott's No. 401 and the Spencerian No. 1 are very good. You will also want a small piece of chamois skin, cotton cloth, or some other suitable article on which to wipe your pen. A dirty pen will not make a fine, neat line.

6. Ink.—Black ink is to be preferred to any other. Select a kind which flows freely and which writes black. The pale green or blue inks which turn black are not good for practice. Carter's, Stafford's, and Caw's inks are reliable.

7. Paper.—A good quality of foolscap is the best for practice. Avoid that with a glossy finish. A hard, firm surface will give the best results. The small extra cost of the best over the poor is not to be considered. For the *best* results, your materials must be of the *best*. *Always keep a blotter under the right hand, so as to keep the paper clean.* Having obtained the pens, paper, penholder, ink, and pen-wiper, we are ready for study and practice.

Before we can profitably begin the use of these materials, we must learn how to take an easy and proper position at the table, and also how to hold the pen correctly. Your success in attaining a good handwriting depends very much on these things. You must, therefore, carefully study and practice the directions here given.

POSITIONS.

8. Position.—A correct position of the body and feet is quite as essential as that of the arms and hands. Carefully study and imitate the position shown in Fig. 1. This is called the "front" position. It is the position to be preferred, and is the one most used. We shall, however, illustrate and explain the "side" position, because for lack of room on narrow desks the side position must occasionally be used. In studying the "front" position, carefully observe the following points: *First*, that the body is nearly square with the table, quite near it but not touching it. *Second*, that the position is nearly erect, the body inclining slightly to the left and leaning a little upon the left arm; this is done to avoid putting any weight upon the right arm. *Third*, that the elbows project about two inches over the edge of the table. *Fourth*, that the arms cross the desk obliquely. *Fifth*, that the feet rest squarely on the floor, the left slightly in advance. This tends to give a firm support to the body. (The illustration does not show this.) *Sixth*, that the paper is nearly in front of the body, and turned slightly to the left; that is, with the long way of the paper in the same direction as the right forearm and hand. *Seventh*, that the fingers of the left hand hold the paper in place. All these having been carefully observed and learned, the next thing to study is the correct position of the right arm and hand, the manner of holding the pen, and the movements for producing good writing.

9. Position of the Arm.—By referring to Fig. 1, you will notice that the arm rests on the muscles of the forearm

just below the elbow. This is the point from which the power of the arm is obtained. *Do not bear any weight on the right arm.*



FIG 1.

10. Position of the Hand. —The position of the hand is one of the most important features in writing. By turning to Fig. 1, you will notice: *First*, that the hand rests on the nails of the third and fourth fingers, these fingers being drawn back toward the palm of the hand. *Second*, that there is an open space between the second and third



... and also points
... direction; that is,
... the wrist and the
... table, and the back
... the hardest things

you will have to acquire. You should, therefore, give them your very closest attention.

11. Holding the Pen.—The next thing for you to learn is the correct manner of holding the pen. Figs. 1 and 2 show the pen in the proper position, from two different



FIG. 3.

views. The position is quite well described as follows: The holder is held between the first and second fingers and the thumb, crossing the second finger at the root of the nail

on the side nearest the thumb, then running backward and upward under the first finger, crossing it either just in front of the knuckle joint or just behind it. Some prefer one way and some the other. If held below the joint, Fig. 3, the pen is less apt to stick into the paper and spatter ink; it will also run over the paper more smoothly than if held in the more upright position in front of the joint. The thumb is placed against the left side of the holder, about opposite the first joint of the first finger. The pen held lightly in this manner, with the third and fourth fingers well drawn back and resting lightly on the tips of the nails, the hand well over to the left, the wrist and the hand clear of the table, the arm propelled with a light, yet strong and springy, movement from the muscles of the forearm, must, when combined with careful study and rightly directed practice, produce good results in the acquirement of a handsome and practical style of penmanship.

12. Side Position.—This position is illustrated in Fig. 3. It is best adapted to narrow desks. Sit erect, with the feet firmly on the floor, the right side to the desk, the right arm, hand, and paper parallel with the front edge, and the left hand at the edge, to hold the paper. The arm should rest lightly on the muscles near the elbow. Keep the wrist elevated and let the hand slide on the nails of the third and fourth fingers. The manner of holding the pen is the same as in the front position. While the front position is the one most generally preferred, it is well to practice in both, that you may be able to write well at any style of desk.

MOVEMENTS.

13. Their Names.—There are four methods of using the arm and the hand in producing writing. These are called **whole-arm**, **muscular** or **forearm**, **finger**, and **combined movement**.

14. Whole-Arm Movement.—This is produced by slightly raising the arm from the table and allowing it to

swing freely from the shoulder. This movement is the one you would use if writing on a blackboard. It is much used by professional penmen in making off-hand capitals, but it is not desirable for our present use.

15. Muscular, or Forearm, Movement.—This is the foundation of all good writing, and you cannot acquire an easy, graceful, and rapid style of writing until you master this movement. Caretully study and practice every detail of it as here given, for without this movement your success as a penman will be small.

This movement is developed by resting the arm on the muscles of the forearm just below the elbow (see Fig. 1). These muscles act as a center of power, propelling the hand, which slides along on the third and fourth fingers (see Fig. 1). The thumb and the fingers must not be used in forming the letters, the whole work being done by the muscles of the forearm.

The acquirement of this movement is the first thing to be given attention in learning to write a free, easy, businesslike style of penmanship.

In studying and practicing for a good movement, be sure that the position of your body, arm, and hand is correct, for no good writing can be done in a cramped position.

Be sure, also, that the muscles of the arm, hand, and fingers are lax and free, as no good writing can be done if the muscles are tense and hard.

Writing must be done with a light touch and an elastic movement, or good results cannot be attained.

For those who find it difficult to get that easy, swinging motion of the hand so necessary in acquiring muscular movement, it is a good plan to practice as follows: Drop the arm and hand by the side, letting it hang listlessly and lifelessly; in this position the muscles will become lax, the fingers will curl up slightly, and naturally assume a position which, if retained when the hand is raised to the table for writing will be the correct position of the hand and fingers. With the arm resting lightly on the table, the back of the hand

facing the ceiling, close the hand, making a fist, and with the arm rolling easily on the muscles near the elbow, practice freely and with force the exercises of Lesson 1. When thus practicing these exercises, use no pen or pencil, and be sure that no part of the hand or wrist touches the table or paper. See that the arm rolls and slides freely in the sleeve and that the sleeve itself does not move.

In addition to the above, it is a good plan to practice retracing the copies, using a dry pen, held correctly, and writing at a good rate of speed.

See that neither the wrist nor the side of the hand touches the paper. Keep the muscles lax and free, steadying the hand by allowing it to rest lightly and slide easily on the tip of the fourth finger, or on the nails of the third and fourth fingers.

Success in writing with the pure muscular movement depends entirely on the command one has over the muscles of the forearm, and the method of practice here outlined will, if thoroughly persevered in and used in practicing all the exercises, do much to produce that complete command of the arm, hand, and pen so necessary to success in becoming a good writer.

16. Finger Movement.—This movement is produced by the action of the first and the second finger, in connection with the thumb. You must guard against this movement, as the tendency at first is to use this movement entirely. Perhaps in your writing up to this time you have used the finger movement quite a good deal, if not entirely. If this is so, you may find it difficult to leave it off, but it must be done, as it is impossible to write a smooth, easy, and rapid hand with the pure finger movement.

17. Combined Movement. This movement is produced by the united action of the muscles of the forearm and the fingers, and it is the one chiefly used by skilful penmen. The muscles, resting on their center below the elbow, propel the hand, which slides easily on the nails of the third and fourth fingers; the first and second fingers and the

thumb, acting together, assist the muscles of the arm in shaping the letters. Care must be taken not to use the fingers too much. Their use is principally in forming the long upward and downward letters; the small letters should be made almost entirely with the pure muscular movement. This combination movement of muscles and fingers is the very best movement for both practical business writing and for fine penmanship.

Many fail to become good writers from lack of study. They realize that they make errors, but are not able to tell just where the fault is. This is because they do not have an accurate picture of the letter in their minds. To be successful in learning to write, you must be able to form a mind picture of the letter you wish to make; you must really see a correct picture of it on the paper before you make a mark with the pen. When you can do this and can readily name the strokes necessary to produce a letter, you will be able to see your errors and to correct them. The copies given with these lessons show the correct forms, and the Instruction Paper explains how they are made, and also points out the places where you will be most likely to fail.

DIMENSIONS.

18. Measurements.—Now a few words as to measurement and space. The height of the small letter *i* is taken as the standard of measurement, and is called a *space*. The height and width of all the other letters are regulated by the height of the small letter *i*. The following "space-lined" copies illustrate the height of all the small letters. To get letters the proper size for any width of ruling, imagine the space between the ruled lines to be divided into four equal parts, or spaces. Make the small letter *i* the height of one of these spaces, and make the other letters in the same proportion to this *i* as those in the space-lined copies are to the *i* there.

PRINCIPLES.

19. Lines.—Before a successful study of penmanship can be made, you must become familiar with the principles by which letters are formed. There are three lines from which all the letters of the alphabet are formed: two curved lines and one straight stroke. They are named *right curve*, *left curve*, and *straight line*.

20. Curves.—A **right curve** is one which would appear at the right side of a straight line. A **left curve** is one which would appear at the left side of a straight line.

21. Straight Line.—A straight line is one that does not change its direction throughout its entire length. Nearly all the down strokes in small letters are straight lines on the main slant of writing.

22. Slant.—The slant of writing must also have attention, for without a uniform slant our writing will not have a good appearance. The down strokes of all writing should be made on a slant of about 50 degrees from the ruling of the paper. This is called the *main* slant. The slant of the strokes which connect the letters is called the *connective* slant, and is at an angle of about 28 degrees from the ruling of the paper.

DETAILED INSTRUCTIONS.

23. In the following pages each of the small letters, capitals, and figures is described in more or less detail, and the parts needing the most study and practice are pointed out. Study carefully and fix firmly in your mind all that is said about each letter. The combined movement may be used in making all the letters, but use the fingers as little as you can. When in your work you find yourself unable to

form any letter easily and correctly, it is to these instructions that you must look for a remedy for your failing. Therefore, as you value success, do not slight this work.

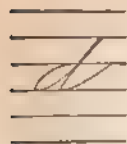
SMALL LETTERS.

24. Be sure to give the oval part of the letter *a* enough slant. Be careful that the downward straight line does not retrace the last upward line of the oval. You will not do this if enough slant is given to the oval. The last three strokes of this letter should form a nearly perfect letter *i*. Do not fail to close this letter at the top.

25. The loop in the letter *b* is the same as in the *l* (see what is said about it). Make the down stroke straight to the base line. The upward and the downward stroke should cross one space above the base line, and below this cross they should make the first two strokes of a perfect letter *i*. The last two lines are the same as the last two lines in the letter *w* (see what is said about it). The loop and also the bottom part of the letter should be one-half space wide. Study carefully the letter *l*. If you fail on the *l*, you will fail in the same place on the loop in this letter. Pay special attention to the last half of this letter, or you will make it too wide.

26. Use the fingers in forming the upper part of the *c*, and be very careful in making the hook at the top; if this is not properly made, the letter may easily be mistaken for the letter *e*. Do not make the top too large. Make the down stroke nearly straight. Do not make too broad a turn at the bottom. See that the finishing line is a full right curve and that it does not extend too far to the right. Give particular attention to these points.

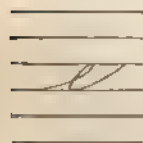
27. The letter *d* is the letter *a* with the top of a *t* added.



Shade only the top part of this letter, and be sure to make the down stroke straight to the line. Give the oval part of the letter plenty of slant, so that the long, straight down stroke will not retrace the up stroke of the oval. Study carefully the special points under both the small

a and the *t*, and apply them to this letter.

28. In the letter *e* make both the turn at the top and the loop small. The upward strokes should be full



right curves; the down stroke should be nearly a straight line, and should not be shaded. The turn at the base should be short, and the base of the finishing line the same as in the *i* and *u*.

Be careful not to close the loop, as then the letter may easily be mistaken for *i* or *c*.

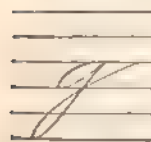
29. The total length of the letter *f* is five spaces, three above the line and two below. The width of each of the loops is one-half space. The first upward and the downward



line cross at the height of the letter *i*, and the upward line of the lower loop joins the downward line one-half space above the base line.

The length of this letter, together with the two loops, makes it one of the most difficult in the alphabet. See that the loops are of the same and the proper width, and that the whole length of the down stroke is on the main slant. Be sure to connect the upward line of the lower loop to the down stroke.

30. The *g* is a letter *a* with the downward stroke extended two spaces below the base line, finishing with



an upward left curve, making an inverted *l*. See that the upper part forms a perfect letter *a*, and that the long, straight, down stroke and the upward curve make the first two lines of a perfect *l*. Do not make the down stroke

more than two spaces long below the base line. See that

the down stroke does not retrace the previous up stroke of the *a*. If shaded anywhere, let it be on the first down stroke.

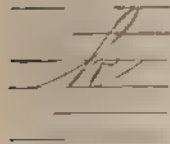
31. The letter *h* is a combination of the *l* loop and the last three strokes of the letter *u*. Make the upward line heavily curved, and the downward stroke perfectly straight to the base line. Let the up and the down stroke cross at the height of the letter *i*; a common error is to cross them too low. Be sure that the downward line in the *u* has the same slant as the long stroke. The making of these long and straight loops is one of the hardest

things in writing. You will do well, therefore, to give this work your best attention, and carefully study and practice the letters having these long, straight, down strokes.

32. Make the down stroke in the letter *i* straight to the line and on the main slant. Be careful not to retrace the first upward line; this is quite a common error. Do not make the turn at the base too broad. Make the dot directly over the letter, one space above the top, and on the same slant as the down stroke. This letter occurs very often in the formation of other letters. You should, therefore, give it your close attention.

33. The letter *j* is composed of the first two strokes of the letter *i* joined to the *l* loop, or, again, the last three strokes of the letter *g*, with a dot over them. Make the dot one space high and on the same slant as the down stroke. Be sure that the down stroke is but two spaces in length below the base line, perfectly straight, and that the last up stroke is a full left curve.

34. After making the first two strokes, which are the same as in the letter *l*, the difficult part of the letter *k* is to form the small horizontal loop. This is made by the united action of the fingers and the muscles. Do not extend the horizontal loop more than one space to the right. See that the first down stroke is on the main slant.



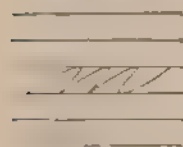
The last down line should be nearly parallel with the main down stroke and one-half space from it. Finish with the last stroke of the letter *n*.

35. The first two strokes of the letter *l* appear in the letters *h*, *b*, *k*, the top of *f*, and, inverted, the last part of *g* and of *y*. Use the combined movement. Make the down stroke straight to the line, crossing the up stroke one space high. Below the point of crossing, the letter should form a perfect letter *i*. If desired, the lower

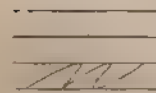


part of the long down stroke may be slightly shaded, to give strength to the letter.

36. The letter *m* is exactly like the letter *n*, except that it has three parts instead of two, the second part being a repetition of the first. Be sure to get all the down strokes parallel, the same height and the same distance apart, and straight. The tops of the three parts should be rounded, and the upward curves parallel. There should be a point at the bottom of the first and the second down stroke, but a round turn at the bottom of the third.



37. In the letter *n* make all the upward lines good, full curves. See that the tops are turns, not points. Both the down strokes should be straight to the line. There should be a point at the bottom of the first down stroke, but a turn at the bottom of the second; other than this the two parts are alike. Be careful to get the down strokes parallel as well as straight.



38. Both sides of the letter *o* should be of the same curvature, and the center of the letter should be on the main slant. A common error is to leave it open at the top; avoid this. The first downward stroke should not retrace the line connecting this letter with the preceding one; be very careful about this, and also see that the down stroke is a full curve; many writers are apt to make this line too straight.

39. Make the first upward stroke in the letter *p* a full curve; many fail by making it a straight line. The down stroke should be a straight line from two spaces above the base line to one and one-half below. It may be shaded a little at the bottom if desired. The last half of this letter is the last three strokes of the letter *n*. In making this part of the letter, lift the pen at the bottom of the long down stroke, and place it on the base line, where the down stroke crosses it, and then finish the letter.

40. Some writers make the letter *q* one and one-half spaces below the line, while others make it two. See that you make the first part of this letter the same as *a*. The downward line should be straight from top to bottom, and on the main slant of writing. The loop of this letter is turned on the opposite side from that of the letter *g*. It joins the main down stroke at the line and then goes upward with a right curve.

41. Use the fingers in making the top of the letter *r*. The errors in forming this letter consist in making the top too pointed or too round, and in making the letter too wide. The sharp, retraced point of this letter should extend above the one-space line. Be sure to make the first up stroke a full curve, or you will not be able to get a sharp point at the top of the letter. Pay special attention to this letter, as you will find it rather difficult to master.

42. The height of the letter *s* is the same as that of the letter *r*. To be successful with this letter, you must make the first line a very full curve. The down line should retrace the up stroke less than one-fourth of a space, and the letter is this much more than one space high. Make the down stroke very heavily curved. See that the finishing dot rests on the up line of the letter, one-fourth of a space above the base line.

43. The full height of the letter *t* is two spaces. Up to one space high it is the letter *i*; above this it is a slightly shaded straight stroke, one space in length. See that the shade is square across the top, and that the heaviest part of the shade is at the highest point. The shade should extend only one-half the length of the letter. Be sure to get the cross of the letter one and one-half spaces above the ruling, and one space long. See that it crosses the letter, instead of being two or three spaces to the right, as we often see it.

44. The letter *u* is practically a letter *i* and the last two strokes of a second letter *i*. This letter should be made entirely with the muscular movement. Make the upward strokes full curves, and the downward ones straight to the line. Have them parallel, one space high, and nearly one space apart. Be sure that neither of the down strokes retraces an up stroke. The tops of this letter should be pointed and the turns at the bottom should be round.

45. Be sure to make the top of the first part of the letter *v* a short turn and not an angle. Do not make the space between the first down stroke and the second up stroke too wide. Remember, it is only one-half a space. The bottom of the letter should be a round turn, not a sharp point. The finishing horizontal curved

line should be turned away from the up stroke at an angle and should be only one-half a space long.

46. The letter *w* is the first four strokes of a *u* with the last strokes of the letter *v* added. See that the down strokes are parallel, and that the top of the third part is only one-half a space from the last down stroke. See that the points at the top of the letter are sharp; do not make the middle one a loop, as is often done. See that the finishing line to the right is only one-half a space long. Do not shade this letter.

47. The letter *x* is peculiar in form, but not a difficult letter to make. The first part made is the last three strokes of a small letter *u*. A straight upward stroke, on the connective slant, is then thrown across the second of these strokes and completes the letter. This stroke should commence on the base line and end at one space high. Study this letter carefully, for if it is not made well it will spoil the appearance of any word in which it occurs.



48. The first part of the letter *y* is the three strokes in the last part of the letter *u*; the last part is the inverted loop, as in the letter *g*. Be careful not to make too much space between the first and the second part. See that the down strokes are straight and parallel. If you make this letter correctly, it will be, if you turn the paper around, a perfect *h*.


49. The first two strokes of the letter *z* are the same as in the letter *u*; then, turning at an angle with a horizontal curve to the right one-fourth of a space in length, finish with the inverted loop, as in the letters *g* and *y*. Be careful not to make the down stroke in the first part curved. Do not make the right horizontal curve too long. Be sure you curve the down stroke of the loop but *very* little.


FIGURES.

50. Main Points. It is exceedingly important to be able to make good figures. A poorly made figure may lead to an expensive mistake. Figures must be made rapidly, as well as good.


To do this requires much careful study and practice. Most beginners make their figures too large. Before practicing a figure, study it until you have a good picture of it in your mind; then train the hand to produce it. All figures extend one space above the line, except the 6, which extends one and one-half. *Shade but very little, if at all.* The 7 and 9 extend one-half space below the line. Make the figures with the combined movement. Observe carefully the following points regarding the making of each figure:

51. This figure should be a short, straight down stroke, one space high. Be careful to get the correct slant.
 If shaded at all, it should be below half the height.
 Do not shade heaviest at the top, or make the stroke a heavy one all the way down. Be sure that this stroke is straight.


52. This is not a difficult figure to make, if the following directions are carefully observed. Be sure to leave one-fourth of a space between the down stroke of the loop and the main down stroke of the figure.
 Many people spoil the figure by running these strokes together. Be careful to have the small loop at the bottom extend well to the left of the top loop.

53. Make the top loop the same as in the 2, but smaller.
 Be sure to have a good quarter space between the first and the second down stroke. Let the middle loop point straight to the left. Finish the letter with a light upward line, or with a dot, about one-third of a space above the line.


54. See that the second down stroke is higher than the first, and somewhat curved, and that the horizontal stroke is a little above the line and nearly parallel with it. The two down strokes should be nearly parallel, and the first may be slightly shaded at the top. The second should not be shaded, and should end on the base line




55. After forming the loop, or an angle, at the bottom of the first stroke, carry the curve upward and to the right (this is where many fail); then bring the curved line well to the left of the first down stroke. Be careful to begin the straight horizontal line, at the top of the figure, at the same place as the first down stroke. Take pains to make this figure very plain; as made by some it is often mistaken for the figure 3.




56. Be careful to get the first stroke on the main slant. Remember what has been said about the height of this figure. See that there is a proper space between the first down stroke in the figure and the down stroke in the oval. This figure may be shaded at the top if desired.





57. See that the first down stroke is a short one, not much more than a dot. A slight double curve in the horizontal line to the right will add quite a little to the neatness of the figure. Be sure that the second and long down stroke is on the main slant. It should extend about one-half space below the base line, and may also be shaded a little.



58. In this figure, make the double curved stroke first, and be sure that the curves are as full as in the copy. See that the upper oval is on the main slant of writing. Notice the bottom loop; it should be long and narrow. Have the upward cross the downward line high enough to divide the figure in the center



- 59.** Keep the bottom of the oval half a space above the line. The first half of this figure is like the body of a letter *a*. See that the downward line does not retrace the last upward line in the oval. The bottom of the down stroke may extend below the base line, and may also be shaded.
- 

- 60.** This figure is really the body part of the letter *o*. The center of this figure should be on the main slant. See that the two curves are equal and that they come together at the top. Avoid the common error of making the downward stroke straight. The down stroke may be slightly shaded if desired.
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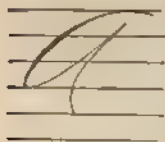
If the above directions are carefully followed, and the copies faithfully studied, you will have no trouble in making figures that will be a credit to you and a source of much pleasure to those who may have to read them. Practice them in lines and in columns, as well as in combinations. The ability to make good figures means the ability to make them in every way in which they are used in business.

CAPITAL LETTERS.

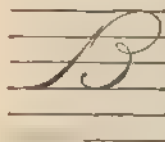
61. How Made.—The capital letters should be made entirely with the muscular movement. To make good capitals you must pay particular attention to the movement. The hand must slide on the tips of the third and fourth fingers, which are held in the manner shown in the figures. Be careful to see that neither the wrist nor the side of the hand touches the paper. Keep the body erect and feet squarely on the floor. Use light shades and a steady, even movement. It is a common fault with beginners to make one part of a letter faster than another. Be careful to avoid this, as it is fatal to good results.

62. This style of letter *A* is almost exactly like the small letter *a* without the first upward stroke. The down stroke

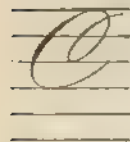
may be shaded a little if desired. The last down, or finishing, stroke is made with a slight curve, to give the letter a graceful appearance. Be sure to give the oval part enough slant. Making it too straight is one of the most common errors. Be careful not to make the oval too wide. Do not shade the last down stroke.



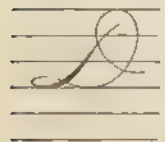
63. In making the letter *B*, be sure that the capital stem has a full curve. The height of the letter is three spaces, the height of the capital stem being two and one-half spaces. Make the ovals in the main part of this letter full and smooth. See that the small loop in the center of the letter points upward. Get the shades as low down as you can; the nearer the bottom of the letter they are, the better the letter will look.



64. Some prefer to make the letter *C* with a large loop, reaching down to within three-fourths of a space of the line; others prefer to have a very short loop, not more than half a space long. We think a loop nearly two spaces long is about the proper size. In making this letter be sure not to use the fingers, for, if you do, you will make a sharp angle in it, and thus spoil the beautiful curves of the letter. Be careful to give the left side of this letter a full curve. Many fail in this, and make it straight. Make full turns at the top and the bottom of the letter, and you will be sure to have good results. Do not make the shade very heavy, but see that it is smooth and bright.

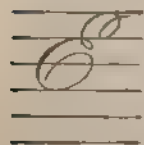


65. Some find it easier to make the loop at the top of the letter *D* very small, while some prefer to make it quite large. Either is permissible, but we think that for rapid business writing the size given in the copy is the better. In this letter, as in the letter *C*, use only the muscular movement. This is made with a double curve and on a slant between the main and connective. This stroke

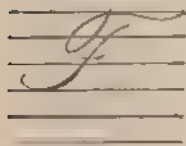


is called the *capital stem stroke*. It occurs in the capitals *B, D, F, G, L, P, R, S,* and *T*; give it, therefore, your thoughtful attention. Be very careful, in this letter, not to get the space between the down stroke and the upward curve too small. If you shade this letter, be sure to keep all the shading below half its height.

66. Use only the muscular movement. The secret of making a good letter *F* is in making the curves full, and in having the small loop in the middle of the letter point across the letter at a right angle to the main slant. Be careful to give the proper space between the first and the second down stroke in the upper part of the letter. See that all the curves are full and broad. The bottom half of this letter is a little larger in every way than the top. The third left curve may be shaded if desired.

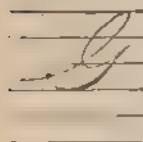


67. The difficult part of the letter *F*, as in the *T*, is to get the capital stem right. A full description of the capital stem and the manner of making it is as follows: This stem should have a little more slant than the down strokes of the other letters. Beginning at the top of the stem, two and one-half spaces above the line, descend about half way with a left curve, verge into a right curve, and continue to the line; carry it one space to the left, then upward one-half space, ending with a dot. Be sure to keep the shade below one-half the height of the stem. See that the dot is carried well to the left. This letter is made exactly like the *T*, with the addition of the cross. This cross is

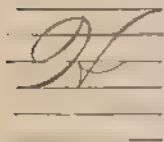


made by a straight horizontal line in the middle of the letter, ending with a downward tick. One-half of the cross-line should be to the left of the letter and one-half to the right. In making the cap to this letter, make a loop about one space to the left of the capital stem and then carry a double curved line across the top of the stem, one-half space above it. Many spoil the cap by making it straight. Do not do this, but make it with full curves.

68. The most common point of failure in the letter *G* is in making the bottom of the loop and oval. This part of the letter should be large and full; too many make it small and pointed. To be successful with this letter, you must make a full curved up stroke, and then, with a full swing, make three-fourths of an oval. Some fail by making the left side of the loop nearly or quite straight. The oval should be two spaces long, coming down to within one space of the base line. The second, or capital-stem, part of the letter should be a little more than half the height of the letter, and should be nearly parallel with the first up stroke. Shade only on the capital-stem stroke, and get the shade below one-half its height.



69. The first part of the letter *H* is nearly the same to the base line as the first stroke in the *W*. The last part of the letter is made by beginning at the full height of the letter and coming down with a full left curve for half its height, then verging into a straight line, continuing to the line on the main slant. Add the connecting stroke, and finish with a horizontal curve. See that the downward line is on the main slant; many fail in this. Be careful not to get the space between the two parts too wide.



70. The full height of the letter *I* is three spaces; the width of the top loop is one space. The width of the bottom loop is one and one-half spaces. The first upward and the downward line should cross one-fourth of a space above the base line. Be careful to get the top loop on the main slant; this is very important. See that the shade is low down; also, that the bottom loop is carried well to the left, and is made full and without a break. Some finish this letter with a dot, one-half a space above the base line, at the same distance to the left as the extreme edge of the full curve.

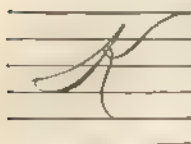


71. The difficult part of the letter *J* is in making the long down stroke. It is made nearly straight, having a very slight curve to the right. Some prefer to make the lower part two spaces below the base line, and some only one and one-half.



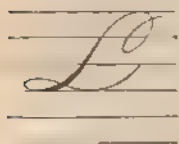
Some also prefer to make the top loop twice as wide as the bottom one, while some prefer to have them alike. We think that, if the top loop is a little larger than the bottom one, it gives a style to the letter not obtained by either of the above. See that all shade is below the base line, and that both of the curved up strokes cross the main down stroke at nearly the same place, one-fourth of a space above the base line, leaving a small three-cornered space in front of the main down stroke.

72. In making the first part of the letter *K*, see that the first stroke is a full right curve, two and one-half spaces high; then descend with a straight line for about one space; then verge into a full right curve, continuing to the base line, then to the left and upward, finishing with a dot on the first stroke of the letter, one-half a space above the base line. The second part of this letter is formed as follows:



Begin at the full height of the letter, three spaces, and two spaces to the right of the first part; descend on nearly the connective slant, with first a left and then a right curve, to the center of the letter; then, forming a small loop pointing upward and encircling the first down stroke of the letter, descend with a graceful curve. Be sure to use both a left and right curve in the first stroke of the second part of the letter. Many fail to do this, and use a right curve all the way down to the small loop, and thus completely spoil the letter. Be sure that the little loop points across the first part of the letter at right angles, or you will not be able to get the last down stroke in its proper place. Notice that the space between the strokes from the loop down gradually increases in width to the base line.

73. The difficult part of the letter *Z* is the capital-stem stroke. The trouble with this is to get it curved enough



The length of the top loop is nearly two spaces; the length of the bottom loop is one and one-half spaces. The width of the top loop should be two-thirds of a space, and the distance between the down stroke and the loop should be nearly one space. See

that the shade is down low and that the stem has the proper slant.

74. Form the first part of the letter *M* the same as the first part of the *W*. The main points to observe in this letter are that the shade is below half of the height of the



letter, that the top of the second down stroke is lower than the top of the first, and that the top of the third down stroke is lower than the top of the second; also, that the down strokes in these lines are on the same slant as the first down stroke in the letter.

They should be parallel, and a little less than one space apart. Let the turns at the top be short but round, the last turn at the bottom the same, and end with a full right curve.

75. The letter *N* is the same as the first and the third part of the *M*. The same things may be said regarding the special points to observe. The downward strokes in this letter should be about three-fourths of a space apart, and parallel.

The shade should be low; the turns should be short but full. Many fail on this letter by making it too wide. Do not do this.



76. The two most common errors in making the letter *O* are the making of the first down stroke too straight, and the turn at the base too short. Be sure to give the same degree of curvature to both sides of the letter. Make the loop at the top full and without a break; a sharp point in

it destroys the appearance of the letter. If this letter is shaded, it should be on the first down stroke, and the ends of the shade should be nicely tapered.

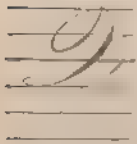
77. The height of the capital stem in the letter *P*, as in all others in which it appears, is two and one-half spaces; the full height of the letter is three spaces; the width of the



letter, measured across at half its height, is two spaces; the width of the right-hand loop is one-half a space. Be sure to get a full curve in the capital stem. See that the width of the space at the left of the stem is the same at both top and bottom. It is a

common error after making the bottom all right to make the top narrow and flat, and at the same time to make the space to the right several times its proper width. Keep all the shade below one space in height.

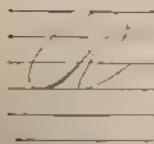
78. The first part of the letter *Q* is the same as the *W*, except that the shade is higher up and the loop smaller. Add the horizontal loop one



space in length, cross the main down stroke, touch the base line, and finish with a right curve one space in height. Study the points in the first stroke of the *W*. Be sure that the

long diameter of the small loop is parallel with the ruling, and that the finishing line is an easy curve.

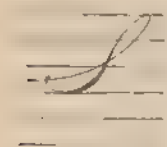
79. The height of the stem in the letter *R* is the same as in the other letters in which it appears.



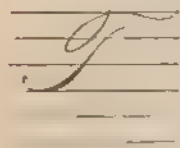
This letter, as far as the small loop, is nearly identical with the letter *P*. The small loop, at one-half the height of the letter, encircles the down stroke and points across it at an angle. Descend from the loop to the base

line with nearly a straight stroke, touching the line and finishing with a full right curve.

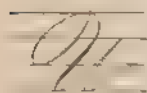
80. The usual difficulty with the letter *S* is to get enough curve to the up stroke, also to get enough curve to the down stroke. Make these curves very full and you will not have very much trouble with this letter. See that the lines cross at one and one-fourth spaces high, neither higher nor lower. Keep all the shade below the crossing point ; the nearer the line you can get it, the better will be the effect.



81. The two parts of the letter *T* are the capital stem and the cap. Make the capital stem just as described for the letter *F*, also the cap. The horizontal part of the cap should be nearly parallel with the line of writing. Note the position of the shade and the ending of the stem. If you follow carefully all that has been said about the capital stem, you will not have much trouble with this letter.



82. The first part of the letters *U* and *V* are alike. The most difficult part of both of these letters is the second down stroke. Form the loop the same as in the *U*, giving the oval a full curve ; descend one space with the curve and then, changing to the straight line, continue neatly to the base line, make a short turn, and ascend with a right curve two spaces high and one space to the right of the first down stroke ; then descend with a straight line to the base line, turn short, and finish with a right curve.



See that the main down stroke is straight for two full spaces. Confine the shade to this straight line ; it is quite a common error to shade too high. See that the down strokes are parallel and that the third down stroke does not retrace the up stroke.

83. Form the first half of the letter *V* to the base line exactly like the *U* ; then turn short and ascend with nearly a straight line one and one-fourth spaces ; then, with a full

it destroys the appearance of the letter. If this letter is shaded, it should be on the first down stroke, and the ends of the shade should be nicely tapered.

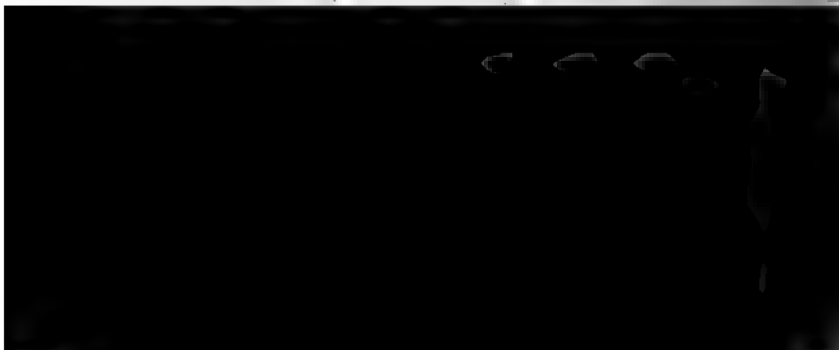
77. The height of the capital stem in the letter *P*, as in all others in which it appears, is two and one-half spaces; the full height of the letter is three spaces; the width of the letter, measured across at half its height is two spaces; the width of the right-hand loop is one-half a space. Be sure to get a curve in the capital stem. See that the width of the space at the left of the stem is the same at both top and bottom. It is a



a common error after making the bottom all right is to make the top narrow and flat, and at the same time to make the space to the right several times its proper width. Be sure the shade below one space in height.

78. The first part of the letter *Q* is the same as the first part of the letter *P*, except that the shade is higher and the loop smaller. Add the horizontal stroke in length, cross the main down stroke at the base line, and finish with a curve one space in height. Study the first stroke of the *W*. Be sure

the long diameter of the small loop is parallel with the main stem, and that the first curve is a easy curve.

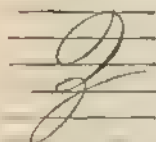


Be sure to get the proper space at the top and bottom between the second and the third down lines. Make the shade as low as possible and leave it pointed at the bottom. See that the second stroke is a left curve all the way from its starting point to the base line, and that it touches the first stroke in the middle.

86. The letter *Y*, to the bottom of the third down stroke, is the letter *U*; then add the loop as in the small *y*. See that both down strokes are on the same slant. The last up stroke should cross the long down stroke at the base line. Watch the shade and the turn at the bottom of the second down stroke. For the further points to observe in this letter, see the capital *U* and small *y*.



87. The first part of the letter *Z* is the same as the first part of *W*, except that, on account of the small loop, the shade is made a little higher. Form the loop, then turn to the right and descend with a slight right curve two spaces; then ascend with a full left curve, crossing the down stroke at the base line, and finishing at three-fourths of a space high.



LESSON PAPERS, NO. 1 TO NO. 16.

COPIES NO. 1, NO. 2, AND NO. 3.*

88. Copy No. 1.—This copy is to be practiced at a good rate of speed and with the pure muscular movement. In making the oval forms in this copy, let the arm rest lightly on the desk; propel the hand by the muscles of the forearm in the direction indicated by the arrows in the copy. Let

* The following copies are actual penwork, written and reproduced by Williams & Rogers, Rochester, N. Y.

LESSON 1



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the hand have a free, rolling motion, moving a little to the right with each revolution, thus giving the exercise the appearance of a coiled spring. Do not make the ovals too round, and see that they are on the same slant as writing. The up strokes and down strokes should be uniform in slant and spacing. Keep the ovals close together, or they will look scrawled. Let the height of the exercise be equal to the full space between two blue lines. Watch carefully that you do not use your fingers. Keep at work on this copy until you can make the exercises fairly well; then begin with No. 2.

89. Copy No. 2.—This copy is also designed to produce a free, rolling, and sliding movement, training the hand to produce forms of different sizes.

Practice this copy carefully with the hand resting lightly on the tips of the nails of the third and fourth fingers, and with the arm rolling freely on the muscles near the elbow. The nails of the third and fourth fingers should actually trace on the paper the same movements as the pen. Make the ovals in the direction indicated by the arrow; see that you make ovals and not rings; observe the gradual taper of the exercise as shown in the copy, and see that the ovals are uniform in slant and spacing; retrace each oval at least ten times, then slide to the next size. Make all exercises at a good rate of speed. After you begin an exercise, do not take the pen from the paper until you have completed the five ovals. Work on this copy until you can produce a good sample; then take up No. 3.

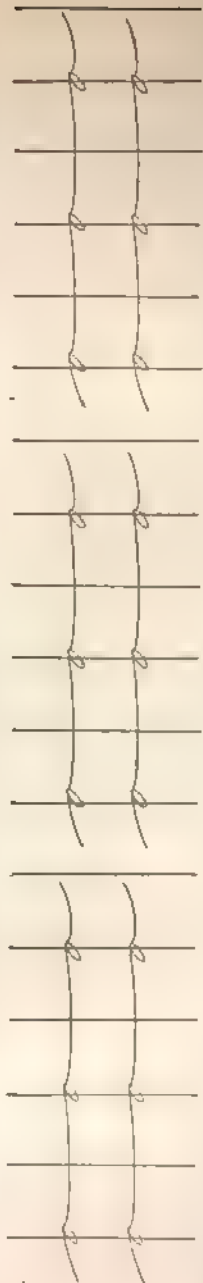
90. Copy No. 3.—This is the same as copy No. 2, with a reversed rolling motion. Practice this exactly as directed for No. 2, and do not leave it until you can make the ovals of different sizes nicely and without taking the pen off the paper. These copies are designed to train the hand and the eye to act together. The ability to produce ovals of various sizes will have much to do with your success in making capital letters; therefore, give these copies close attention.

LESSON 2

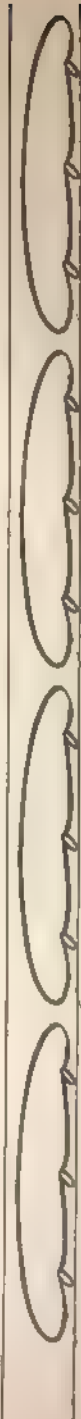
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5



6



Review the copies in this lesson and send us a sample of your work, one-third of a page of each of the exercises; then take up Lesson No. 2.

COPIES NO. 4, NO. 5, AND NO. 6.

91. Copy No. 4.—This is a combination of copies No. 1 and No. 2, and is designed to give you further practice on the rolling and the sliding movement, grading the rolling movement from large to small ovals. Practice this exercise carefully until you can make it well and of a uniform taper. This will train your hand to produce characters of almost any size, which is a desirable thing for a writer to be able to do. See that the slant and spacing are correct, use a free movement, and write at a good rate of speed. Note the gradual taper of the exercise. Work on this exercise until you can produce it fairly well, and then take up copy No. 5.

92. Copy No. 5.—This copy is designed to give you more drill on the sliding movement, and at the same time to train you in making the small letter *o*. In practicing this copy, be sure that the hand slides freely on the nails of the third and fourth fingers, and that the movement is an easy one. Make the first up-stroke a full curve, then form the letter *o* with the muscles of the arm, assisting, if necessary, with the fingers, to give the oval a full round shape. Be sure to close the *o*'s at the top and be careful not to make a loop in doing so. Let the hand slide freely between the letters and allow the connecting line to sag a little. Finish the exercise with a free, swinging up stroke.

You will get the best results with this copy, if you practice writing it lengthwise of the paper, across the lines, making a letter *o* at every other line. This will train you to space the letters evenly, and the large space between the letters will help you to acquire that free, sliding movement so necessary to rapid and easy writing. Study your copy carefully, and imitate it closely. Always watch your position and practice this as you have the others until you can produce good copies; then proceed with the next copy.

Handwriting practice lines showing cursive letter formation and tracing exercises.

Handwriting practice lines showing cursive letter formation and tracing exercises.

Handwriting practice lines showing cursive letter formation and tracing exercises.

Handwriting practice lines showing cursive letter formation and tracing exercises.

93. Copy No. 6.—This copy is designed to further perfect you in the sliding movement, and also to give you more practice in forming the letter *o*. Keep the *o*'s clear in the center and do not make the return line, over the top, too far above the *o*'s. The practice of making one copy well and then retracing it, is one which will produce the best results in the least possible time, and is an exercise on which we will dwell quite a little during the lessons. When you are fairly well satisfied with your work on the copies in this lesson, prepare one-third of a page of each of them and send it to us for correction; then proceed with Lesson No. 3.

COPIES NO. 7, NO. 8, AND NO. 9.

94. Copy No. 7.—This copy is designed to give you a drill in the practice of upward curves and straight down strokes of various lengths. This is a valuable copy and you will do well to give it a great deal of time and study. Work on it faithfully, always being sure that your body, arm, and hand are in a correct position. Watch yourself that you do the work with a free muscular movement and avoid using the fingers. See that the down strokes are all on the same slant, that the curved strokes are parallel, and that the turns at the bottom are short but round. No shading.

95. Copy No. 8.—This copy is also designed to produce a free, sliding movement of the hand, and at the same time to give you a drill on the short down strokes, making the letter *i*. These two things go a long way toward producing good writing, and you will do well to give them your best attention. Dwell on this lesson quite a little and review it at frequent intervals. Practice this copy by writing it across the blue lines, as shown in the copy-slip. Make the four letters without lifting the pen. Swing the hand freely from the bottom of one letter to the top of the next without using the fingers or wrist. See that the down strokes are straight and all on the same slant, with slightly rounded turns at the bottom. No shading.

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11



96. Copy No. 9.—This copy is in the same class as the two previous ones, right curved up strokes and straight down strokes making the letter *u*. Work on this, being careful to get the letters rounded at the bottom and pointed at the top. Let the downward stroke separate from the upward one so sharply that the open space between the two will extend to the top of the letter. Practice them in single lines of letters one space high, and take particular pains that the down strokes all slant alike and that the letters are all of the same height. The small, short letters form a large part of all writing, and your ability as a penman will depend largely on how well you make them. Carefully study their forms and the principles from which they are made. Thorough, honest work will produce good results, but without it you cannot expect to succeed.

COPIES NO. 10, NO. 11, AND NO. 12.

97. Copy No. 10.—This is a copy of upward left curves and straight down strokes, and in this copy we have the two main strokes of the letters *m* and *n*. Give the upward stroke a good full curve and make the downward strokes perfectly straight to the base line. Let the tops of these letters be rounded and the points at the bottom sharp. Practice them in connected rows, as in the copy, and then in single letters, as you would use them in writing *m*'s and *n*'s. The exercises in this copy and in copy No. 9 are very important ones, as they are made up of right and left curves and straight down lines, which are the elements of writing. Give them, therefore, your best thought and study, for you will find that your success with all writing depends upon how well you make these strokes. Try to get each set of them uniform and parallel, both in the curves and in the straight down strokes. Watch the turns and make those that should be rounded, round, and the pointed ones, pointed. Strive for uniformity, as this is one of the greatest points in good writing. No shading. When you have learned to

LESSON 8

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make copies No. 9 and No. 10 accurately and easily, you have got a good start towards becoming a good writer.

98. Copy No. 11.—This is a combination of the previous copies making the word *union*. Before beginning practice on this copy, turn to the explanations of detail for each of these letters, as given in the Instruction Paper, under Detailed Instructions for Small Letters. From a study of the points contained therein and a careful examination of the copy, you will gain much valuable knowledge relative to their correct proportions. Practice this copy across the blue lines and thus accustom yourself to write with a uniform spacing. This uniformity will have much to do with the handsome appearance of your writing. Be particular to watch yourself that you make the curves full and the down strokes straight. Swing freely between the letters that you may not become cramped in your work. Work on this copy until you can execute it freely and quite well before you go on with the next.

99. Copy No. 12.—This copy is the same word as the previous copy, but it is lined off into spaces and heights, so that you may see how correctly written it is and how to test your own writing. Practice this word with the down strokes one space apart, and draw lines through your own writing to see how all the letters slant and how evenly you have spaced the letters. Work on this faithfully and your reward will be a great gain in the correctness with which you will write.

COPIES NO. 13, NO. 14, AND NO. 15.

100. Copy No. 13.—This lesson brings into our work the letters *n*, *m*, *v*, *x*, *o*, and *a*. You will notice that all these letters begin with a left curve. Study under the Detailed Instructions all about these letters and compare the copies with these Instructions. This is a way in which you can gain a thorough knowledge of the true proportions and forms of the letters, and as good writing can only come to those who know how the letters are formed from

to send your to the following address

[illegible]

18

1888

the principles, your success depends greatly upon your work in this direction. In the second line of the copy there are given several combinations of letters, making a few common words. The particular things to be observed in making these combinations are the uniformity of slant, height, round and sharp points where needed and the total absence of shade. Be very careful about the lines connecting the letters. See that both the right and left curves appear and that you do not make the letters any further apart when using this double curved connecting line than when you use simply a single right or left upward curve.

101. Copy No. 14.—This is the letter *e* in various sizes. Practice on them all and in the order given. The particular points for attention are given under Detailed Instructions. In working on the two words given at the end of the copy pay close attention to all that has just been said regarding slant, space, connecting lines, etc., under copy No. 13.

102. Copy No. 15.—This copy deals with the small letter *c*. Study the Detailed Instructions given for this letter and practice it as given in the copy. Pay special attention to the top of this letter, as it is here that you will fail, if anywhere. Do not attempt the words at the end of the copy until you have studied the letters *r* and *s* under the Detailed Instructions for these letters and have practiced on them. These two words are likely to give you some trouble, for the letters *r* and *s* are quite hard for most writers to make at first, but if you will carefully follow the directions given you are not likely to have any very serious trouble.

COPIES NO. 16, NO. 17, AND NO. 18.

103. Copy No. 16.—This copy is principally devoted to the letter *a* and the letters of which it forms a part; namely, *d*, *g*, and *q*. Study the Detailed Instructions regarding this letter and practice it alone before trying it in the combinations. When you are able to make a single letter *a* well, try it in the same manner as you practiced

LESSON 2

9.

19

..... across immenses immensities across worlds.

04

ityphg ityphg ityphg ityphg ityphg
 ityphg ityphg ityphg ityphg ityphg

21

to do tidy aditto aditto

pink pink pink pink pink

o and *i* in copies No. 5 and No. 8, respectively. When you are able to make them nicely three at a time, in this way study up on the other parts of the *d*, *g*, and *q*, and practice these letters and then the combinations as illustrated in the copy. This copy will require considerable work, as it is a combination of letters all of which require quite a good deal of study and practice to produce them in correct forms. Do not, however, become discouraged, but persevere, and success will come in due time. There are many letters on which you will find you need to put more study, and in this review you will be more likely to get them as you wish than you did the first time you went over them.

104. Copy No. 17.—This is on the letter *r*. You gave this copy some study a while ago, but now you will devote a little more time to it. You will find this letter quite easy if you will closely follow the directions given under the Detailed Instructions. Carefully compare your work with the copy, and study just where you can improve.

105. Copy No. 18.—This is on the letter *s*. You will find it of great benefit to you if you will practice this letter in the same way as you have the *i*, *o*, and *a*. In fact, this may be said of all the letters. We really believe that the practice of making each letter in groups by itself, writing them across the ruled lines as in copy No. 5 for a page, and then retracing the exercise as in copy No. 6, will do more to produce a good, free, easy movement than any other one thing you can work on. In writing the words at the end of the copies No. 17 and No. 18, be very careful that you follow out all the points regarding each letter given you in the Detailed Instructions and all that has been said heretofore on the letters here given in the copies.

COPIES NO. 19, NO. 20, AND NO. 21.

106. Copy No. 19.—This lesson is a review of the letters *i*, *u*, *w*, *e*, *c*, *r*, and *s*. Carefully review your study of the points regarding these letters and again

LESSON 8

He'll all his high horse life all
 not his not and not his not

in his every day life

to his every day life

practice them in groups by themselves. In practicing them in the combinations given in the lesson, compare your work carefully with the copy. See that you do not shade any of them and that the bottom of each letter is as in the copy.

107. Copy No. 20.—This is a copy on which you must put a great deal of hard study before you can expect to have the success you would like. Carefully study the Detailed Instructions regarding each letter given in this copy, and also study the spaced combination given at the beginning of the copy. Do your very best with this copy, carefully watching the curved lines and the straight down strokes. You may shade a little on the *t* and *p*, on the second down stroke in the *h* and on the first down curve in the *g*. While still working on this copy take up also copy No. 21.

108. Copy No. 21.—This will give you more of a drill on the *t* and *p*. Be sure in your work that you do not become cramped. Always keep a good position and do your work with a free movement. As to copy No. 21, there is little to be said that has not been explained in the Detailed Instructions.

COPIES NO. 22, NO. 23, AND NO. 24.

109. Copy No. 22.—The object of this copy is to perfect you on the long loops, both above and below the line. Study carefully all that has been said under *l* in the Detailed Instructions. Practice this one letter until you are fairly well satisfied with the results and then take up the next letter. Study this as you did the *l* and apply all your thought to it until you know that you are doing well with it, then take the next, and so on until you have gone through the list.

110. Copy No. 23.—This is to give you practice in combining long loop letters and small, or one-space, letters. Be very careful to have all down strokes straight, evenly spaced and parallel; all up strokes should be good curves; shade but little, if any. In all double letters, see that

time, not like that! Stand!
stand! Stand! Stand!

up! Stand! Stand! Stand!
stand! Stand! Stand! Stand!

up for jumps over the dog

both are exactly alike. See that the finishing line of the *y* in *highly*, as well as that of the *g* in *liking*, is free and easy.

111. Copy No. 24.—This is for a further drill along the same line. The copies are rather difficult, but with study, practice, and patience you will master them, and you will find the work on them will help you very much in your study of other letters.

Practice drawing lines through your writing to test its accuracy of spacing, slant, size, and shape. To gain success you must study as well as practice.

COPIES NO. 25, NO. 26, AND NO. 27.

112. Copy No. 25.—This is a copy on which you can afford to spend considerable time. It is one which will try you on what you have learned and will give you a good drill in writing on the proper slant. Practice drawing long lines through your writing, as in the word *union*, copy No. 11, but draw them longer. Be sure to draw them on the slant on which you have written the down strokes of the letters, or you will not get the full benefit of the test. Do not favor your writing. In this copy try not only for the shape of the letters, but for the general appearance. You will find it as hard to make the writing appear well as to produce fairly good shaped letters.

113. Copy No. 26.—This is the entire alphabet in rotation, giving several styles of some of the letters. Practice them in order and try to excel on each letter before taking up the next. In this work pay careful attention to all you have been over and try to apply it to your work. Try to have your writing appear well, as well as to have the letters formed correctly. Smooth, uniform writing produced with a free, easy movement will look better than the most correctly formed letters produced by a slow, dragging movement.

7

1

1719
1122
23456

10/10/10


114. Copy No. 27.—This copy also contains the entire alphabet arranged in the form of a sentence. Practice writing this sentence carefully, looking for your errors and studying to correct them. Shade only where the copy is shaded. Write the same size as the copy and do all your work with a free, easy swing and always keep yourself in a good position for writing. Select for an extra lesson those letters on which you think you need to put the most study and practice, and work on these as you would on any lesson which we might give you.

COPIES NO. 28, NO. 29, AND NO. 30.

115. Copy No. 28.—The copy is designed to give you a drill in writing large and small. It is often very convenient to be able to write any size we wish. For an ordinary writer to write as well at any size as at the one he is used to is impossible, but with a little practice a good writer will soon learn to do so, and will find the ability much to his advantage. Therefore, practice it well and faithfully.

116. Copy No. 29.—This copy is devoted to figures. A good writer must be able to make good figures. With regard to this copy we shall say but little, for we think the explanation of each figure under Detailed Instructions is sufficient for your guidance in their study. Practice them in rotation and in the different sizes as given. Also practice them in columns, for this is the way in which they are very much used. A good way of doing this is to turn your paper around and write them between the rulings, writing three figures in each space. This will give you about the size that you will use them in any book work that you may have to do. Actual practice is the most valuable, and this is what we have designed to give you.

117. Copy No. 30. This copy is devoted to signs and characters. The only instruction we shall give you regarding these is that you follow the copy very closely and apply to these exercises the knowledge that you have gained in



your study up to this time. By carefully watching your work with reference to spacing, slant, height, width, shade, and size, you will be able to make these characters without any trouble.

If you have been faithful in your work up to this time, you should be able to make all the small letters very well and also the figures. We shall devote more or less time to them in connection with the work on capitals, but singly no more attention will be paid to them in these lessons. You will, of course, continue to work on them by yourself and to study to improve your work, for only by so doing can you expect to become an expert penman, and we hope it is with this idea in view that you have undertaken this work. Keep thoroughly interested in your work. In the following lessons try to do even better than you have done up to this time. The next work will be on the capitals, and you will find it far more interesting than the work so far taken up.

COPIES NO. 31, NO. 32, AND NO. 33.

118. Copy No. 31.—We are now to begin the study of capital letters. To do good work on capitals it is necessary that you have a very free movement. This can only be obtained by careful and persistent practice of those exercises that tend to develop freedom for the hand and an easy style of handling the pen. Copies No. 1, No. 2, No. 3, and No. 4 are particularly useful in this respect. Copy No. 31, with which we begin this lesson, is another very much used for this purpose and one which will produce good results. It is really capital *O*'s made in a line. Study under the Detailed Instructions all that is said about the capital *O*, and then practice this copy carefully. Write with a free, easy movement, no matter what the results so far as shape of the letters goes, but under no circumstances write carelessly. Better not write at all than to write carelessly. One of the secrets of easy writing is to keep the arm properly adjusted all the time. Write this copy in sets of three

222222222222

222222222222

222222222222

O's. Do not shade. Be sure to get them as near like the copies as you possibly can. This is a grand exercise and one which you will appreciate more and more as you become accustomed to use it.

119. Copy No. 32.—This is an exercise which will help you to get the first stroke in the capital *H*. It will also aid in the practice of shading for the down stroke of the *W*, *N*, *M*, *U*, *V*, *X*, *Z*, and all other letters in which a right curve, shaded low down, appears. Practice them in sets, as given in the copy. Always be sure that your movement is free and easy, and your position correct.

120. Copy No. 33.—This is the letter *H* made in sets. The principal objects of this lesson are to teach you the correct form of the letter, to drill you on the sliding movement, and to train you to make the letters with that smoothness of line that so marks the work of the expert penman. Be sure that you curve the first two lines enough, and see that the finishing dot is above the base line. The last part of the letter should be higher than the first and should curve at the top. Study the copy closely. The fingers may assist the muscles in the finishing parts of the letter, but must not be used to any great extent.

COPIES NO. 34, NO. 35, AND NO. 36.

121. Copy No. 34.—This copy is an important one, for ten letters begin with this loop. The motion should be the same in all the loops; the only difference in them is the size. See that the lines cross each other at the top of the loop. Study your copies. You must learn to see the forms of the letters on the paper before you can expect to have the greatest success with your writing. Practice this copy very carefully. Imitate it closely and do not be satisfied with anything but the very best you can do.



122. Copy No. 35.—The first two strokes, in the second and third letters in this copy, are very much like those in copy No. 32. Make the horizontal loop parallel with the line, ascend with a very full right curve, and circle the top with a full and graceful loop, finishing the letter with an easy swing to the right. Make the letters in sets, as shown in the copy. Keep the shade well down to the bottom of the letter. To make smooth work you must use a free, easy movement. No matter if at first you may not do as well as you expect; later you will be glad that you followed the directions.

123. Copy No. 36.—This is the capital *J* several times retraced. Be very careful that you use a free and easy swing in making this letter. Make the top larger than the bottom. See that the first and last upward lines cross each other just above the base line, making a very small three-cornered space in front of the long down stroke. Make this exercise at a fair rate of speed. Do not draw the letter; in retracing the letter see how near the original lines you can keep. This is a practice that will give you confidence, and it will train the hand so that when you put it to the paper to make a certain letter, it will do it almost alone, so mechanical have become the motions used.

COPIES NO. 37, NO. 38, AND NO. 39.

124. Copy No. 37.—This copy is designed to give you more practice on the letter *J*. The practice of making the capital letters in sets by themselves is a good one. If you would practice it with all the letters of the alphabet you would find it would help you very much in the acquirement of a free and easy style of writing. In making the letter *J* be sure to have the top loop a little larger than the bottom one. Also, endeavor to form a small three-cornered space, at the base line, in front of the main down stroke and between the two upward curves. While this space is not

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2. F. B. C. C.

1859

actually necessary to the letter, it will be noticed that all the capital *J*'s that have this space in them will be the best *J*'s that you make. Be sure that the upward curves cross each other at the base line. Keep all the shade below the base line, and take pains that you make it smooth and uniform. Work faithfully on this exercise and you will be well paid not only in the improvement that you will make in the capital *J*, but in the more complete control of the hand that this practice will bring to you.

125. Copy No. 38.—This is the letter *Y* made in the same manner as the *J* in the preceding copy. Study carefully under Detailed Instructions all that has been said about this letter and the most common faults that are made in producing it. Practice this copy by making the letters in sets of three. Carefully watch the shade, that you get it smooth and in the right place. Look out also for the loop at the beginning of the letter, and for the turn at the bottom of the shade. See that the last down stroke separates from the preceding up stroke at an angle; see that it is straight from top to bottom, and that the loop is good and full and that it finishes with a free, swinging curve.

126. Copy No. 39. This copy deals with words beginning with capitals. This style of copy is one of great value, as it is in connection with small letters that you will find your greatest use for the capital letters. Endeavor to imitate the copies of the letters, as here given, as closely as possible. Practice drawing lines through your writing to test its accuracy as to spacing, height, slant, shape, and size. This is a most excellent practice and one from which we get some of our best results.

In this copy, practice first each word alone and then two or more together. Keep in your mind all that you have learned about each letter and apply it to this work, if you would look for the best results. Some of the final letters, like the *t* in *suit* and *y* in *gay*, are not the same as explained in the Detailed Instructions, but if you will study copy No. 26 you will find them there, and you will be able to see

43

K L M N O P Q R S T

44

U V W X Y Z A B C D E F

45

G H I J K L M N O P Q R S

46

Tudor City, London, N. York, N. Y.
 Albany, N. Y. (Albany, N. Y.)

the main points of difference between them and those explained. These styles of endings are given that you may have a little more variety from which to select and in this way develop your individual taste.

COPIES NO. 40, NO. 41, AND NO. 42.

127. Copy No. 40.—This copy is in the same style as the last and has the same purpose in view. All that has been said about the previous one applies to this, and that, we think, covers the whole ground.

128. Copies No. 41 and No. 42.—These are the standard capital letters of the alphabet and are the ones from which the Detailed Instructions regarding capitals were taken. Practice each carefully, being very particular about the smooth shades, clean cut lines, and full curves. Study well the special points about each letter before beginning practice on it. The only way to success is to thoroughly know what you are about to do before you begin work, no matter whether it be writing or anything else.

COPIES NO. 43, NO. 44, NO. 45, AND NO. 46.

129. Copies No. 43, No. 44, and No. 45.—These are the same as those in the last copy, and we cannot say more about them than is given in the Detailed Instructions and what has just been said. Give them your best attention, dwelling on each until you are well satisfied with the result, and then take up the next.

130. Copy No. 46.—This shows a handsome business-like style of writing a person's name and address. Practice not only this one copy, but many others in this same style, and send us quite a collection for inspection. You have now got where the fruits of your patient practice and hard work are going to show. Do not falter now if the results on your first trials of these names are not all you would wish. Keep at them, and the final results will show how well you have mastered the art.

LESSON 10

is called a cursive or cursive body-writing
and is called with the pen.

1. National J. B. B.

UJXKLMNOPQRSTUVWXYZ, 10

uopqrstuvwxyz. abcdefghijklmnopqrstuvwxyz

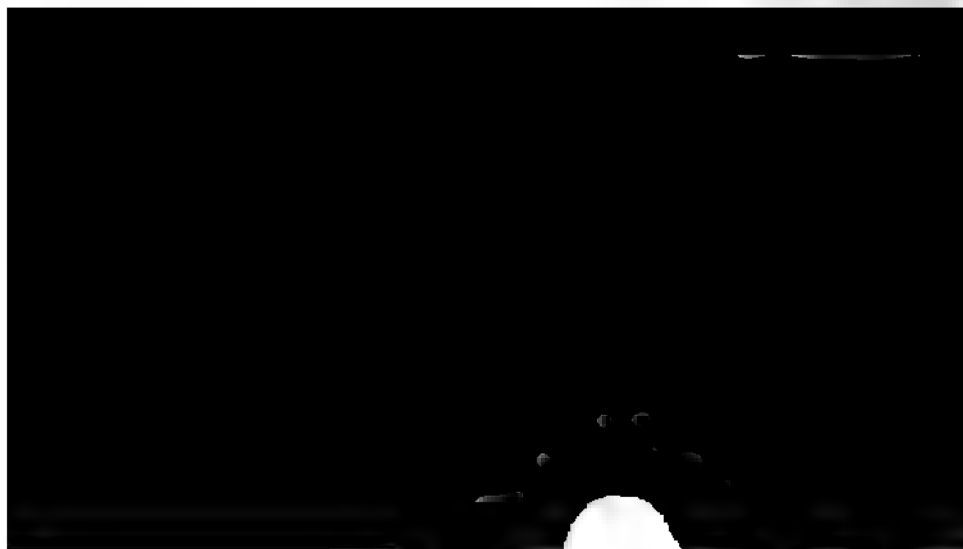
COPIES NO. 47, NO. 48, NO. 49, AND NO. 50.

131. Copy No. 47.—This copy explains itself. It is designed to give you that most necessary of all practices, that of writing several lines and having them all show up well. This shows the skill of the true penman. Many can write a few words well and also names and addresses, but when called upon to write a page they are not able to do so in the same good style as on a few words or names. Work hard on this and it will pay you well.

132. Copy No. 48.—This copy shows a style of writing which is adapted for show purposes. It is very plain and yet handsome. It is much used by bookkeepers for writing "headings" in their ledgers. It is the same as any other writing, except that each down stroke is shaded a little, thus producing a strong look and bringing the words out very clear and plain. You will find many uses for this style of writing; therefore, study it thoroughly.

133. Copies No. 49 and No. 50.—These illustrate a printing alphabet. It is very convenient to have at your command an alphabet similar to this for use in marking packages, tags, sales tickets, price cards, and many other like things in which writing would not show off to so good advantage or would not be so attractive. The pen is held in the same position as in writing, but the paper is turned around to the left so as to give the printing a backhand slant. The shades must be bright and smooth, and taper to fine points. Keep the down strokes evenly spaced and have all the shades of the same weight. See that you get the small letters all of the same height and be sure that the slant is uniform. The many uses to which this copy can be put will pay well for many hours of careful, persistent practice. Do not, therefore, become discouraged if at first the printing is not all that you would wish; but keep at it, and you will find yourself master of a style of marking that will soon be the envy of many

If you have up to this time carefully followed all directions, you should now be able to write a good, plain, business hand. We trust that you will keep up your interest in the work, at least to an extent sufficient to keep yourself in working practice. When a person is able to write well it is merely a matter of practice for him to become a very fair professional penman. Aside from the pleasure it gives one to be able to sit down and write a letter in a handsome style, it is a valuable accomplishment for any one to have, as the ability to write well is much appreciated by all.



VERTICAL PENMANSHIP.

PRELIMINARY INSTRUCTIONS.

GENERAL REMARKS.

1. Acquirements.—The acquirement of a handsome, yet plain and practical, style of penmanship is the duty of every one. It is not necessary that all of us become professional penmen, but *it is necessary* that we all have a style of penmanship easily and rapidly written, and plain to read. It is thought by some that a handsome style of penmanship is a gift. This is not so; *it is an acquirement*, and any one who will *intelligently* follow the course of study given in this Paper, with that effort necessary to master any other study, will acquire a style of penmanship that will be valuable both as an accomplishment and as a business qualification.

The course of instruction given in this Paper is designed to produce *practical results*, and includes everything needful for the acquirement of a plain and rapid handwriting.

2. Specimen.—Before reading any further, we desire you to write two pages of specimens, each containing one set of capitals, one set of small letters, one set of figures, and the following short letter:

§ 13

For notice of the copyright, see page printed lately following the title page.

Your address and the current date.

*The International Correspondence Schools,
Scranton, Pa.*

Gentlemen:

*This is a fair specimen of my writing
before beginning your course of penmanship lessons by your
correspondence method.* *Yours truly,*

Your signature.

One set of these specimens you keep; the other send immediately to us.

METHOD OF PRACTICE.

3. Routine.—There are ten lessons in this Paper, and that you may get the most benefit from them it is desirable that you take them up in their order and closely follow the directions given, and that you intelligently practice for a time each day, making that time as long as possible. To be successful, you must become thoroughly interested in your work and keep so.

Beginning on page 29 of this Paper are given the copies for this course of lessons and the instructions for studying and practicing them. Lesson 1 is composed of five copies. Study these copies carefully, and practice on them as directed until you are fairly well satisfied with the results; then mail us a sample of your work, giving at least three lines of each copy.

While waiting for the return of these specimens, you may work on Lesson 2. Practice on these copies until you have good results; then send us specimens of your work as before. While waiting for the return of Lesson 2, take up Lesson 3. Unless directed differently, this will be the order of work throughout the Course. NEVER SEND US MORE THAN ONE LESSON AT A TIME.

MATERIALS.

4. Penholders.—Before beginning the study and practice of these lessons, you must obtain suitable materials with which to work. For practical business writing, a common

straight holder with a taper stem, like the one shown in the figures, made entirely of wood, or of wood with a cork or a rubber tip, is generally preferred, and is certainly the best. Avoid all holders with a polished metal piece at the bottom. The smooth, polished metal is very difficult to hold, and the student will acquire the habit of pinching the holder, which will contract the muscles and thus make the whole arm and hand rigid, and entirely prevent the acquirement of that free and easy movement which is absolutely necessary for good writing.

Many "professional" penmen use the "oblique" holder. For their class of work it has many advantages, but for business writing it is not to be recommended.

5. Pens. For practice and study it is best to use moderately fine steel pens, even if their use is not to be continued. No difficulty will be experienced in changing from a fine to a coarse pen later on, if one is preferred. With a fine pen you will make the lines more accurately and be able to locate your errors with more certainty.

Its use will also cultivate that lightness of touch which is actually necessary to good writing. Gillott's No. 404 and the Spencerian No. 1 are very good. You will also want a small piece of chamois-skin, cotton cloth, or some other suitable article on which to wipe your pen. A dirty pen will not make a fine, neat line.

6. Ink.—Black ink is to be preferred to any other. Select a kind that flows freely and writes black. The pale green or blue inks which turn black are not good for practice. Carter's, Stafford's, and Caw's inks are reliable.

7. Paper.—A good quality of foolscap is the best for practice. Avoid that with a glossy finish. A hard, firm surface will give the best results. The small extra cost of the best over the poor is not to be considered. For the *best* results, your materials must be of the *best*. *Always keep a blotter under the right hand, so as to keep the paper clean.* Having obtained the pens, paper, penholder, ink, and pen-wiper, we are ready for study and practice.

Before we can profitably begin the use of these materials, we must learn how to take an easy and proper position at the table, and also how to hold the pen correctly. Your success in attaining a good handwriting depends very much on these things. You must, therefore, carefully study and practice the directions here given.

POSITIONS.

8. Position.—A correct position of the body and feet is quite as essential as that of the arms and hands. Carefully study and imitate the position shown in Fig. 1. This is called the "front" position. It is the position to be preferred, and is the one most used. We shall, however, illustrate and explain the "side" position, because for lack of room on narrow desks the side position must occasionally be used. In studying the "front" position, carefully observe the following points: *First*, that the body is nearly square with the table, quite near it, but not touching it. *Second*, that the position is nearly erect, the body inclining slightly to the left and leaning a little upon the left arm; this is done to avoid putting any weight upon the right arm. *Third*, that the elbows project about two inches over the edge of the table. *Fourth*, that the arms cross the desk obliquely. *Fifth*, that the feet rest squarely on the floor, the left slightly in advance. This tends to give a firm support to the body. (The illustration does not show this.) *Sixth*, that the paper is nearly in front of the body, the bottom edge parallel with the edge of the table, or turned slightly to the left—that is, with the long way of the paper in the same direction as the right forearm and hand—provided vertical down strokes can be made easily with the paper in this position. *Seventh*, that the fingers of the left hand hold the paper in place. All these having been carefully observed and learned, the next thing to study is the correct position of the right arm and hand, the manner of holding the pen, and the movements for producing good writing.

9. Position of the Arm.—By referring to Fig. 1, you will notice that the arm rests on the muscles of the forearm just below the elbow. This is the point from which the power of the arm is obtained. *Do not bear any weight on the right arm.*



FIG. 1.

10. Position of the Hand.—The position of the hand is one of the most important features in writing. By turning to Fig. 1, you will notice: *First*, that the hand rests on

the nails of the third and fourth fingers, these fingers being drawn back toward the palm of the hand. *Second*, that there is an open space between the second and third fingers. *Third*, that the wrist and the side of the hand are



FIG 2.

kept clear from the desk. *Fourth*, that the hand is turned well over to the left, so that its back faces the ceiling. This turning of the hand brings the arm on the thickest and most muscular part of the forearm, and also points the top of the

penholder in the proper direction; that is, directly over the right shoulder. Keeping the wrist and the side of the hand free and clear from the table, and the back of the hand facing the ceiling, are two of the hardest things you will have to acquire. You should, therefore, give them your very closest attention.



FIG. 3.

11. Holding the Pen.—The next thing for you to learn is the correct manner of holding the pen. Figs. 1 and 2 show the pen in the proper position, from two different views. The

position is quite well described as follows: The holder is held between the first and second fingers and the thumb, crossing the second finger at the root of the nail on the side nearest the thumb, then running backwards and upwards under the first finger, crossing it either just in front of the knuckle joint or just behind it. Some prefer one way and some the other. If held below the joint, Fig. 3, the pen is less apt to stick into the paper and spatter ink; it will also run over the paper more smoothly than if held in the more upright position in front of the joint. The thumb is placed against the left side of the holder, about opposite the first joint of the first finger. The pen held lightly in this manner, with the third and fourth fingers well drawn back and resting lightly on the tips of the nails, the hand well over to the left, the wrist and the hand clear of the table, the arm propelled with a light, yet strong and springy, movement from the muscles of the forearm, must, when combined with careful study and rightly directed practice, produce good results in the acquirement of a handsome and practical style of penmanship.

12. Side Position.—This position is illustrated in Fig. 3. It is best adapted to narrow desks. Sit erect, with the feet firmly on the floor, the right side to the desk, the right arm, hand, and paper parallel with the front edge, and the left hand at the edge, to hold the paper. The arm should rest lightly on the muscles near the elbow. Keep the wrist elevated, and let the hand slide on the nails of the third and fourth fingers. The manner of holding the pen is the same as in the front position. While the front position is the one most generally preferred, it is well to practice in both, that you may be able to write well at any style of desk.

MOVEMENTS.

13. Their Names.—There are four methods of using the arm and the hand in producing writing. These are called whole-arm, muscular or forearm, finger, and combined movements.

14. Whole-Arm Movement.—This is produced by slightly raising the arm from the table and allowing it to swing freely from the shoulder. This is the movement you would use if writing on a blackboard. It is much used by professional penmen in making offhand capitals, but is not desirable for our present use.

15. Muscular, or Forearm, Movement.—This is the foundation of all good writing, and you cannot acquire an easy, graceful, and rapid style of writing until you master this movement. Carefully study and practice every detail of it as here given, for without this movement your success as a penman will be small.

This movement is developed by resting the arm on the muscles of the forearm just below the elbow (see Fig. 1). These muscles act as a center of power, propelling the hand, which slides along on the third and fourth fingers (see Fig. 1). The thumb and the fingers must not be used in forming the letters, the whole work being done by the muscles of the forearm.

The acquirement of this movement is the first thing to be given attention in learning to write a free, easy, business-like style of penmanship.

In studying and practicing for a good movement, be sure that the position of your body, arm, and hand is correct, for no good writing can be done in a cramped position.

Be sure, also, that the muscles of the arm, hand, and fingers are lax and free, as no good writing can be done if the muscles are tense and hard.

Writing must be done with a light touch and an elastic movement, or good results cannot be attained.

For those who find it difficult to get that easy, swinging motion of the hand so necessary in acquiring muscular movement, it is a good plan to practice as follows: Drop the arm and hand by the side, letting it hang listlessly and lifelessly; in this position the muscles will become lax, the fingers will curl up slightly, and naturally assume a position which, if retained when the hand is raised to the table for writing, will be the correct position of the hand and fingers.

With the arm resting lightly on the table, the back of the hand facing the ceiling, close the hand, making a fist, and, with the arm rolling easily on the muscles near the elbow, practice freely and with force the exercises of Lesson 1. When thus practicing these exercises, use no pen or pencil, and be sure that no part of the hand or wrist touches the table or paper. See that the arm rolls and slides freely in the sleeve and that the sleeve itself does not move.

In addition to the above, it is a good plan to practice retracing the copies, using a dry pen, held correctly, and writing at a good rate of speed.

See that neither the wrist nor the side of the hand touches the paper. Keep the muscles lax and free, steadying the hand by allowing it to rest lightly and slide easily on the tip of the fourth finger, or on the nails of the third and fourth fingers.

Success in writing with the pure muscular movement depends entirely on the command one has over the muscles of the forearm, and the method of practice here outlined will, if thoroughly persevered in and used in practicing all the exercises, do much to produce that complete command of the arm, hand, and pen so necessary to success in becoming a good writer.

16. Finger Movement.—This movement is produced by the action of the first and second fingers, in connection with the thumb. Perhaps in your writing up to this time you have used the finger movement quite a good deal, if not entirely. If this is so and it is thought impracticable to give the necessary time to thoroughly train the arm and hand to the use of the muscular movement, the finger-and-wrist movement may still be used, and with much better success in vertical writing than could be had with it in the slant style. If the finger movement is used, the position of the right arm, hand, and paper will be somewhat different from those just described. The bottom of the paper should be nearly or quite parallel with the front edge of the desk. The right forearm rests partly on its side, the back of the

hand facing towards the right instead of upwards. With the hand in this position, the natural action of the fingers will produce vertical down strokes in the writing. In forming the oval and extended letters, a slight action of the wrist or forearm is combined with the finger movement. The arm and hand held in this position will not slide along the desk as easily as when more muscular movement is used ; it must, therefore, rest very lightly on the desk, and must be slightly raised and moved to the right as often as it becomes difficult to write easily along the line.

17. Combined Movement.—This movement is produced by the united action of the muscles of the forearm and the fingers, and it is the one chiefly used by skilful penmen. The muscles, resting on their center below the elbow, propel the hand, which slides easily on the nails of the third and fourth fingers ; the first and second fingers and the thumb, acting together, assist the muscles of the arm in shaping the letters. Care must be taken not to use the fingers too much. Their use is principally in forming the long upward and downward letters ; the small letters should be made almost entirely with the pure muscular movement. This combination movement of muscles and fingers is the very best movement for both practical business writing and for fine penmanship.

Many fail to become good writers from lack of study. They realize that they make errors, but are not able to tell just where the fault is. This is because they do not have an accurate picture of the letter in their minds. To be successful in learning to write, you must be able to form a mind picture of the letter you wish to make ; you must really see a correct picture of it on the paper before you make a mark with the pen. When you can do this and can readily name the strokes necessary to produce a letter, you will be able to see your errors and to correct them. The copies given with these lessons show the correct forms, and the Instruction Paper explains how they are made, and also points out the places where you will be most likely to fail.

NOTES

12. ~~XXXXXXXXXX~~

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10. The first part of the paper is devoted to the study of the properties of the function $f(x)$ defined by the equation $f(x) = \int_0^x f(t) dt$. It is shown that $f(x)$ is a continuous function and that it satisfies the differential equation $f'(x) = f(x)$. The second part of the paper is devoted to the study of the properties of the function $g(x)$ defined by the equation $g(x) = \int_0^x g(t) dt$. It is shown that $g(x)$ is a continuous function and that it satisfies the differential equation $g'(x) = g(x)$.



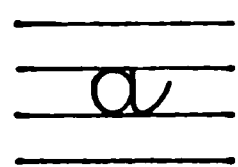
and, if a good style of writing has been acquired, and it slants a little forward or backward, it is not advisable to spend much time in trying to change it to a strictly vertical style, but better to put all the time and attention on improving the movement and forms of the writer's letters.

DETAILED INSTRUCTIONS.

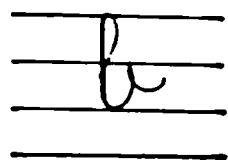
23. In the following pages each of the small letters, capitals, and figures is described in more or less detail, and the parts needing the most study and practice are pointed out. Study carefully and fix firmly in your mind all that is said about each letter. The combined movement may be used in making all the letters. When in your work you find yourself unable to form any letter easily and correctly, it is to these instructions that you must look for a remedy for your failing. Therefore, as you value success, do not slight this work.

SMALL LETTERS.

24. The body of this letter should be a perfect oval. The down stroke should be one space high, straight and vertical. Note carefully how it joins the oval or body of the letter. Do not fail to close the letter at the top. Take pains to make this letter like the copy; then it will not be mistaken for an *o*.

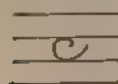


25. The loop in the letter *b* is the same as in the *l*. Make the down stroke straight to the base line. The upward and the downward stroke should cross one space above the base line, and below this cross they should make the first two strokes of a perfect letter *i*. The last two lines are the same as the last two lines in the letter *w*. Compare the width of the loop and the bottom part of the letter. Study carefully the letter *l*. If you fail on the *l*, you will fail in the same place on the loop

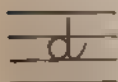


in this letter. Pay special attention to the last half of this letter, or you will make it too wide.

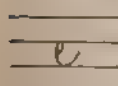
26. Note how round this letter is; also, the slight hook-like finish at the top. Do not close the letter too much, or it will look like *o*. Make a broad turn at the bottom, and finish with a right-curve up stroke. In connecting a preceding letter to a *c*, it is necessary to raise the pen in order to avoid making a loop in the top of the *c*.



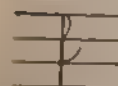
27. The letter *d* is the letter *a* with the top of a *t* added. Shade only the top part of this letter, and be sure to make the down stroke straight to the line. Some make a loop in the top of this letter; avoid it if possible. Study carefully the special points under both the small *a* and the *t*, and apply them to this letter.



28. The first stroke is a right curve; note carefully how and where it joins the rest of the letter. In making this letter, when it follows another, the pen is raised at the end of the connecting stroke; the right-hand side of the loop is next made, the pen passing upwards around the top of the letter, and then down between the connecting line and the stroke already made, forming the back and the base of the letter. If desired, the *c* can be made without raising the pen, as in the slanting style; but if so made, it should be round and full. Try both styles and make your own selection. Be careful not to close the loop, as then the letter may easily be mistaken for *t*.

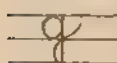


29. The total length of the letter *f* is three spaces, two above the line and one below. The width of the loop is the same as in *t*. The first upward and the downward line cross at the height of the letter *t*, and the finishing line joins the downward line at the base line. See that the whole length of the



down stroke is straight and vertical. Be sure to connect the finishing line to the down stroke.

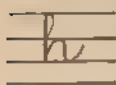
- 30.** The *g* is a letter *a* with the down stroke extended one space below the base line, finishing with an upward left curve, making an inverted *l* loop.



See that the upper part forms a perfect letter *a*, and that the down stroke is straight and vertical.

Do not make the down stroke much more than one space long below the base line. If shaded anywhere, let it be on the down stroke.

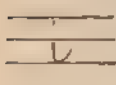
- 31.** The letter *h* is a combination of the *l* loop and the last three strokes of the letter *n*. Make the upward line heavily curved, and the downward stroke perfectly straight to the base line. Let the up and the down stroke cross



at the height of the letter *i*; a common error is to cross them too low. Be sure that the downward line in the *n* is vertical. The making of these long and straight loops is one of the hardest

things in writing. You will do well, therefore, to give this work your best attention, and carefully study and practice the letters having these long, straight down strokes.

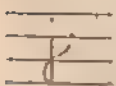
- 32.** Make the down stroke in the letter *i* straight to the line and vertical. Be careful not to retrace the first upward line; this is quite a common error.



Make the dot directly over the letter, and one space above the top. This letter occurs very often in the formation of other letters. You should,

therefore, give it your close attention.

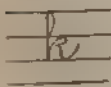
- 33.** The letter *j* is composed of the first two strokes of the letter *i* joined to an inverted *l* loop, or, again,



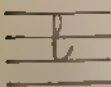
the last two strokes of the letter *g*, with a dot over them. Make the dot one space above the down stroke and directly over it. Be sure that the down stroke is but two spaces in length, one

above and one below the base line, perfectly straight, and that the last up stroke is a full left curve.

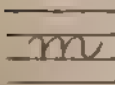
34. After making the first two strokes, which are the same as in the letter *l*, the difficult part of the letter *k* is to form the small horizontal loop. This is made by the united action of the fingers and the muscles. Do not extend the horizontal loop more than one space to the right. The last down line should be nearly parallel with the main down stroke and one-half space from it. Finish with the last stroke of the letter *n*.



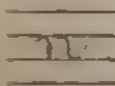
35. The first two strokes of the letter *l* appear in the letters *b*, *h*, *k*, the top of *f*, and, inverted, in the last part of *g*, *j*, and *y*. Use the combined movement. Make the down stroke straight to the line, crossing the up stroke one space high. Below the point of crossing, the letter should form a perfect letter *i*. If desired, the lower part of the long down stroke may be slightly shaded, to give strength to the letter.



36. The letter *m* is exactly like the letter *n*, except that it has three parts instead of two, the second part being a repetition of the first. Be sure to get all the down strokes parallel, the same height, the same distance apart, and straight. The tops of the three parts should be rounded, and the upward curves parallel. There should be a point at the bottom of the first and the second down stroke, but a round turn at the bottom of the third.



37. In the letter *n* make all the upward lines good, full curves. See that the tops are turns, not points. Both the down strokes should be straight to the line. There should be a point at the bottom of the first down stroke, but a turn at the bottom of the second; other than this the two parts are alike. Be careful to get the down strokes parallel as well as equal.



38. The letter *o* should be nearly or quite round. A common error is to leave it open at the top; avoid this. The first downward stroke should not make a loop with the line connecting this letter to the preceding one; be very careful about this, and also see that the down stroke is a full curve. Many writers are apt to make this line too straight.

39. Make the first upward stroke in the letter *p*, when it follows another letter, a full curve; many fail by making it a straight line. The down stroke should be a straight line from two spaces above the base line to one below. It may be shaded a little at the bottom if desired. Study the last half of this letter carefully; it must be made quickly to be made well. Note how closely it resembles an inverted *c*.

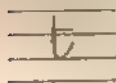
40. The first part of the letter *q* is the same as *a*. The downward line should be straight from top to bottom, and extend one space below the base line. The loop of this letter is turned on the opposite side from that of the letter *g*. It joins the main down stroke at the line and then goes upward with a right curve.

41. Use the fingers in making the top of the letter *r*. The errors in forming this letter consist in making the top too pointed or too round, and in making the letter too wide. The sharp, retraced point of this letter should extend above the one-space line. Be sure to make the first up stroke properly, or you will not be able to get a sharp point at the top of the letter. Pay special attention to this letter, as you will find it rather difficult to master.

42. The height of the letter *s* is the same as that of the letter *r*. To be successful with this letter, you must make the first line nearly straight. The down line should retrace the up stroke less than one-fourth of a space, and the letter is this much more than

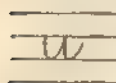
one space high. The down stroke should be very heavily curved. Study carefully how the letter is finished.

43. The full height of the letter *t* is nearly two spaces. Up to one space high it is the letter *i*; above this it is a slightly shaded straight stroke, one space in length. If shaded, see that the shade is square across the top, and that the heaviest part of the shade is at the highest point. The shade should extend only one-half the length of the letter.

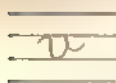


Be sure to get the cross of the letter one and one-half spaces above the ruling, and one space long. See that it crosses the letter, instead of being two or three spaces to the right, as we often see it.

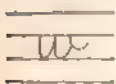
44. The letter *u* is practically a letter *i* and the last two strokes of a second letter *i*. This letter should be made entirely with the muscular movement. Make the upward strokes full curves, and the downward ones straight to the line. Have them parallel, one space high, and one space apart. Be sure that neither of the down strokes retraces an up stroke. The tops of this letter should be pointed and the turns at the bottom should be round.



45. Be sure to make the top of the first part of the letter *v* a round turn and not an angle. Do not make the space between the first down stroke and the second up stroke too wide. The bottom of the letter should be a round turn, not a sharp point. The finishing horizontal curved line should be turned away from the up stroke at an angle and should be only one-half a space long.



46. The letter *w* is the first four strokes of a *u* with the last stroke of the letter *v* added. See that the down strokes are parallel, and that the top of the third part is only one-half a space from the last down stroke. See that the points at the top of the letter are sharp; do not make the middle one a loop, as



is often done. See that the finishing line to the right is only one-half a space long. Do not shade this letter.

47. The letter *x* is peculiar in form. The first part of it resembles an inverted *c*, while the last part is nearly two-thirds of the back of a *c*. Study its form and proportions carefully, and endeavor to produce a neat and graceful letter. Be careful not to make this letter too wide; it should be no wider than its height.

48. The first part of the letter *y* is the three strokes in the last part of the letter *u*; the last part is the inverted loop, as in the letter *g*. Be careful not to make too much space between the first and the second part. See that the down strokes are straight and parallel. If you make this letter correctly, it will be, if you turn the paper around, a perfect *h*.

49. The first two strokes of the letter *z* are nearly the same as in the letter *x*; then, turning at an angle with a short horizontal curve to the right, finish with the inverted loop, as in the letters *g* and *y*. Be sure to make the down stroke in the first part curved. Do not make the right horizontal curve too long. Be sure you curve the down stroke of the loop but *very* little.

FIGURES.

50. Main Points.—It is exceedingly important to be able to make good figures. A poorly made figure may lead to an expensive mistake. Figures must be made rapidly, as well as good.

To do this requires much careful study and practice. Most beginners make their figures too large. Before practicing a figure, study it until you have a good picture of it in your mind; then train the hand to produce it. All figures extend one and one-half spaces above the line,

except the 6, which extends two. *Shade but very little, if at all.* The 7 and 9 extend one-half space below the line. Make the figures with the combined movement. Observe carefully the following points regarding the making of each figure.

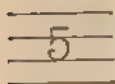
51. This figure should be a short, straight down stroke, one and one-half spaces high. If shaded at all, it should be below half the height. Do not shade heaviest at the top, or make the stroke a heavy one all the way down. Be sure that this stroke is straight. Some writers precede the down stroke with a short upward tick; do not do this.

52. This is not a difficult figure to make, if the following directions are carefully observed. Be sure to leave three-fourths of a space between the first down stroke or tick and the main down stroke of the figure. Many people spoil the figure by making these strokes too near together. Be careful to have the small loop at the bottom extend well to the left of the top loop.

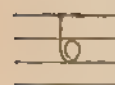
53. Make the top loop the same as in the 2, but smaller. Be sure to have a good quarter space between the first and the second down stroke. Let the middle loop point straight to the left. Finish the letter with a light upward line, or with a dot, about one-third of a space above the line. If this figure is shaded at all, it should be on the last down stroke.

54. See that the second down stroke is no higher than the first, and that the horizontal stroke is a little above the line and nearly parallel with it. The down strokes should be parallel and vertical. The second should end on the base line. If it is so desired, the first down stroke may be shaded a little.

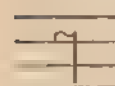
55. After forming the angle at the bottom of the first stroke, carry the curve upwards and to the right (this is where many fail); then continue the curve, forming seven-eighths of an oval and ending a little above the base line and a little to the left of the first down stroke. Begin the straight horizontal line at the same place as the first down stroke. Make the figure plainly, so that it will not be taken for a 3.



56. Be careful to get the first stroke straight and vertical. Remember what has been said about the height of this figure. See that there is a proper space between the first down stroke in the figure and the down stroke in the oval. Do not make the oval too wide, and do not connect it to the down stroke. This figure may be shaded at the top if desired.



57. See that the first down stroke is a short one, not much more than a dot. A slight double curve in the horizontal line to the right will add quite a little to the neatness of the figure. Be sure that the second and long down stroke is vertical. It should extend about one-half space below the base line, and may also be shaded a little.



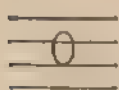
58. In this figure, make the double curved stroke first, and be sure that the curves are as full as in the copy. Notice the bottom loop; it should be larger than the top one. Have the upward cross the downward line a little above the center. This is a peculiar and rather difficult figure; give it plenty of study and practice.



59. The bottom of the oval should be one-half a space above the base line. The first half of this figure is like the body of a letter a. The bottom of the down stroke may extend one-half a space below the base line, and may also, if desired, be shaded a little. Be sure that the down stroke is straight and vertical.



60. A vertical line drawn through the long diameter of this figure should divide it exactly in the center.



See that the two curves are equal and that they come together at the top. Avoid the common error of making the downward stroke straight.

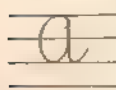
The down stroke may be slightly shaded if desired. Some writers prefer a small circle instead of an oval; try both and see which you like better.

If the above directions are carefully followed, and the copies faithfully studied, you will have no trouble in making figures that will be a credit to you and a source of much pleasure to those who may have to read them. Practice them in lines and in columns, as well as in combinations. The ability to make good figures means the ability to make them in every way in which they are used in business.

CAPITAL LETTERS.

61. How Made.—The capital letters should be made almost entirely with the muscular movement. To make good capitals you must pay particular attention to the movement. The hand must slide on the tips of the third and fourth fingers, which are held in the manner shown in the figures. Be careful to see that neither the wrist nor the side of the hand touches the paper. Keep the body erect and the feet squarely on the floor. Use light shades and a steady, even movement. It is a common fault with beginners to make one part of a letter faster than another. Be careful to avoid this, as it is fatal to good results.

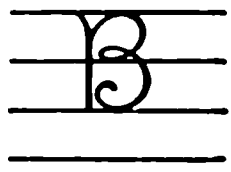
62. This style of letter *A* is almost exactly like the small letter *a*. Note the hook-like beginning and how



it connects with the last down stroke. The first down stroke should be a full curve, and may be shaded a little if desired. The last down, or finishing, stroke ends with a slight curve, to give

the letter a graceful appearance. Be careful not to make the oval too wide.

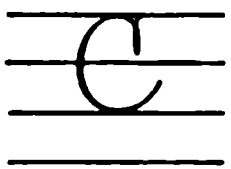
63. In making the letter *B*, be sure that the first down stroke is vertical. The height of the letter is two spaces.



Make the ovals in the main part of this letter full and smooth. See that the small loop in the center of the letter points straight to the left.

Do not make a loop at the bottom of the first down stroke, and see that both ovals are about the same size. Ovals, to be well made, must be made quickly.

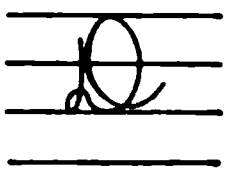
64. Some prefer to begin the letter *C* with a large loop; others prefer to have a very short loop, not much more than a dot. In making this letter be sure not to use



the fingers, for if you do you will make a sharp angle in it, and thus spoil the beautiful curve of the letter. Be careful to give the left side of this letter a full curve. Many fail in this, and make it straight. Make full turns at the top and

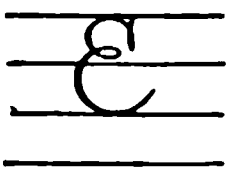
bottom of the letter, and you will be sure to have good results. Do not make the shade very heavy, but see that it is smooth and bright.

65. In making this letter use the muscular movement as much as possible. Study carefully the proportions of each part of the letter, noting particularly the position and shape of the small loop



at the bottom of the first down stroke. To make this letter well will require a great deal of practice; do not, therefore, slight it.

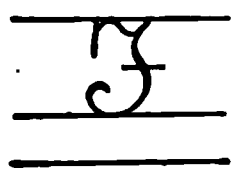
66. Use only the muscular movement. The secret of making a good letter *E* is in making the ovals full, and in having the small loop in the middle of the letter



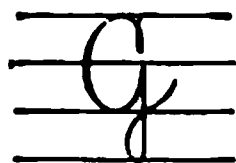
point across the letter at a right angle. Be careful to give the proper space between the first and the second down stroke in the upper part of the letter. See that all the curves are full and

broad. The bottom half of this letter is larger in every way than the top.

67. In this letter make the top first, then the capital stem. In making the top of this letter, make a very small loop or tick about one space to the left of the capital stem, and then carry a double curved line to the right, to the top of the stem. Many spoil the top by making it straight. Do not do this, but make it with full curves. A description of the capital stem and the manner of making it is as follows: Beginning at the top of the stem, two spaces above the line; descend about half way with a left curve, verge into a right curve, and continue to the line; carry it one space to the left, then upwards one-half space, ending with a dot.



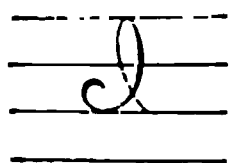
68. The most common point of failure in the *G* is in making the bottom of the oval. It should be large and full. Too many make it small and pointed. Some fail by making the left side of the loop nearly or quite straight. The oval should be two spaces long, coming down to the base line. The second part of the letter should be two spaces long and like an inverted *L*.



69. The first part of the letter *H* is a vertical stem ending with a small oval on the base line. The last part of the letter is made by beginning at the full height of the letter and coming down with a full left curve for nearly half its height, then verging into a straight line continuing to the base line. Add the connecting stroke, and finish with a horizontal curve. See that both the downward lines are vertical. Be careful not to get the space between the two parts too wide.

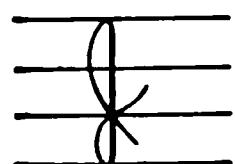


70. The full height of the letter *I* is two spaces; the width of the top loop is nearly one space. The width of the bottom oval is one and one-half spaces. The first upward and the downward line should cross one-fourth of a space above the base line. See that the bottom loop is carried well to the left and is made without a break. Some finish this



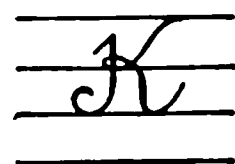
letter with a dot, one-half a space above the base line, at nearly the same distance to the left of the extreme edge of the full curve.

71. The difficult part of the letter *J* is in making the long, vertical down stroke. Some prefer to make the lower part two spaces below the base line, and some only one.



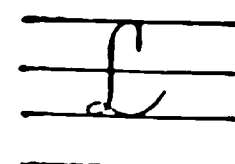
Some also prefer to make the top loop twice as wide as the bottom one, while some prefer to have them alike. We think that, if the top loop is a little larger than the bottom one, it gives a style to the letter not obtained by either of the above. See that both of the curved up strokes cross the main down stroke at nearly the same place, just above the base line and in front of the main down stroke.

72. The first part of the letter *K* is exactly like that of *H*. The second part of this letter is formed as follows: Begin at the full height of the letter, about two spaces, and two spaces to the right of the first part; descend on nearly



the connective slant, with first a left and then a right curve, to the center of the letter; then, forming a small loop pointing to the left and encircling the first down stroke of the letter, descend with a vertical line, and finish with a graceful curve. Be sure to use both a left and a right curve in the first stroke of the second part of the letter. Many fail to do this, and use a right curve all the way down to the small loop, and thus completely spoil the letter. Be sure that the little loop points across the first part of the letter at right angles, or you will not be able to get the last down stroke in its proper place.

73. The down stroke in this letter must be vertical.

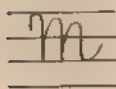


Note the wide hook at the top, and study the position and formation of the small horizontal loop at the bottom. See that the finishing line is heavily curved and parallel with the base line.

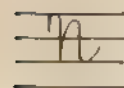
The most common points of failure are in the down stroke and horizontal loop.

74. The main points to observe in this letter are that the top of the second down stroke is lower than the top of the first, and that the top of the third down stroke is lower than the top of the second; also, that the down strokes in these lines are vertical, parallel, and a little less than one space apart.

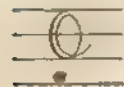
Let the turns at the top be short but round, the last turn at the bottom the same, and end with a full right curve.



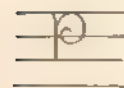
75. The letter *N* is the same as the first and the third part of the *M*. The same things may be said regarding the special points to observe. The downward strokes in this letter should be about three-fourths of a space apart, and parallel. The turns should be short but full. Many fail on this letter by making it too wide. Do not do this.



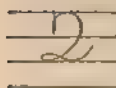
76. The two most common errors in making the letter *O* are the making of the first down stroke too straight and the turn at the base too short. Be sure to give the same degree of curvature to both sides of the letter. Make the letter round and without a break; a sharp point in it destroys the appearance of the letter. If this letter is shaded, it should be on the first down stroke, and the ends of the shade should be nicely tapered.



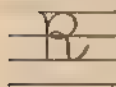
77. The height of the capital stem in the letter *P*, as in all others in which it appears, is two spaces. The oval at the right should be nearly round, about one and one-half spaces in diameter, and finished with a dot a short distance from the stem. This letter to the finishing dot is like *B* and *R*. Study these three letters together and compare them carefully.



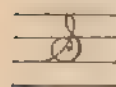
78. The first part of the letter *Q* is the same as the figure 2, but larger. Add the horizontal loop one space in length, cross the main down stroke, touch the base line, and finish with a right curve. Study the first part very carefully. Be sure that the long diameter of the small loop is parallel with the ruling, and that the finishing line is an easy curve.



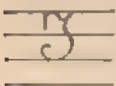
79. The height of the stem in the letter *R* is the same as in the other letters in which it appears. This letter, as far as the small loop, is nearly identical with the letter *P*. The small loop, at one-half the height of the letter, should point across the down stroke at an angle. Descend from the loop to the base line with a straight stroke, touch the line, and finish with a full right curve.



80. The usual difficulty with the letter *S* is to get enough curve to the up stroke, also to get enough curve to the down stroke. Make these curves very full and you will not have very much trouble with this letter. See that the lines cross at about one and one-fourth spaces high. Keep all the shade below the crossing point; the nearer the line you can get it, the better will be the effect. Finish the letter with an oval and dot as shown.

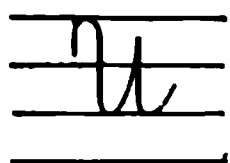


81. The two parts of the letter *T* are the capital stem and the top. Make the capital stem just as described for the letter *F*, also the top. The horizontal part of the top should be nearly parallel with the line of writing. Note the position of the shade and the ending of the stem. This is a difficult letter; give it plenty of study and practice.

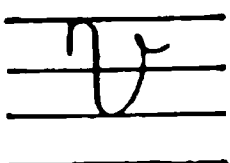


82. The first part of the letters *U*, *V*, and *W* are alike. The most difficult part of these letters is the down stroke.

Form the hook, giving the oval a full curve; descend with a straight line nearly to the base line, make a short turn, and ascend with a right curve one and one-half spaces high and one space to the right of the first down stroke; then descend with a straight line to the base line, turn short, and finish with a right curve. See that the main down stroke is straight for two full spaces. Confine the shade to this straight line; it is quite a common error to shade too high. See that the down strokes are parallel and that the second down stroke does not retrace the up stroke.

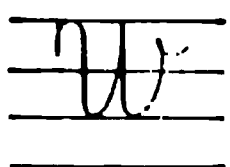


83. Form the first half of the letter *V* to the base line exactly like the *U*; then turn short and ascend with a right curve to one and one-half spaces above the base line; finish with a short horizontal curve to the right. The width of this letter at one-half its height is one space. See



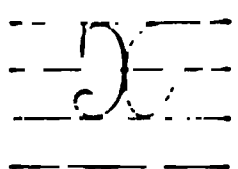
that the down stroke is straight and vertical; it is a common error to make it curved.

84. This is the letter *W* with the second down stroke as high as the first and the last part of a *V* added.



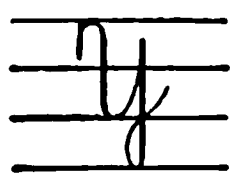
Both down strokes should be straight and vertical. Study and compare this letter and the *U* and *V*, carefully noting their proportions and points of similarity. Do not make a loop in the middle of this letter; it spoils it.

85. This capital letter is made exactly like the small letter *x*. Note the full curves in each part; study



how the parts differ in form. Give special attention to the beginning and the ending hooks, dots, and curved lines. See that the second down stroke meets but does not cross the first down stroke. This is a difficult letter and needs careful study.

86. The letter *Y* is the letter *U* with an inverted *l* loop added. See that both down strokes are vertical.



The last up stroke should cross the long down stroke at the base line. Watch the shades and the turn at the bottom of the first down stroke.

For other points to observe in this letter, see the capital *U* and small *y*.

87. The first part of the letter *Z* is nearly the same as the first part of the letter *Q*, except that the down stroke is not carried quite so far to the left.



Form the loop, then turn to the right and descend with a slight right curve one space; then ascend with a full left curve, crossing the down line at the base line and finishing at one space high.

the base line and finishing at one space high.

LESSON PAPERS.

LESSON NO. 1.

Vertical writing takes its name from the vertical down strokes which appear in almost every letter. Special care should, therefore, be taken in this style of writing to see that the down strokes are made as nearly vertical as possible. If, however, better writing can be done with the down strokes slanting slightly forwards or backwards, it is allowable, and such writing will be accepted by us and graded according to its quality.

In practicing the exercises in this Lesson, be sure that your position is correct and that your arm moves easily and freely, for no good writing can be done in a cramped position.

Line No. 1.—The characters in this line are the figure *1* and the small letters *i* and *u*. In making the figure *1* see that it is at right angles to the line on which the writing is being done. Make all the lines in the *i* and the *u* of the same weight or strength, and keep the down strokes in the *u* parallel, the same height, and vertical. See that the dot

LESSON I.

1 | | | | | i i i i i u u u u u u u u u

2 n n n n n m m m m m m m m m m m m m

3 u u u u u w w w w w w w w w w w w w

4 i u i n i u i u n i u i n i u i n i u i

5 u i n u i n u i n u i n u i n u i n u i n u i n

for *i* is directly over the down stroke and about one space above it.

Line 2.—Study the *m* and *n* carefully. Read what is said about these letters under Detailed Instructions in the Instruction Paper. Note what strokes are straight, what parts are sharp points, and where the round turns occur.

Try to write with a free, rolling motion, so that the tops of *m* and *n* will have somewhat the appearance of folded ribbon, and so that the lines will have that smooth and graceful appearance which is so pleasing in good writing.

Line 3.—Turn to the Detailed Instructions for *v* and *w*, in the Instruction Paper, and study what is said about these letters and try to carry it out in your practice. These are not particularly difficult letters, but need careful study and considerable practice.

Lines 4 and 5.—All the exercises in these lines are combinations of the letters previously introduced. These combinations are of frequent occurrence, and should have careful study. Try to write them in such a manner as to give the writing a neat and pleasing appearance. In such words as *nin*, *unn*, etc., the more like a folded ribbon you can make the writing look, the better it will be. Study the figures carefully and practice from those on the side of the copy-slips. Good figures are a necessity in all kinds of business, and the ability to make them well and quickly is a valuable accomplishment.

LESSON NO. 2.

The exercises in this Lesson are a drill on extended letters, both above and below the line. Study each letter carefully, and in making them follow closely the Detailed Instructions, as given for each in the Instruction Paper.

Line 1.—See that the down stroke in *t* is vertical for its entire length. Note where the cross line is placed; be sure to have it cross the letter instead of being placed to one

side of it. The loop in *l* should be open and clear. Do not make this letter too tall; it is usually made the same height as *t*, although it may be made a little taller if desired.

Line 2.—The letter *j* is quite easy to make; see that the down stroke is vertical, that the loop is open, and that the dot is directly over the down stroke and nearly one space above it. The most difficult part of the *k* is the small horizontal loop in the last part of the letter. Study carefully its position and shape, also the Detailed Instructions for making it, in the Instruction Paper.

Line 3.—The letters *h* and *b* are the same height, and to the bottom of the first down stroke they are alike. The bottom, or last, part of the *h* should be like the strokes of the small *n*. Take pains with the last part of the *b*; it should be open, and like the last part of a *v*.

Line 4.—Study the *f* carefully. Note how the last part connects with the first, and see that the down stroke is straight for its entire length. Study the *y* carefully, and compare it with an inverted *h*; see that both down strokes are parallel and vertical.

Line 5.—All down strokes should be vertical and the same distance apart. Study carefully the height and spacing; also, take pains to have the sharp points and round turns in their proper places.

Use a free movement in all your work, and try to get good, smooth lines. Write at a good rate of speed; it will make your writing smooth and give it that easy, flowing appearance so pleasing in good writing.

LESSON NO. 3.

Line 1.—Be sure that all the *o*'s are round, and where they connect do not allow the down stroke to retrace the connecting line. Note the small hook in the top of the *c*. Be careful not to make this hook in such a manner as to make the letter look like *e*. Do not close the letter too

LESSON III.

- 1 a a a o o o c c c c c c e e e e e e
- 2 a a a a a a d d d d d d q b o 7 q b o 7
- 3 q q q q q q q q q q q q q q q receive
- 4 good a o l a n g c a u c u s c o c a i n e f e e d e r e g g
- 5 b a n a n a a c a c i a d e d u c t e d b a g g a g e q u o t e

much, or it may be mistaken for *o*. The *e* in vertical writing may be made as illustrated, or as in the slanted style. In the copy here given, the last stroke of the preceding letter touches the *e* nearer the top than where the loop begins. This makes it necessary for us to take the pen off at the end of the connecting stroke and begin again at the point where the loop begins, and passing upwards, then downwards, forming the loop and connecting the letter with the last stroke of the preceding one. Practice both styles of this letter, and see which you like better, then use that style in your work.

Line 2.—The *a* and the bottom of the *d* are alike. See that the body of the *a* is round, and that the down stroke is vertical. The down stroke in *d* should be straight for its entire length. The down stroke in the figures 9, 6, and 7 should also be straight and vertical. The ovals in 9 and 6 should be round and full. The cipher is a full oval but not quite a circle. If a circle is used for the cipher it should rest on the line, and be about the size of the oval in the 9. Study the top of 7 very carefully. If it is found very difficult to make the double curve in the horizontal stroke, then this stroke may be made straight. Care must be taken to make the figures plain and legible.

Line 3.—The top of the *g* and the *q* is made like *a*, and the bottom of the *g* is like an inverted *l*. The down stroke in *q* is like that in *g*, but the loop is turned to the right-hand instead of the left-hand side. Both letters require considerable practice.

Lines 4 and 5.—The words in these lines are made up of letters previously given in these Lessons. Each letter should appear to stand by itself, the connecting lines being made in such a way as to give the letters the appearance of having been written separately and then connected together. Study carefully where sharp points are to be used and where round turns should appear. Make each letter and part of a word at the same rate of speed. Keep all lines of the same strength. Make all loop letters open and plain. Be careful not to make a loop in a letter where none is needed. Always

LESSON V.

aha CC Chicago G G Gardiner

hany EE Elmira D D Detroit

via BBB Buffalo RRR Reading

hour too soon than a minute later.

penmanship must be well written

write with a free movement and at a good rate of speed; this will make the lines smooth and give a pleasing appearance to the writing.

LESSON NO. 4.

Line 1.—The first part of *x* and *z* and the last part of *p* are quite similar. In the *x* note how the two parts join, and how the curves at the top and bottom separate. See that the connecting line between the letters is a compound curve, and that in making it you do not separate the letters too much. The loop in the *z* should be but one space long. Study its position carefully, and in making it see that both sides have the same amount of curvature.

Line 2.—Study what is said in the Instruction Paper regarding *r* and *s*, and try to carry it out in your practice on these letters. Both these letters are rather difficult to make, and must, therefore, have a great deal of careful study.

Lines 3 and 4.—The words in these lines are rather difficult combinations; therefore, study them carefully and compare each letter with the Detailed Instructions for that letter as given in the Instruction Paper. Note how each letter is, connected with the others, and watch the height and the spacing.

Line 5.—This sentence brings into use quite a variety of words, and is one that will produce very good results if given proper study and practice. Pay special attention to the loop letters, and take pains to have all down strokes vertical and parallel. Do your best with this Lesson and let us see just how well you can do. Always use a free movement and write at a good rate of speed, for if you slowly draw the letters you will not get the smooth lines that add so much to the appearance of writing.

LESSON V.

aha CC Chicago G G Gardiner

any EE Elmira D D Detroit

via BBB Buffalo R R R Reading

hour too soon than a minute late.

penmanship must be well written

LESSON NO. 5.

Lines 1 and 2.—All capital letters should be made with a free, swinging movement and at a good rate of speed; smooth lines cannot be had if the lines are slowly drawn with the finger movement. In the words given, do not take the pen off between the capital and the rest of the word; when once you begin a word, write it without stopping until it is finished. Watch the spacing and height of each letter; see that the down strokes are parallel.

Line 3.—There are a great many vertical strokes in this line. Examine your work carefully, and see if you have made them all correctly.

Lines 4 and 5.—In writing these sentences endeavor to make each word look as smooth and graceful as in the copies. This can only be done by writing at a fair rate of speed and making graceful turns at the top of such letters as *m* and *n* and at the bottom of *i* and *u*, and by making the *a* and *o*, and all letters of which they form a part, with an easy, rolling movement. Hold the pen easily, let the arm rest lightly and move freely and without effort. Practice thoughtfully and you will not have any trouble in making presentable work

LESSON NO. 6.

Line 1.—See that the first parts of the *H* and the *K* are alike. Make the tops of the *n*'s and the first part of the *y* round. See that the *o*'s are perfect circles and that the body of the *s* is broad. Study the capital *M* carefully, and see that the down strokes are vertical and parallel.

Line 2.—The first part of *N*, *V*, and *U* are exactly like the first part of *M*. Study the last part of each of these letters carefully, and endeavor to make them as near like the copies as possible. The last word on this line is an excellent copy to practice from, as the small letters in it are

LESSON VI.

Yukie KK Kingston MM Maine
 Uada UU Vermont UU Unadilla
 Ugan Y Yokohama Y Jamaica
 Uings determined the winner Y Y
 Uter of the ninth instant at hand.

quite full and round, and enough extended letters are given to illustrate the height and spacing of this class of letters.

Line 3.—Compare the capitals *W* and *Y* with the letters *U* and *V*. The small *k* is rather a difficult letter and will need careful attention. Study it closely and note what is said regarding it under Detailed Instructions.

Lines 4 and 5. Careful practice on the words in these lines will do much to produce that flowing, rolling movement which makes this style of writing so plain to read and so pleasing in appearance. Always compare your work with the copies, and criticise it carefully. Write at a good rate of speed, and use an easy, rolling movement.

LESSON NO. 7.

Line 1.—The last part of the capital *I* should be a full right curve. The loop in the top of this letter should be vertical. Note how it compares with the loop in the small *i*. The first part of *X* and *Q* are alike. See that the curves in these letters are unbroken and that the lines are smooth. Note how these letters connect with the rest of the word.

Line 2.—The first part of *Z* is almost like that of *X* and *Q*. The difficult parts of this letter are the horizontal loop and the loop below the line. Study the copy carefully, also what is said under Detailed Instructions for this letter. *T* and *F* are exactly alike, except the horizontal stroke across the center of *F*. Review the Detailed Instructions for these letters.

Line 3. See that the hook at the top of *L* is quite broad, that the down stroke is straight and vertical, and that the small loop at the bottom of the letter is parallel with the line of writing.

LESSON VII.

- 1 I I I Illinois X X Xenia 2 2 2 Quebec
- 2 Z Z Zamesville 3 3 Trenton 3 3 Fresno
- 3 L L L Louisiana & & & Scranton X 3 3
- 4 Sacornation on foot becomes monotonous
- 5 A specimen of my business penmanship.

LESSON VIII.

Telegraph operators and reporters are the fastest writers in the world, and they use forms that slant but little, usually not more than ten degrees either forward or backward from the perpendicular. It seems, therefore, that writing with a slight slant is faster and more natural than writing with much slant.

Lines 4 and 5. Each of the words in these sentences should be written from beginning to end without raising the pen. As each word is written, study it carefully to see if all the letters are properly formed and if all straight down strokes are vertical. *o*, *e*, and *a* should be almost perfect circles, and should be made fast enough to get good, smooth lines. Always compare your writing with the copy.

LESSON NO. 8.

The object of this Lesson is to give practice in writing several lines in succession. This is often called *body* writing, and is the severest test of good penmanship. Many people are able to write some letters, or a few words, well, but when a number of lines or a page is written, their inability to write the same good style continuously is forcibly brought to their attention. Writing of the class in this Lesson should be done quite rapidly in order to get smooth, clean-cut lines. Careful attention must also be paid to the forms of the letters, their height, and spacing. The proper use of rounded turns and sharp points undoubtedly adds more to the appearance of writing than any other one thing.

Study the copies carefully, and devote a great deal of thoughtful, conscientious practice to them, being careful that the desire to produce too correct forms does not induce too cramped and slow a movement; otherwise the acquirement of a smooth and rapid handwriting will be very much delayed, if not entirely prevented.

LESSON NO. 9.

This Lesson is a valuable one, as it introduces actual business forms, the correct use of which adds much to the appearance of a written communication.

The first part of this Lesson introduces a letter heading, and not only shows how it should be written, but illustrates the proper arrangement and punctuation. Practice on this

LESSON IX.

Scranton, Pa., Jan 12, 1899.
 Mr W R. Wallace,
 Bristol, Conn.

Dear Sir Your letter of the 5th instant is

Mr. W R. Wallace,
 Bristol,
 14 Ocean Ave. Conn

The Chicago Gas Co.,
 Chicago, Ill.
 Rooms 314-317,
 Masonic Temple.

LESSON X.

 (Your Residence and Date.)

ational Correspondence Schools
ven. This is a specimen of my
ir completing your Course of les-
manship.

Yours truly,

 and Number.

 (Your Name.)

section of the Lesson until you can write it well; then vary it by making such changes as will give you practice on the headings and addresses you most often use. The second part shows the correct style for addressing envelopes. Vary this practice by using such names and addresses as you often write, and then return to the copy.

Put double the amount of study and practice on this Lesson that you have on any of the others, and let us see how much improvement you can make and how well you can do.

LESSON NO. 10.

This is the last Lesson in the Paper, and for it we have selected a copy that is not only valuable because of the excellent practice it affords, but also because it will give you a specimen, prepared under instruction, to compare with that written at the beginning of the course of lessons. Note carefully the arrangement of the different parts of the exercise, paying particular attention to those which you are to add, and endeavor to follow this arrangement in your work.

Only thoughtful, conscientious practice can produce the best results. Apply yourself earnestly, and you will be successful in making satisfactory improvement and acquiring a good handwriting.

A SERIES
OF
QUESTIONS AND EXAMPLES
RELATING TO THE SUBJECTS
TREATED OF IN THIS VOLUME.

It will be noticed that the questions and examples contained in the following pages are divided into sections corresponding to the sections of the text of the preceding pages, so that each section has a headline which is the same as the headline of the section to which the questions refer. No attempt should be made to answer any questions or to work any examples until the corresponding part of the text has been carefully studied.

ARITHMETIC.

- (1) What is arithmetic ?
- (2) In what two ways may numbers be represented ?
- (3) What is a number ? a unit ?
- (4) What is the difference between a concrete number and an abstract number ?
- (5) What is meant by the *order* of a figure ?
- (6) What is the reading of a number called ?
- (7) Express, in both Arabic notation and Roman notation, seven thousand five hundred three.
- (8) For what purpose are ciphers used ?
- (9) What is an integer ?
- (10) Write each of the following numbers in words:
(*a*) 980; (*b*) 605; (*c*) 28,281; (*d*) 9,006,042; (*e*) 850,317,-002; (*f*) 700,004.
- (11) Express in Roman notation the following numbers:
(*a*) 76; (*b*) 353; (*c*) 1,732; (*d*) 1,496; (*e*) 1,888.
- (12) (*a*) What is addition ? (*b*) What is the sign used to indicate addition, and what is it called ?
- (13) Represent in figures the following expressions:
(*a*) Seven thousand six hundred; (*b*) eighty-one thousand four hundred two; (*c*) five million four thousand seven; (*d*) one hundred eight million ten thousand one; (*e*) ten million six; (*f*) thirty thousand ten.

§ 1

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(14) What is the sum of $3,290 + 504 + 865,403 + 2,074 + 81 + 7$?
Ans. 871,359.

(15) Find the sum of $83,027 + 46,928 + 4,769 + 81,987 + 46,729 + 479,897 + 627 + 14,896 + 987,649$.
Ans. 1,746,509.

(16) Multiply 29,800 by 390. Ans. 11,622,000

(17) A man owes \$1,000; he pays \$329 the first year, and \$438 the second year. How much does he still owe?
Ans. \$233.

(18) Find the difference between $23,896 + 4,982 + 96,875 + 59,674$ and $31,627 + 54,892 + 6,925 + 8,976$. Ans. 83,007.

(19) Multiply 8,765 by 987, and from the product subtract $4,695 \times 823$.
Ans. 4,787,070.

(20) The greater of two numbers is 1,004, and their difference is 49. What is their sum? Ans. 1,959.

(21) Find the sums of the numbers composing the columns below:

(a)	(b)	(c)	(d)	(e)
5714	5823	6287	72584	1753862
8965	6986	3575	37682	5713286
7258	2575	6982	49587	9822568
9843	1829	9384	31675	7389837
7867	3746	1629	28579	9485796
5327	3585	7802	34682	2928368
<u>9217</u>	<u>2633</u>	<u>4713</u>	<u>47386</u>	<u>9886377</u>

Ans. $\left\{ \begin{array}{l} (a) \quad 54,191. \\ (b) \quad 26,577. \\ (c) \quad 40,372. \\ (d) \quad 302,175. \\ (e) \quad 46,980,094. \end{array} \right.$

(22) Divide 8,329,678 by 9,692. Ans. $859\frac{4258}{9692}$.

(23) Multiply 986 by 729, and divide the product by 469.
Ans. $1,532\frac{288}{469}$.

(24) What is the value of $87,964,201 \div 849$?
Ans. $103,609\frac{148}{849}$.

(25) A drover bought 36 oxen at \$24 each, and 23 cows at \$96 each. How much did they all cost? Ans. \$3,072.

(26) Find the value of $938 \times 672 \times 497$. Ans. 313,276,992.

(27) From 839,672 subtract 47 times 989. Ans. 793,189.

(28) Solve the following by cancelation:

$$(a) \frac{72 \times 48 \times 28 \times 5}{96 \times 15 \times 7 \times 6} = ? \quad (b) \frac{80 \times 60 \times 50 \times 16 \times 14}{70 \times 50 \times 24 \times 20} = ?$$

$$\text{Ans. } \begin{cases} (a) & 8. \\ (b) & 32. \end{cases}$$

(29) Divide $2,937 \times 864$ by 923. Ans. $2,749\frac{44}{11}$.

(30) A certain number, multiplied by 468 and the product divided by 349, gives a quotient of $1,201\frac{1}{11}$. Find the number. Ans. 896.

(31) Multiply 8,976 by 4,298, and subtract 98,765 from the product. Ans. 38,480,083.

(32) Divide the sum of $8,269 + 86,955 + 42,301 + 287,695$ by 1,492. Ans. 285.

(33) An engine and boiler in a manufactory are worth \$3,240; the building is worth three times as much, plus \$1,200, and the tools are worth twice as much as the building, plus \$1,875. (a) What is the value of the building and tools? (b) What is the value of the whole plant?

$$\text{Ans. } \begin{cases} (a) & \$34,689. \\ (b) & \$37,935. \end{cases}$$

(34) In the following examples, subtract the upper numbers from the lower:

(a)	(b)	(c)	(d)
43259867	557351092	9877166431	969798
<u>60095001</u>	<u>842196340</u>	<u>10009876542</u>	<u>43932176</u>

$$\text{Ans. } \begin{cases} (a) & 16,835,134. \\ (b) & 284,845,248. \\ (c) & 132,710,111. \\ (d) & 42,962,378. \end{cases}$$

(35) The divisor is 1,389, the quotient is 748, and the remainder is 1,263. What is the dividend? Ans. 1,040,235.

(36) The multiplicand is 4,896 and the product, 3,862,944. Find the multiplier. Ans. 789.

(37) In the two examples below, add the two columns of figures belonging to each example, and find their difference by the method described in Art. 56.

(a)		(b)	
54085	38509	847593	668721
72342	19714	288792	967842
88700	38887	989341	476976
11643	53141	540623	887674
<u>29872</u>	25167	249623	<u>694593</u>
	76905	474954	
	<u>28788</u>	<u>313671</u>	

Ans. $\begin{cases} (a) & 24,469. \\ (b) & 8,791. \end{cases}$

(38) A man left \$25,375 to his widow, and \$12,450 to each of his seven children. How much did they all receive?

Ans. \$112,525.

(39) If I sell my farm for \$375 an acre, I would get \$31,734 more for it than if it sold for \$246 an acre. How many acres in the farm?

Ans. 246 acres.

(40) Solve the following by cancelation:

$$(a) \frac{231 \times 96 \times 192 \times 45}{11 \times 24 \times 48 \times 15 \times 7} = ?$$

$$(b) \frac{42 \times 64 \times 125 \times 98 \times 144}{14 \times 16 \times 25 \times 49 \times 36 \times 20} = ?$$

Ans. $\begin{cases} (a) & 144. \\ (b) & 24. \end{cases}$

ARITHMETIC.

- (1) Find the sum of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, and $1\frac{1}{2}$. Ans. $4\frac{1}{2}$.
- (2) What is the value of $3\frac{3}{4} + 6\frac{7}{8} - 4\frac{5}{8}$? Ans. $5\frac{1}{2}$.
- (3) A man bought $12\frac{3}{4}$ tons of coal. How much was left after he had burned $5\frac{1}{2}$ tons? Ans. $6\frac{1}{2}$ tons.
- (4) How many dresses can be made of $53\frac{1}{2}$ yards of gingnam, if each dress contains $6\frac{3}{4}$ yards? Ans. 8 dresses.
- (5) Find the product of $83\frac{2}{3} \times 67\frac{1}{3}$. Ans. 5,672.
- (6) Divide $4\frac{1}{2} \times 6\frac{3}{4}$ by $1\frac{1}{2}$. Ans. $16\frac{1}{2}$.
- (7) How many loads of wood, each consisting of $1\frac{1}{2}$ cords, are there in 56 cords? Ans. 32 loads.
- (8) At \$ $3\frac{7}{8}$ a yard, what must be paid for 24 yards of velvet? Ans. \$93.
- (9) Simplify $\frac{4\frac{7}{8} \times 3\frac{2}{3} \times 3\frac{1}{5}}{1\frac{5}{8} \times 4\frac{2}{5}}$. Ans. 8.
- (10) Multiply $8\frac{1}{20}$ by $\frac{1}{40}$. Ans. $\frac{1}{10}$.
- (11) Divide $43\frac{5}{7}$ by $5\frac{2}{7}$. Ans. $8\frac{1}{2}$.
- (12) Subtract from 100 the sum of $8\frac{3}{4}$, $7\frac{5}{8}$, and $10\frac{1}{2}$. Ans. $72\frac{1}{2}$.
- (13) A cask containing 40 gallons leaks at the rate of $3\frac{3}{4}$ gallons an hour. In how many hours will it be empty? Ans. $10\frac{2}{3}$ hr.
- (14) Find the value of $\frac{65}{100} \times 4\frac{2}{3} \times 6\frac{1}{2}$. Ans. $19\frac{1}{2}$.

§ 2

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(15) The circumference of a wheel is $\frac{25}{11}$ times its diameter. What is the circumference of a wheel whose diameter is $8\frac{2}{3}$ feet? Ans. $27\frac{17}{33}$ ft.

(16) What does a man get per day who earns $\$97\frac{1}{2}$ in 26 days? Ans. $\$3.75$.

(17) A company of soldiers consume $165\frac{1}{4}$ pounds of meat, which is an allowance of $1\frac{1}{2}$ pounds each. How many soldiers are in the company? Ans. 102 soldiers.

(18) A father and his son together weigh 312 pounds. How heavy is each, if the son weighs $\frac{2}{5}$ as much as the father?

Ans. $\left\{ \begin{array}{l} \text{Son, } 120 \text{ lb.} \\ \text{Father, } 192 \text{ lb.} \end{array} \right.$

(19) What is the value of $8\frac{1}{2} \times 1\frac{1}{4}$? Ans. 11.

(20) Find the diameter of a wheel having a circumference of $28\frac{1}{2}$ feet. (See example 15.) Ans. $9\frac{51}{16}$ ft.

(21) In a field of $4\frac{1}{2}$ acres there were raised 96 bushels of wheat. What was the average number of bushels per acre? Ans. 20 bu.

(22) How many yards of cloth can be bought for $\$91$ at $\$2\frac{1}{2}$ per yard? Ans. 104 yd.

(23) How much is $4\frac{1}{2}$ times $\frac{2}{3}$ of $6\frac{1}{2}$? Ans. $20\frac{7}{7}$.

(24) Find the cost of $24\frac{1}{2}$ tons of hay at $\$23\frac{1}{2}$ a ton. Ans. $\$587\frac{1}{2}$.

(25) Three men in partnership gain $\$3,696$. In dividing the profits, one gets $\frac{1}{3}$ of the amount, the second gets $\frac{2}{5}$ of the remainder, and the third gets what still remains. Find the share of each. Ans. $\$1,232$; $\$985\frac{1}{5}$; $\$1,478\frac{2}{5}$.

(26) If 25 turkeys cost $\$18\frac{3}{4}$, what was the cost of one turkey? Ans. $\$3\frac{3}{4}$.

(27) A man paid $\$357\frac{1}{2}$ for $9\frac{3}{4}$ acres of land. What did he pay per acre? Ans. $\$36\frac{3}{4}$.

(28) The sum of two numbers multiplied by $18\frac{1}{2}$ is $296\frac{1}{2}$. One of the numbers is $12\frac{1}{4}$. Find the other number. Ans. $3\frac{1}{2}$.

(29) If the difference between $28\frac{1}{2}$ and $14\frac{5}{8}$ be multiplied by a certain number, the product will be $49\frac{1}{8}$. Find the multiplier. Ans. $3\frac{3}{8}$.

(30) By how much must $20\frac{2}{3}$ be increased so that the sum shall be contained in $114\frac{3}{8}$, $3\frac{3}{4}$ times? Ans. $9\frac{5}{8}$.

(31) How many days will it take a man to cut 45 cords of wood, if he can cut $3\frac{3}{4}$ cords per day? Ans. 12 da.

(32) At $\$6\frac{3}{4}$ per barrel, how many barrels of flour can be bought for $\$81$? Ans. 12 bbl.

(33) If $12\frac{7}{8}$ times the difference between two numbers is $41\frac{1}{5}$, and one of the numbers is 21, what is the other number? Ans. $24\frac{1}{5}$ or $17\frac{4}{5}$.

(34) What is the circumference of a wheel that goes 108 feet in turning $5\frac{3}{4}$ times? Ans. $18\frac{1}{2}\frac{3}{8}$ ft.

(35) Simplify $\frac{4\frac{2}{3} \times 5\frac{1}{7} \times 9\frac{5}{8}}{3\frac{1}{2} \times 8\frac{5}{14}}$. Ans. $7\frac{35}{611}$.

(36) Multiply $42\frac{3}{8}$ by $19\frac{1}{2}$. Ans. $822\frac{3}{2}$.

(37) Add $\frac{1}{8}$, $\frac{1}{4}$, $\frac{2}{5}$, and $\frac{5}{8}$. Ans. $1\frac{49}{80}$.

(38) From $67\frac{5}{8}$ take $48\frac{3}{4}$. Ans. $18\frac{3}{4}$.

(39) If a man travels $85\frac{5}{12}$ miles in one day, $78\frac{9}{5}$ miles the next day, and $125\frac{1}{5}$ miles the third day, how far did he travel in the three days? Ans. $289\frac{11}{20}$ mi.

(40) Bought $211\frac{1}{4}$ pounds of old lead for $1\frac{7}{8}$ cents per pound. Sold a part of it for $2\frac{1}{2}$ cents per pound, receiving for it the same amount as I paid for the whole. How many pounds did I have left? Ans. $52\frac{1}{8}$ lb.

ARITHMETIC.

- (1) Express $\frac{17}{32}$ as a decimal fraction. Ans. .53125.
- (2) Change $2\frac{1}{2} \times 3\frac{1}{2} \div 8$ to a decimal. Ans. 1.0417—.
- (3) Change .285714 to a common fraction in its simplest form. Ans. $\frac{2}{7}$.
- (4) Reduce 2.0418 to a mixed number. Ans. $2\frac{23}{250}$.
- (5) By the method of aliquot parts, multiply 448 by $62\frac{1}{2}$.
Ans. 28,000.
- (6) Find the wages of 25 men for 16 days at \$2.75 per day.
Ans. \$1,100.
- (7) How many tons of coal at \$2 87 $\frac{1}{2}$ a ton should be received in exchange for 128 cords of wood at \$1.62 $\frac{1}{2}$ a cord, and \$436 in money? Ans. 224 T.
- (8) A dealer bought peaches at \$1.87 $\frac{1}{2}$ a basket, and sold them so as to lose \$12.50 on 100 baskets. How much did he receive per basket? Ans. \$1 75.
- (9) Reduce to decimals and add the following fractions:
 $\frac{1}{4}, \frac{1}{6}, \frac{1}{11}, \frac{1}{12}$. Ans. 2.84375.
- (10) If the diameter of the earth is 7,912 miles, and its circumference is 3.1416 times as much, how many miles are in its circumference? Ans. 24,856.3392 mi.
- (11) How many yards of cloth at \$2.37 $\frac{1}{2}$ a yard can be bought for \$133? Ans. 56 yd.
- (12) Find the value of $\frac{1}{3}$ of $\frac{2}{3}$ of .0168. Ans. .00672.

$\frac{8}{3}$

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(13) If $\frac{3}{4}$ of a ton of hay cost \$14 $\frac{1}{2}$, how much will a ton cost? Ans. \$19.

(14) The sum of .37 of a number and .23 of the same number is 33.6. Find the number. Ans. 56.

(15) If $\frac{2}{3}$ of a number be subtracted from .8 of the number, the remainder is 87.8. What is the number? Ans. 658.5.

(16) If \$8.20 more is paid for 14 $\frac{1}{2}$ yards of cloth than for 9 $\frac{1}{2}$ yards, what is the price per yard? Ans. \$1.60

(17) How many feet in .5625 of a mile, there being 5,280 feet in one mile? Ans. 2,970 ft.

(18) Change 4,620 feet to the decimal of a mile. Ans. .875 mi.

(19) A falling body falls 16.083 $\frac{1}{3}$ times as many feet as the square of the number of seconds during which it is falling. Find the distance through which it will fall in 6 seconds. Ans. 579 ft.

(20) How long must a man work at \$3.87 $\frac{1}{2}$ per day to earn \$155? Ans. 40 da.

(21) The circumference of a wheel is 18 $\frac{1}{2}$ feet; how many times will it turn in going one mile? Ans. 288 times.

(22) If .45 of the price of a farm was \$4,230, what was its price? Ans. \$9,400.

(23) If 18.75 yards of sheeting were bought at \$.18 $\frac{3}{4}$ a yard, what did it cost? Ans. \$3.5156, or \$3.52.

(24) Find the cost of 6 barrels of flour at \$6.375 a barrel, 28 bushels of potatoes at \$.875 per bushel, and 120 pounds of sugar at \$.03 $\frac{1}{2}$ a pound. Ans. \$66.50.

(25) A lot cost \$3,300, and this was .165 of the cost of the house erected upon it. Find what was paid for the house. Ans. \$20,000.

(26) A man whose daily pay was \$1.87 $\frac{1}{2}$ had his wages advanced to \$2.25. Counting 300 working days in a year, find the annual increase in his salary. Ans. \$112.50.

(27) A man paid .143 of his annual salary to his butcher, .347 of it to his grocer, .256 of it for clothing, and .154 for other expenses. How much did he save, if his butcher received \$327.47? Ans. \$229.

(28) At \$18.75 a ton what must be paid for 23.75 tons of hay? Ans. \$445 31+.

(29) What will be the cost of 37.625 M shingles at \$7.28 per M? Ans. \$273 91.

(30) A farm was bought at \$87.50 per acre and sold at \$112.50, by which there was a gain of \$3,670. How many acres were there? Ans. 146.8 A.

(31) If $\frac{1}{8}$ of my crop of wheat is 368.5 bushels, how many bushels are in the whole crop? Ans. 2,926.75 bu.

(32) Find the cost of 51 pairs of shoes at \$3.37½ per pair.
Use method of aliquot parts. Ans. \$172.125.

(33) Cast iron is 7.21 times as heavy as water. Assuming that a cubic foot of water weighs 62½ pounds, what will be the weight of 4.86 cubic feet of cast iron? Use method of aliquot parts. Ans. 2,190.0375 lb.

(34) If a cubic inch of water weighs .03617 pound, what will be the weight of 231 cubic inches? Ans. 8.355+ lb.

(35) Using the method of aliquot parts, divide (a) 475 by $19\frac{1}{2}$; (b) 25 by $16\frac{2}{3}$.
 Ans. $\begin{cases} (a) & 38. \\ (b) & 1\frac{1}{2}. \end{cases}$

(36) Reduce $\frac{5}{13}$ to a repeating decimal. **Ans.** $.38461\bar{5}$.

(37) Reduce $.35\bar{1}$ to a common fraction. Ans. $\frac{13}{37}$.

(38) Change .30208 $\bar{3}$ to a complex fraction whose denominator is 24. Ans. $\frac{7\frac{1}{2}}{24}$.

ARITHMETIC.

- (1) How many inches in 83.75 yards? Ans. 3,015 in.
- (2) Change .67 of a mile to integers of lower denomination. Ans. 214 rd. 2 yd. 7.2 in.
- (3) How many telegraph poles 120 feet apart will be required for a distance of 15 miles? Ans. 660 poles.
- (4) How many miles apart are two points on the earth's equator, one being at $23^{\circ} 28'$ west longitude, and the other at $5^{\circ} 22'$ east longitude? (See example 16.) Ans. 1,994.11 mi.
- (5) How many yards of lining $\frac{7}{8}$ of a yard wide will be required to line a piece of goods 28 yards long and $\frac{3}{4}$ of a yard wide? Ans. 24 yd.
- (6) What must be paid for a rectangular block of granite, its dimensions being 3 ft. \times $4\frac{1}{2}$ ft. \times 6 ft., at $\$1.37\frac{1}{2}$ per cubic foot? Ans. $\$111.375$.
- (7) Allowing 1 bunch of laths for each 3 square yards of wall surface, what must be paid at $\$.37\frac{1}{2}$ a bunch to lath the four walls and the ceiling of a room 20 ft. \times 30 ft., and 12 feet high, no deduction being made for openings? Ans. $\$25$.
- (8) Find to four decimal places the number of times that 2 ft. 8 in. must be repeated to be equal to the circumference of a flywheel 12 feet in diameter. Ans. 14.1372.
- (9) Change $\frac{5}{8}$ of a year (365 da. 5 hr. 48 min. 49.7 sec.) to integers of lower denomination. Ans. 228 da. 6 hr. 38 min. 1.0625 sec.

§ 4

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(10) How many acres in a rectangular field 48 rd. \times 72 rd.?

Ans. 21 6 A.

(11) How many square chains in 12.625 A.?

Ans. 126.25 sq. ch.

(12) If 5 T. 875 lb. of coal cost \$9.13 $\frac{1}{2}$, how much is it a ton? Use the long ton.

Ans. \$1.69.

(13) A fish weighing 5 lb. 12 oz. was bought for \$1.15. What was the price per pound?

Ans. \$.20.

(14) How many times will a wheel of a bicycle turn in going 3.5 miles, its diameter being 28 inches?

Ans. 2,521.008+.

(15) How many 8-inch furrows, each 40 rods long, are equivalent to an acre?

Ans. 99.

(16) Assuming that a degree of latitude is 69 16 miles, how far from the equator is a place whose latitude is 42° 20' 30"?

Ans. 2,928.3497—mi.

(17) A cubical cistern is 8 ft. 6 in. in each of its three dimensions. How many gallons will it hold?

Ans. 4,594—gal.

(18) Find the exact time in years and days between Jan. 14, 1898, and July 23, 1900.

Ans. 2 yr. 190 da.

(19) What decimal fraction of an entire circle in 60° 40' 30"?

Ans. .1685.

(20) How many bushels will a bin contain whose dimensions are 6 ft. \times 6 $\frac{1}{2}$ ft. \times 8 ft.?

Ans. 250.7+ bu.

(21) How many doses of opium each of 3 $\frac{1}{2}$ gr. are there in an avoirdupois pound?

Ans. 2,000.

(22) Multiply 3 lb 39 35 92 gr.15 by 9.

Ans. 34 lb. 33 35 gr.15.

(23) A family used a quantity of sugar at the rate of 8 $\frac{1}{2}$ ounces per day and it lasted 80 days. How many pounds were there in all?

Ans. 43 $\frac{1}{2}$ lb.

(24) Divide 75 rd. 4 yd. 2 ft. 8 in. by 36.

Ans. 2 rd. 0 yd. 1 ft. 9 $\frac{2}{3}$ in.

- (25) Find the difference between 25 ft. square and 75 sq. ft.
Ans. 550 sq. ft.
- (26) A rectangular stick of lumber 36 ft. long and 10 in. thick contains 40 cu. ft. How wide is it? Ans. 16 in.
- (27) A vessel having a capacity of 100 bushels will hold how many gallons? Ans. 930.92— gal
- (28) By the ordinary method of subtracting denominate numbers, find the age of a man born Nov. 9, 1842, who died Apr. 2, 1889. Ans. 46 yr. 4 mo. 23 da.
- (29) (a) How many square feet in a board 18 75 ft. long and 2 ft. 10 in. wide? (b) What will it cost at 8 cents a square foot?
Ans. $\begin{cases} (a) & 53\ 125 \text{ sq. ft.} \\ (b) & \$4.25. \end{cases}$
- (30) (a) What fractional part of 63 gallons is 18 gal. 3 qt. 1 pt.? (b) What decimal part? Ans. $\begin{cases} (a) & \frac{11}{14}. \\ (b) & .2996+. \end{cases}$
- (31) What is one of 7 equal parts of a solar year?
Ans. 52 da. 4 hr. 15 min. 32.81+ sec.
- (32) A grocer bought 6 bushels of cranberries and sold them at 25 cents for 3 quarts. His gain was \$4. What did they cost him per bushel? Ans. \$2.
- (33) A student spent 8 hours and 40 minutes daily in study for 4 years. How much time did he study in all, if school was in session 5 days each week for 40 weeks each year? Ans. 6,933½ hr.
- (34) From £21 5s. 3d. take £13 9s. 6d. Ans. £7 15s. 9d.
- (35) Find the sum of 21 A. 67 sq. ch. 3 sq. rd. 21 sq. li.; 28 A. 78 sq. ch. 2 sq. rd. 23 sq. li.; 47 A. 6 sq. ch. 7 sq. rd. 18 sq. li.; 56 A. 59 sq. ch. 3 sq. rd. 16 sq. li.; 25 A. 38 sq. ch. 3 sq. rd. 23 sq. li.; 46 A. 75 sq. ch. 2 sq. rd. 21 sq. li.
Ans. 255 A. 4 sq. ch. 4 sq. rd. 122 sq. li.
- (36) If 16 square miles be equally divided into 62 farms, how much land will each farm contain?
Ans. 165 A. 25 sq. rd. 24 sq. yd. 3 sq. ft. 80+ sq. in.

(37) How many bushels of apples are contained in 9 barrels, if each barrel contains 2 bu. 3 pk. 6 qt.?

Ans. 26.4375 bu.

(38) What is the sum of $11^{\circ} 16' 12''$; $13^{\circ} 19' 30''$; $20^{\circ} 25''$; $26' 29''$; $10^{\circ} 17' 11''$?

Ans. $55^{\circ} 19' 47''$.

(39) Reduce 4,763,254 links to miles.

Ans. 595 mi. 32 ch. 2 rd. 4 li.

ARITHMETIC.

(1) How many kilos of water will a tank contain that is 3 m. \times 2 m. \times 3 m. ? Ans. 18,000 K.

(2) How many hectares of land in a rectangular field 300 meters long and 200 meters wide ? Ans. 6 Ha.

(3) Find the cost of 1.5 tonnes of sugar at $2\frac{1}{2}$ cents a pound. Ans. \$82.67+.

(4) How much will 25.875 kilograms of opium cost at 10 cents a grain ? Ans. \$39,930.30.

(5) At 25 cents a square yard, find how much must be paid to plaster a ceiling 12 meters long and 10 meters wide. Ans. \$35.88.

(6) What would be the value of the 5-cent nickel coins that, placed side by side, would make a row 50 meters long, the diameter of the 5-cent nickel coin being 2 centimeters ? Ans. \$125.

(7) How many centimeters in 18.725 meters ? Ans. 1,872.5 cm.

(8) Find the cost, at 12 cents a quart, of 1,000 liters of cranberries. Ans. \$108.94.

(9) How many cubic meters in a rectangular block of marble 3 meters long, 2 meters wide, and 1.5 meters high ? Ans. 9 cu. m.

(10) Change 56.8 meters to yards. Ans. 62.1+ yd.

(25) Find the sum of 2.75 K., 439.17 Dg., and 38,295 g., and express the result in hektograms. Ans. 453.367 Hg.

(26) Find the number of liters in 100 gallons of water.
Ans. 378.546— l.

(27) A gold eagle of the United States weighs 258 grains Troy; what would be the value of a kilo of these coins?
Ans. \$598.14—.

(28) If a cubic foot of marble weighs 168.75 lb., what will be the weight of a marble block, the dimensions of which are 2.5 m. \times 2.8 m. \times 3.75 m.?
Ans. 156,432 lb., nearly.

(29) Find the square root of 4,322,241. Ans. 2,079.

(30) What is the value of $\sqrt{7,921} \times \sqrt{9,604}$? Ans. 8,722.

(31) Find, to four decimal places, the square root of 19.
Ans. 4.3589.

(32) Bought a farm of 200 hectares at \$250 a hektare, and sold it at \$150 an acre. Find the gain or loss.
Ans. \$24,130 gain.

(33) A wall 23.5 m. long, .5 m. thick, and 2.4 m. high was built at \$5.25 per m³. How much did it cost?
Ans. \$148.05.

(34) Find the cube root of $\frac{1}{3}$, to five decimal places.
Ans. .95647.

(35) Multiply the cube root of 571,787 by the square root of 15,625. Ans. 10,375.

(36) What is the nearest whole number of city lots, 25 ft by 100 ft., that are equivalent to a hektare of land?
Ans. 43 lots.

(37) A rectangular field 100 m. long contains 1 acre. How wide is it?
Ans. 8 0488— rd.

(38) A fish weighing 50 Kg. was bought at 30 cents a Kg. and sold at 18 cents a pound. Find the gain or loss.
Ans. \$4.84+ gain.

ARITHMETIC.

(1) Find the area of a parallelogram having a base of 12 ft. 8 in. and an altitude of 8 ft. 6 in. Ans. $107\frac{2}{3}$ sq. ft.

(2) A rectangle has a base of 35 rd. and an area of $3\frac{1}{2}$ A. Find the altitude. Ans. 16 rd.

(3) A square has the same area as a rectangle having a base of $56\frac{1}{2}$ rd. and an altitude of $16\frac{1}{2}$ rd. Find the side of the square. Ans. $30\frac{1}{2}$ rd.

(4) The perimeter of a room is 140 ft., and its width is $\frac{2}{3}$ of its length. What are its dimensions? Ans. $30' \times 40'$.

(5) What is the length in feet of a side of a square field that has an area of 1 acre? Ans. $208.7+$ ft.

(6) Making no allowance for openings, what must be paid for plastering the sides and ceiling of a room 24 ft. 6 in. by 30 ft. 8 in. and 12 ft. high, at 25 cents per square yard? Ans. \$57.65.

(7) What will it cost, at 28 cents a square yard, to sod a lawn 60 yd. long and 120 ft. wide? Ans. \$672.

(8) The perimeter of a field is 600 yd., and its length is 80 yd. more than its width. How many acres in the field? Ans. $4.32-A$.

(9) The perpendicular of a right-angled triangle is 55.7 ft. long, and its hypotenuse is 100.9 ft. long; find the length of the base. Ans. $84.13+$ ft.

(20) Find the volume of a frustum of a cone, the diameter of its lower base being 20 in., its upper base 18 in., and its altitude 2 ft. Ans. 6,810.988 cu. in.

(21) How many bushels will a granary hold that is 12 ft. long, 5 ft. wide, and 4 ft. deep? Ans. 192.9— bu.

(22) The altitude of the frustum of a square pyramid is 6 ft., and the lengths of the sides of its lower and upper bases are 5 ft. and 4 ft., respectively. Find the volume of the frustum. Ans. 122 cu. ft.

(23) What is the length of a side of a square equal in area to two squares, one being 7 ft. square, and the other 10 ft. square? Ans. 12.207 ft.

(24) How many 3-foot cubes are together equal in volume to a 5-foot cube? Ans. 4.63, nearly.

(25) A cylindrical tank 12 ft. high and 40 in. in diameter is full of oil worth 8 cents a gallon. What is the value of the oil? Ans. \$62.67.

(26) How many perches of masonry in the foundation wall of a building 150 ft. long and 100 ft. wide, the wall being 8 ft. high and 18 in. thick? Ans. 239.52— pch.

(27) How many bricks in a wall 4 ft. thick, 90 ft. long, and 36 ft. high, counting 574 bricks in a cubic yard of wall? Ans. 275,520 bricks.

(28) What is the capacity in heaped bushels of a bin 12 ft. by 8 ft. by 6 ft.? Ans. 362.24— bu.

(29) Find the area of a circle whose circumference is 1,000 ft. Ans. 79,577.285+ sq. ft.

(30) How much must be paid for a sheet of copper 26 ft. long and 23 in. wide at 15 cents per square foot? Ans. \$7.48—.

(31) (a) Find the area of the largest square that can be cut from a circle of zinc 48 in. in diameter. (b) How much zinc is cut off? Notice that the diameter is the diagonal of the square.

Ans. $\begin{cases} (a) & 1,152 \text{ sq. in.} \\ (b) & 657.5616 \text{ sq. in.} \end{cases}$

(43) How many single rolls and how many double rolls of paper would be required to paper the walls of a room 16 ft. by 20 ft. 6 in., allowing for a baseboard 7 in. high, 5 doors 3 ft. by 7 ft., and 3 windows 3 ft. by 6 ft., the walls of the room being 11 ft. high? Also, how many double rolls of border 18 in. wide are required?

Ans. $\left\{ \begin{array}{l} 7 \text{ double rolls.} \\ 17 \text{ single rolls.} \\ 2 \text{ double rolls border.} \end{array} \right.$

(44) At \$1.25 per yard, what will be the cost of carpeting a room 15 ft. by 17 ft., with Brussels carpet, allowing 1 foot on each strip for waste and matching?

Ans. \$52.50.

(45) What would be the cost of erecting the walls of a building 145 ft. by 75 ft., and 30 ft. high; the walls being 3 bricks (= 1 foot) thick, allowance to be made for 110 windows, each 7 ft. by 3 ft. 6 in.; for 4 doors, each 8 ft. by 10 ft.; and 2 doors, each 6 ft. by 8 ft.; the bricks to cost \$5.60 per M, and the laying to cost \$1.45 per M?

Ans. $\left\{ \begin{array}{l} \$1,299.23 \text{ for bricks.} \\ \$336.41 \text{ for laying.} \end{array} \right.$

(46) At $12\frac{1}{2}$ cents per square yard, what will it cost to paint the walls and ceiling of a hall 75 ft. long, 52 ft. wide, and 20 ft. high, no allowance being made for windows and doors?

Ans. \$124.72.

(47) (a) How many shingles are required for a roof which measures 55 ft. by 19 ft. on one side, and 55 ft. by 26 ft. on the other side, the exposed portion of the shingle being 4 in. by 5 in.? (b) Referring to Art. 45, how many bundles of shingles are required?

Ans. $\left\{ \begin{array}{l} (a) \quad 17,820 \text{ shingles.} \\ (b) \quad 72 \text{ bundles.} \end{array} \right.$

(48) At 21 cents per square yard, what would be the cost of plastering the walls and ceilings of the 4 rooms of example 42, no allowance being made for doors and windows, etc.?

Ans. \$76.55.

(49) (a) How many perches of stone are required to build the walls of a church, 80 ft. by 40 ft. outside, the walls being 36 ft. high and 3 ft. thick, there being 12 windows, each

6 ft. by 12 ft., and 3 doors, each 7 ft. by 10 ft.? (b) What is the cost of laying the walls, at \$3.75 per perch?

Ans. $\begin{cases} (a) & 864.7 \text{ pch.} \\ (b) & \$3,927.27. \end{cases}$

(50) (a) How many thousand feet of lumber are contained in a pile having 48 layers of boards 18 ft. long, the width of the layers being 12 ft. and the thickness of the board $1\frac{1}{2}$ in.? (b) What is its value, at \$19.65 per M?

Ans. $\begin{cases} (a) & 15.552 \text{ M.} \\ (b) & \$305.60. \end{cases}$

ARITHMETIC.

(1) Divide: (*a*) the ratio of $24 : 18$ by 2; (*b*) the ratio of $\frac{3}{4} : \frac{4}{5}$ by 5; (*c*) the ratio of $.2 : .05$ by 4; (*d*) the inverse ratio of $1 : .2$ by $\frac{1}{2}$; (*e*) the inverse ratio of $12.5 : 125$ by $\frac{3}{2}$.

$$\text{Ans. } \left\{ \begin{array}{ll} (a) & \frac{2}{3}. \\ (b) & \frac{8}{15}. \\ (c) & 1. \\ (d) & .4. \\ (e) & 6\frac{2}{3}. \end{array} \right.$$

(2) Multiply: (*a*) the ratio of $\frac{4}{5} : \frac{3}{4}$ by 3; (*b*) the ratio of $1\frac{1}{2} : 2\frac{1}{2}$ by 10; (*c*) the ratio of the volume of a gallon to a cubic foot by 27; (*d*) the inverse ratio of a yard to a rod by 4; (*e*) the inverse ratio $33\frac{1}{3} : 187\frac{1}{2}$ by 4.

$$\text{Ans. } \left\{ \begin{array}{ll} (a) & 5\frac{1}{5}. \\ (b) & 6. \\ (c) & 3.6+. \\ (d) & 22. \\ (e) & 22.5. \end{array} \right.$$

(3) Find the sum of: (*a*) the ratio of $\frac{2}{3} : \frac{3}{4}$ and $5 : 1\frac{1}{2}$; (*b*) the ratio of $.5 : 1.2$ and $\frac{3}{4} : 1\frac{7}{8}$; (*c*) the ratio $\frac{2}{3} : .8$ and $1\frac{1}{2} : 4\frac{1}{2}$; (*d*) the ratio of $2.3 : 11\frac{1}{2}$ and the inverse ratio of $.8 : 1.2$; (*e*) the inverse ratio of $4.2 : 21$ and the direct ratio of $4 : 13\frac{1}{3}$.

$$\text{Ans. } \left\{ \begin{array}{ll} (a) & 4\frac{2}{3}. \\ (b) & \frac{49}{80}. \\ (c) & 1\frac{1}{6}. \\ (d) & 1.7. \\ (e) & 5.3. \end{array} \right.$$

- (4) What is the square root of the inverse ratio of
 (a) $7.2 : 5$? What is the cube root of the inverse ratio of
 (b) $16 : 54$?

$$\text{Ans. } \begin{cases} (a) & \frac{5}{6}. \\ (b) & 1.5. \end{cases}$$

- (5) What is the square of the inverse ratio of (a) $\frac{3}{4} : \frac{3}{5}$?
 What is the cube of the ratio of (b) $4.2 : .14$?

$$\text{Ans. } \begin{cases} (a) & 5.0625. \\ (b) & 27,000. \end{cases}$$

- (6) What is the value of $\frac{3}{4}$ of the inverse ratio of (a)
 $52 : 41.6$? Find the value of the inverse ratio of (b) a bushel
 to a cubic foot.

$$\text{Ans. } \begin{cases} (a) & 6. \\ (b) & .804. \end{cases}$$

- (7) Explain (a) why the following is not a correct propor-
 tion, $20 : 23 = 39 : 45$; (b) how much must be added to 23
 to make it correct?

$$\text{Ans. (b) } \frac{1}{14}.$$

- (8) If 3 is to 5 inversely as x is to 8, what is the value
 of x ?

$$\text{Ans } 13\frac{1}{2}.$$

- (9) Express in integers the ratio of 2 lb. 3 oz. to 5 lb.
 $1\frac{1}{2}$ oz.

$$\text{Ans. } 70 : 163.$$

- (10) Which is greater, and how much, $3 : 8$ or $5 : 12$?

$$\text{Ans. The latter, } \frac{1}{24}.$$

- (11) Find the value of x in

$$\left. \begin{array}{c} \frac{1\frac{2}{3}}{\frac{2}{3}} \\ 7 \end{array} \right| \left. \begin{array}{c} \frac{5}{6} \\ \frac{5}{6} = x \\ 10 \end{array} \right| 20 \quad \text{Ans. } x = 22.4.$$

- (12) Find the value of x in each of the following propor-
 tions: (a) $3 : .5 = x : .9$; (b) $3 : x = 7 : 19$; (c) $5\frac{1}{2} : 7.2$
 $= 4.8 : .6x$; (d) $\frac{3}{4}x : \frac{2}{3} = \frac{5}{8} : \frac{7}{12}$; (e) $9 : x = x : 16$.

$$\text{Ans. } \begin{cases} (a) & 5\frac{3}{2}. \\ (b) & 8\frac{1}{2}. \\ (c) & 10.4\frac{8}{11}. \\ (d) & \frac{20}{11}. \\ (e) & 12. \end{cases}$$

- (13) If x and 4 are inversely as $3 : 6$, what is the value
 of x ?

$$\text{Ans. } 8.$$

(14) If the prices of grindstones vary directly as their weights and their weights vary as their thickness multiplied by the squares of their diameters, what must be paid for a grindstone 4 feet in diameter and 9 inches thick when \$5 is paid for one that is 2 feet in diameter and 5 inches thick?

Ans. \$36.

(15) What is the value of x in the proportion $2^x : 3^x = x^2 : 5^2$?

Ans. 10.541—.

(16) Find in integers the ratio of 3 gal. 2 qt. 1 pt. to 10 gal.

Ans. 29 : 80.

(17) The temperature being constant, the volume of a given weight of air varies inversely as the pressure of the air. If a pound of air has a volume of 10 cubic feet at a pressure of 17.8 pounds per square inch, what will be the volume if the pressure is 67.2 pounds per square inch, the temperature remaining the same?

Ans. 2.65 cu. ft., nearly.

(18) If 5 be added to the antecedents of the proportion, $2 : 5 = 6 : 15$, what change of the first consequent will correct the proportion? What change of the second consequent?

Ans. $\left\{ \begin{array}{l} 4\frac{8}{11} \text{ must be added.} \\ 7\frac{1}{11} \text{ must be subtracted.} \end{array} \right.$

(19) If a cylinder whose diameter is to its height as 3 : 5 will hold 100 gallons, how many gallons will a cylinder hold whose diameter in the same units is to its height as 5 : 9? (See Art. 50.)

Ans. 500 gal.

(20) Two cylinders have equal volumes. The diameter of the first is 15 in. and its length is 49 in.; what is the diameter of the second if its length is 36 in.?

Ans. 17.5 in.

(21) A certain light at the distance of 4 rods has an intensity of 20; what will be its intensity at the distance of 5 rods? (See example 1, Art. 42.)

Ans. 12.8.

(22) The volume of a sphere having a diameter of 6 inches is 113.0976 cubic inches; what is the volume of a sphere having a diameter of 12 inches? The volumes vary as the cubes of the diameters.

Ans. 904.7808 cu. in.

(23) A block of Quincy granite $3' \times 4' \times 5'$ weighs 9,945 pounds; what is the weight of a block $5' \times 8' \times 10'$?

Ans. 66,300 lb.

(24) What added to the first term, or what subtracted from the second term, will form from $3 : 7 = 8 : 12$ a correct proportion?

Ans. $1\frac{1}{2}$; $2\frac{1}{2}$.

(25) If 5 men in 9 days of 10 hours each can build a wall 100 feet long, 6 feet high, and 18 inches thick, in how many days of 9 hours each will 8 men build a wall 120 feet long, 8 feet high, and 20 inches thick?

Ans. 11 da. 1 hr.

(26) The time of oscillation of a pendulum varies directly as the square root of the length of the pendulum. If at a certain place a pendulum 39 inches long oscillates once in a second, how long must a pendulum be to oscillate once in two seconds at the same place?

Ans. 156 in.

(27) By the method of proportional parts, divide a gain of \$4,750 among three men in the ratios of 2, 3, and 5.

Ans. $\left\{ \begin{array}{l} \$ 950, \\ \$1,425, \\ \$2,375. \end{array} \right.$

(28) Arrange the fractions $\frac{2}{3}$, $\frac{5}{7}$, $\frac{1}{2}$, $\frac{1}{3}$, in a correct proportion and change to a form containing no fractions. Prove your work to be correct.

(29) Knowing that the amounts of water that can be discharged through pipes vary as the times, the velocities of flow, and the squares of the diameters of the pipes, find how much water will flow through a 5-inch pipe in 2 hours with a velocity of 44 feet per second, if a 4-inch pipe will discharge 5,385 6 gallons in 50 minutes, when the flow is at the rate of 33 feet per second.

Ans. 26,928 gal.

(30) If the antecedents of the proportion $4 : 8 = 6 : 12$, be increased by 10, what change must be made in the first consequent?

Ans. $2\frac{1}{2}$ must be added.

(31) Divide 100 into parts proportional to $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$.

$$\text{Ans. } \begin{cases} 26\frac{2}{3}, \\ 34\frac{1}{3}, \\ 39\frac{5}{6}. \end{cases}$$

(32) The load that a beam or joist will safely support varies as the breadth, as the square of the depth, and inversely as the length of the beam. If a beam 20 feet long, 3 inches wide, and 6 inches deep will support a load of 6,000 pounds, how many pounds will a beam 15 feet long, 4 inches wide, and 5 inches deep support under the same circumstances?

Ans. 7,407.41 lb.

(33) A fish 20 inches long weighs 10 pounds. Find the weight of a fish of similar shape 30 inches long, knowing that their weights vary as the cubes of their lengths.

Ans. $33\frac{1}{2}$ lb.

(34) If a man in correct proportion and 5 feet 6 inches in height weighs 140 pounds, what should be the weight of a similarly shaped man 6 feet high? (See example 33.)

Ans. 181.76—1b.

(35) Knowing that the interest for \$250 for 2 years 6 months at 8% is \$50, find by proportion the interest of \$450 for 3 years 10 months at 6%.

Ans. \$103.50

(36) Rectangles vary in area directly as their bases and altitudes. If \$100 is paid for a rectangular piece of ground whose dimensions are 9 rd. and 8 rd., what should be paid for a piece of ground equally valuable, having dimensions 20 rd. and 16 rd.?

Ans. \$444.44+.

(37) The distances through which bodies will fall vary as the squares of the times during which they fall. If a body falls 144 feet in 3 seconds, how far will it fall in 5 seconds?

Ans. 400 ft.

(38) Knowing that the weights of spheres of similar material vary as the cubes of their radii, and that a sphere of

lead 12 inches in diameter weighs 371.75 pounds, find the weight of a leaden sphere 30 inches in diameter.

Ans. 5,808.59+ lb.

(39) If a pendulum 39 inches long oscillates once per second, in what time will a pendulum oscillate that is 20 inches long? (See example 26.)

Ans. .7161 sec.

(40) The capacities of bins vary as the products of their three dimensions. Divide 3,700 bushels of grain among three bins in proportion to their capacities, their dimensions being as 2, 3, and 5; 3, 4, and 6; and 4, 5, and 6.

**Ans. { 500 bu.
1,200 bu.
2,000 bu.**

ARITHMETIC.

- (1) Express decimally: $\frac{1}{2}\%$, $\frac{3}{4}\%$, $1\frac{3}{8}\%$, $\frac{5}{8}$ of 1% , $\frac{3}{8}$ of $\frac{3}{4}\%$.
- (2) A farmer planted $6\frac{1}{4}\%$ as many potatoes as his crop amounted to; this was 1,575 bushels more than he planted. How many bushels in his crop? Ans. 1,680 bu.
- (3) What is $37\frac{1}{2}\%$ of \$448.64? Ans. \$168.24.
- (4) I pay $37\frac{1}{2}\%$ of my month's salary to my grocer, $12\frac{1}{2}\%$ of it to my butcher, and $16\frac{2}{3}\%$ of it for other expenses, and then have \$90 of it left. How much do I get per year? Ans. \$3,240.
- (5) What number is it 23% of which exceeds $12\frac{1}{2}\%$ of it by 315? Ans. 3,000.
- (6) Pure water consists, by weight, of 2 parts of hydrogen and 16 parts of oxygen. Of what per cent. of each is water composed? Ans. $\left\{ \begin{array}{l} H, 11\frac{1}{8}\% \\ O, 88\frac{7}{8}\% \end{array} \right.$
- (7) A merchant bought some goods at 20% below list price, and sold them so as to gain 30%. At what per cent. above list price did he sell them? Ans. 4%.
- (8) If 30% of $\frac{2}{3}$ of the distance from New York to Philadelphia is $10\frac{1}{3}$ miles, what is the entire distance? Ans. 90 miles.

§ 8

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(9) Two horses were sold at \$120 each. If by the transaction 25% was gained on one and 25% lost on the other, how much was gained or lost? Ans. \$16 lost.

(10) What per cent. of 15 tons of coal have I used when I have $9\frac{1}{4}$ tons left? Ans. $38\frac{1}{4}\%$.

(11) A farm and the buildings on it cost \$13,691.70, and the cost of the buildings is 65% of that of the farm. Find the cost of each. Ans. $\left\{ \begin{array}{l} \text{Buildings, } \$5,393.70. \\ \text{Farm, } \$8,298.00. \end{array} \right.$

(12) A town having 6,375 inhabitants increased in population 28% in five years. How many inhabitants were there after the increase? Ans. 8,160.

(13) A bought a horse and sold it to B at an advance of 20%; B sold it to C and gained 25%. What did each man pay for it if it cost C \$60 more than it cost A? Ans. $\left\{ \begin{array}{l} \text{A, } \$120. \\ \text{B, } \$144. \\ \text{C, } \$180. \end{array} \right.$

(14) A man invested \$180 in each of two speculations. By one of them he gained 30% and by the other he lost 25%. Did he gain or did he lose, and how much? Ans. \$9 gain.

(15) In a certain specimen of lead ore composed of lead and sulphur, the ratio of lead to sulphur is 206.95 : 32.06. Find the per cent. of each ingredient.

Ans. $\left\{ \begin{array}{l} \text{Lead, } 86.5863+\%. \\ \text{Sulphur, } 13.4137-\%. \end{array} \right.$

(16) A merchant bought 200 barrels of flour at \$5.40 per barrel. He sold 65% of it at \$5.90 and the remainder at \$6.50. Find his gain per cent. Ans. $13\frac{1}{2}\%$.

(17) A speculator, after losing 35% of his money and 10% of the remainder, had \$17,550 left. How much did he have at first? Ans. \$30,000.

(18) After a serial discount of 40%, 20%, and 10% was deducted from the list price of a piano, the selling price was \$216. What was the list price? Ans. \$500.

(19) A man earned 20% more during the second of three years than he did the first, and 25% more during the third year than he earned the second. In the three years he earned in all \$7,400. What sum did he earn each year?

Ans. $\left\{ \begin{array}{l} \text{First year, \$2,000.} \\ \text{Second year, \$2,400.} \\ \text{Third year, \$3,000.} \end{array} \right.$

(20) Divide \$0,459 among three men in the proportion of 20%, 30%, and 40%.

Ans. $\left\{ \begin{array}{l} \$2,102. \\ \$3,153. \\ \$4,204. \end{array} \right.$

(21) If I should sell my horse for \$140, my gain would be 12%; for what must I sell him to gain 28%? Ans. \$160.

(22) What is the ad valorem duty at 22% on 18,600 yards of silk invoiced at 8 francs per yard? Ans. \$6,317.96.

(23) A commission merchant sold some goods for \$8,407.96. His commission was 2%, for guaranteeing payment he charged $1\frac{1}{2}\%$, and his other charges were \$275; how much should he send his principal? Ans. \$7,838.68.

(24) Find A's tax, knowing that the tax rate is $12\frac{1}{2}$ mills on 100 cents, that his assessment on real estate is \$7,800, on personal property \$5,640, and that he pays for 3 polls at \$.75 each. Ans. \$170.25.

(25) By selling some goods for \$32.40 more than cost, a gain of 18% was realized. For how much were they sold? Ans. \$212.40.

(26) How much better on a bill amounting to \$987.50 is a discount of 40% than discounts of 20% and 20%? Ans. \$39.50.

(27) A boy buys oranges, and always receives for 3 of them as much as he pays for 5. What is his gain per cent.? Give reasons for your solution. Ans. $66\frac{2}{3}\%$.

(28) If the premium for insuring a store is \$267 when the rate is $1\frac{1}{2}\%$, what is the amount of insurance? Ans. \$17,800.

(29) When the rate is $\frac{3}{4}$ of 1%, and the amount to be insured is \$75,375, what will be the premium?

Ans. \$565.31+.

(30) If the rate of duty is 28% ad valorem, what will be the duty on a lot of silk invoiced at £378 15 s.? Ans. \$516.04.

(31) A man's tax on his personal property and real estate, when the rate is $28\frac{1}{2}$ mills on 100 cents, is \$637.83. If the real estate is assessed at \$18,600, find the assessment on the personal property.

Ans. \$3,780.

(32) After deducting $2\frac{1}{2}\%$ commission and 1% for insurance, a commission merchant remitted \$6,613 as the proceeds from a sale of peaches. For how much were the peaches sold?

Ans. \$6,800.

(33) A commission merchant received \$520.45 to buy peaches after deducting expenses. Drayage cost \$18.75, storage, \$8.50, and his commission was $2\frac{1}{2}\%$. At \$.75 a crate, how many crates did he buy?

Ans. 640.

(34) The value of a store is to that of its contents as 9 : 11. If both are insured for $\frac{1}{2}$ of their value when the rate of premium is $1\frac{1}{2}$ cents on a dollar, the premium is \$372. Find the value of each.

Ans. $\left\{ \begin{array}{l} \text{Contents, } \$17,050. \\ \text{Store, } \$13,950. \end{array} \right.$

(35) A merchant purchased a bill of goods, on which he received a serial discount of 20%, 15%, and 5%. If, instead, he had been given a discount of 20%, 10%, and 10%, his bill would have been \$25.48 more. Find the amount of the bill without discount.

Ans. \$12,740.

(36) A life-insurance company took a risk at $\frac{7}{8}\%$ annual premium for \$25,000. The company reinsured 28% of their risk at $\frac{1}{2}\%$ in a second company, and 48% of it at $\frac{3}{4}\%$ in a third company. What was the annual gain of the first company by reinsuring?

Ans. \$41.25.

(37) The duty at 35% ad valorem on a consignment of silk velvet invoiced at 40 francs per yard is \$2,364.25. How many yards were there in the consignment?

Ans. 875 yd.

(38) A discount of 20% on $\frac{3}{4}$ of the amount of a bill, and 15% on the remainder is \$324.50 better than a discount of 16% on the whole bill. Find the amount of the bill.

Ans. \$11,800.

(39) A dealer receives \$33.90 more for a piano by allowing a serial discount of 20%, 10%, and 10% instead of one of 25%, 15%, and 5% from the list price. What is the list price?

Ans. \$800.

(40) A merchant sold 20% of his stock at a gain of 20%, 30% of the remainder at 1.25 of its cost, and what still remained he sold at $\frac{7}{8}$ of its cost. Did he gain or lose, and what per cent.?

Ans. 3% gain.

ARITHMETIC.

(1) By the method of Art. 10, find the interest (*a*) of \$875 for 3 yr. 5 mo. 24 da. at 5%; (*b*) of \$1,468.50 for 2 yr. 11 mo. 20 da. at $4\frac{1}{2}\%$.

Ans. $\begin{cases} (a) & \$152.40. \\ (b) & \$196.41. \end{cases}$

(2) By the method of Art. 12, find the interest (*a*) of \$439.48 at $3\frac{1}{2}\%$ for 2 yr. 9 mo. 21 da.; (*b*) of \$5,628.40 at $3\frac{1}{2}\%$ for 5 yr. 7 mo. 29 da.

Ans. $\begin{cases} (a) & \$43.20. \\ (b) & \$1,211.39. \end{cases}$

(3) By the six-per-cent. method, find the interest (*a*) of \$4,280 for 3 yr. 3 mo. 3 da. at 6%; (*b*) of \$5,670.80 for 1 yr. 11 mo. 27 da. at $4\frac{1}{2}\%$.

Ans. $\begin{cases} (a) & \$836.74. \\ (b) & \$508.25. \end{cases}$

(4) Knowing that the interest of a certain principal for a certain time is \$834.72 at 6%, what would be the interest (*a*) of the same principal at 9% for half the time? (*b*) of $\frac{2}{3}$ of the same principal at 6% for $\frac{1}{2}$ the time?

Ans. $\begin{cases} (a) & \$626.04. \\ (b) & \$278.24. \end{cases}$

(5) If an investment at 5% yields an income of \$4,060 annually, what should it yield (*a*) at $4\frac{1}{2}\%$? (*b*) at $4\frac{2}{3}\%$?

Ans. $\begin{cases} (a) & \$3,654. \\ (b) & \$3,789.33. \end{cases}$

(6) By the 60-day method, find the interest of \$4,689.50 at 6% for 87 days.

Ans. \$68.00.

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(7) By what must a principal be multiplied to give the interest for 2 yr. 9 mo. 18 da. (a) at 5%? (b) at $4\frac{1}{2}\%$?

Ans. $\begin{cases} (a) & .14. \\ (b) & .126. \end{cases}$

(8) Find, in the shortest way, how much more the interest of \$6,600 will be for 3 yr. 8 mo. 15 da. at $5\frac{1}{2}\%$ than at $4\frac{2}{3}\%$.

Ans. \$203.96.

(9) If, in finding the interest at 6% of any sum of money, the multiplier is .2365, what is the time?

Ans. 3 yr. 11 mo. 9 da.

(10) What will be the exact interest of \$24,000 for the first four months of the year 1900 at 6%? Ans. \$473.12.

(11) How much more at 6% is the exact interest of a debt of \$100,000 for February, 1896, than for the same month in 1897? Ans. \$15.14.

(12) Find the exact interest of \$14,600 at 6% from July 1, 1898, to April 25, 1900. Ans. \$1,591.20.

(13) Find the interest (a) of \$4,780 at 8% for 5 yr. 3 mo.; (b) of \$8,690 at $7\frac{1}{2}\%$ for 4 yr. 9 mo. 20 da.

Ans. $\begin{cases} (a) & \$2,007.60. \\ (b) & \$3,132.02. \end{cases}$

(14) In 3 yr. 5 mo. 18 da. at $4\frac{1}{2}\%$, what part of the principal equals the interest? Ans. .156.

(15) What is the exact interest (a) of \$2,375 for 2 yr. 234 da. at $4\frac{1}{2}\%$? (b) of \$468.90 for the first 140 days of a leap year at 5%?

Ans. $\begin{cases} (a) & \$282.27. \\ (b) & \$8.97. \end{cases}$

(16) Find the exact interest at 7% of \$928.60 from April 8, 1899, to October 17, 1899. Ans. \$34.19.

(17) What is the compound interest of \$3,690 for 3 yr. 7 mo. 20 da. at 6%, interest being compounded semiannually? Ans. \$886.05.

(18) What is the difference between the exact and the ordinary interest of \$3,600 at 6% for 270 days of a common year?
Ans. \$2.22.

(19) When payable annually, what is the interest at 6% of \$8,000 for 4 yr. 9 mo.?
Ans. \$2,539.20.

(20) The amount of a certain principal at 5% for 2 yr. 3 mo. 18 da. is \$535.20; what is the principal?
Ans. \$480.

(21) At 6%, what principal, in 3 yr. 5 mo. 20 da., will yield \$150 interest?
Ans. \$720.

(22) What is the present worth of \$1,260 due in 6 mo. 24 da., money being worth 5% a year?
Ans. \$1,225.28.

(23) Find the compound interest of \$240 at 5% for 4 yr. 5 mo. 20 da.
Ans. \$58.61.

(24) What is the face of a 90-day note that will yield \$1,200 proceeds when discounted at 6%?
Ans. \$1,218.89.

(25) Find the bank discount of a note for \$2,400, due in 60 days, the rate of discount being 6%.
Ans. \$25.20.

(26) Write, in proper form, a 90-day note which, when discounted at a bank at 5%, will yield \$6,000 proceeds. Give the work for finding the face.
Ans. Face, \$6,078.51+.

(27) What principal, compounded at 4% annually, will amount to \$8,000 in 5 yr. 9 mo.?
Ans. \$6,383.90.

(28) By the United States rule for partial payments, find what will be due at the time of settlement of the following note:

<p>\$2,960 $\frac{90}{100}$. <i>On demand, I promise to pay William Draper Two Thousand Nine Hundred Sixty Dollars, value received, with interest at 6%.</i> Erastus Masters.</p>	<p><i>Boston, Mass., April 1, 1899.</i></p>
--	---

Indorsements: July 1, 1899, \$400; Dec. 15, 1899, \$50;
Jan. 21, 1900, \$680, Mar. 24, 1900, \$800.

Settled, Apr. 25, 1900.

Ans. \$1,187.91.

(29) A 90-day note for \$1,200, dated July 1, 1900, and bearing interest at 8%, was discounted at 6% at a bank, Sept. 1, 1900. Find the proceeds. Ans. \$1,218.47.

(30) A 60-day note, discounted at date, at 6%, yields the bank \$45.80. What is the face of the note?

Ans. \$4,361.90.

(31) In what time will \$860 give \$139.32 interest, at 6%?

Ans. 2 yr. 8 mo. 12 da.

(32) A note for \$2,920, bearing interest at 5%, is dated April 1, 1900. At what date will the exact interest amount to \$58.40?

Ans. Aug. 25, 1900.

(33) What is the exact interest of \$10,000 from Feb. 22, 1900, to July 4, 1900, at $4\frac{1}{2}\%$?

Ans. \$162.74.

(34) A demand note for \$12,000 with interest at 6% is dated May 10, 1894, and bears the following indorsements: Sept. 1, 1894, \$200; Jan. 9, 1895, \$960; Mar. 1, 1895, \$150; May 1, 1895, \$500. Settled, Sept. 1, 1895. Which is better for the lender, and by which will he receive the more interest—the merchants' rule or the United States rule, exact interest being computed?

Ans. U. S. rule, \$28.02.

(35) At what rate per cent will \$1,280, in 2 yr. 5 mo. 24 da., give \$148.31 interest?

Ans. $4\frac{2}{3}\%$.

(36) A bank discounts a 90-day note for \$10,860 at the rate of 6% per annum. How much more does it receive than is equitable?

Ans. \$2.57.

(37) The bank discount on a 30-day note, bearing interest at 8%, is \$66, the rate of discount being 6% per annum. Find the face of the note, if the note was discounted on the day it was made.

Ans. \$11,912.64.

(38) When interest is to be paid annually at 6%, but is not paid, what is due, in 5 yr. 7 mo. 24 da., on a note for \$8,000?

Ans. \$11,093.60.

(39) I lend equal sums to each of two men, for which one pays me $5\frac{1}{2}\%$ and the other $4\frac{1}{3}\%$ per annum. At the end of

1 yr. 9 mo. 18 da., the interest due me from the first exceeds the interest due from the second by \$42. What did I lend to each?
Ans. \$2,000.

(40) If the interest of $\frac{3}{4}$ of A's money added to that of $\frac{2}{3}$ of B's in 4 yr. at $5\frac{1}{2}\%$ is \$1,320, how much has each, if $\frac{3}{4}$ of A's equals $\frac{2}{3}$ of B's?
Ans. $\left\{ \begin{array}{l} \text{A, } \$4,000. \\ \text{B, } \$4,500. \end{array} \right.$



ARITHMETIC.

(1) What must be paid for 480 shares of Northern Central stock bought at $113\frac{5}{8}$, brokerage being $\frac{1}{8}\%$?

Ans. \$54,600.

(2) How much must be invested in stocks paying $3\frac{1}{2}\%$ semiannual dividends to yield a yearly income of \$5,600, if they are bought at $108\frac{1}{2}$, brokerage being $\frac{1}{8}\%$?

Ans. \$86,900.

(3) How many shares of stock selling at $79\frac{3}{4}$ can be bought for \$38,979, brokerage being $\frac{1}{8}\%$?

Ans. 488.

(4) A broker at a commission of $\frac{1}{4}\%$ receives \$50 for buying stock at $110\frac{1}{2}$. (a) How many shares does he buy? (b) What do they all cost without brokerage?

Ans. $\left\{ \begin{array}{l} (a) \quad 200. \\ (b) \quad \$22,100. \end{array} \right.$

(5) What will be the cost of 125 shares of bank stock bought at $114\frac{1}{8}$, brokerage being $\frac{1}{8}\%$?

Ans. \$14,281.25.

(6) How much must be invested in stock paying a quarterly dividend of 2% to yield an annual income of \$4,800, their market price being $112\frac{3}{8}$ and brokerage $\frac{1}{8}\%$?

Ans. \$67,500.

(7) A man pays 120 for stock that yields a quarterly dividend of 3% . What per cent. per annum does he receive on his investment?

Ans. 10% .

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(8) Which is better, and by how much per cent. — to invest in stock at 110 that pays an annual dividend of 10%, or in stock at 90 that pays 8% a year?

Ans. The former, by $\frac{2}{3}\%$.

(9) Find the cost of a sight draft on San Francisco for \$4,875, exchange being at $1\frac{1}{2}\%$ discount. Ans. \$4,807.97.

(10) What must be paid for a sight draft for \$5,600 on New Orleans when exchange is at $1\frac{1}{2}\%$ premium?

Ans. \$5,663.

(11) What will be the cost of a draft for \$8,000 payable in 60 days after sight, exchange being at $1\frac{1}{2}\%$ premium, and money being worth 6% interest? Ans. \$8,036.

(12) Find the cost of a 90-day draft for \$6,800 when money is worth 5% interest and exchange is at $1\frac{1}{4}\%$ discount.

Ans. \$6,627.17—.

(13) Exchange being at $\frac{1}{4}\%$ premium, find the cost of a sight draft for \$2,800. Ans. \$2,821.

(14) Find the face of a 60-day draft costing \$12,000 when exchange is at $1\frac{1}{2}\%$ discount and money is worth 6% interest.

Ans. \$12,314.01.

(15) What is the face of a sight draft costing \$10,000 when exchange is at $1\frac{1}{4}\%$ discount? Ans. \$10,126.58.

(16) What is the cost in New York of a draft on London for £400 15 s. when exchange is \$4.855? Ans. \$1,945.64.

(17) What must be paid in New York for a draft on Paris for 20,000 francs when \$1 is equal in exchange to 5.22 francs? Ans. \$3,831.42.

(18) If the exchange value of a guilder is \$40 $\frac{3}{4}$, what will be the cost of a draft on Amsterdam for 6,000 guilders?

Ans. \$2,422.50.

(19) A draft on Berlin cost \$1,541 when the exchange value of reichsmarks was quoted at 94 $\frac{1}{2}$. For how many reichsmarks was its face? Ans. 1,628.53 reichsmarks.

(20) What will be the cost of a draft on Paris through London for 10,000 francs, exchange between New York and London being \$4.84, and between London and Paris 24 85 ?

Ans. \$1,947 69.

(21) Find the cost in New York of a draft for 10,000 reichsmarks on Berlin through London and Amsterdam, exchange between New York and London being \$4.85, between London and Amsterdam 11.85 guilders to the £, and between Amsterdam and Berlin .42½ reichsmarks to the guilder, and brokerage at intermediate places of exchange being ½%.

Ans. \$9,654.27+.

(22) How many bushels of oats are worth 30 bushels of wheat, when 3 bushels of corn are worth 5 bushels of oats, 7 bushels of corn are worth 4 bushels of rye, and 8 bushels of rye are worth 5 bushels of wheat ?

Ans. 140 bu.

(23) If 11.36 cu. in. of gold weigh as much as 19.3 cu. in. of lead, 10.5 cu. in. of lead as much as 11.36 cu. in. of silver, 14.24 cu. in. of silver as much as 21 cu. in. of zinc, and 2.58 cu. in. of zinc as much as 7.12 cu. in. of aluminum, how many cu. in. of aluminum weigh as much as 1 cu. in. of gold ?

Ans. 7.48 cu. in.

(24) A in 4 days can earn as much as B in 5 days, B in 3 days as much as C in 7 days, C in 6 days as much as D in 11 days, and D in 5 days as much as E in 9 days. How much will A earn while E earns \$100 ?

Ans. \$962.50.

(25) Four men, A, B, C, and D, engage in a bicycle race of 50 miles. A, who wins, goes 12 miles while B goes 11.5, B goes 10 miles while C goes 9½, and C goes 9 miles while D goes 8½. How far has D gone at the end of the race ?

Ans. 42.99 mi.

(26) A lends B at one time \$320 for 5 months, and at another time \$450 for 8 months. How long should B lend A \$500 to repay the favor ?

Ans. 10 mo. 12 da.

(27) In what time will the interest of \$1,000 be the same as the interest of \$800 for 30 days, \$500 for 20 days, and \$400 for 60 days, the rate in each case being the same ?

Ans. 58 da.

(28) A sold B a bill of goods as follows: silk for \$1,200 on 30 days' credit, cloth for \$3,600 on 60 days' credit, and cotton goods for \$1,200 on 90 days' credit. In what time can the three debts be equitably paid at once? Ans. 60 days.

(29) I owe my bank \$360 due July 1, \$560 due Aug. 10, \$800 due Sept. 15, and \$1,000 due Oct. 1. When may I pay the four debts at once without loss to either party?

Ans. Sept. 3.

(30) Find the equated time for paying in one payment the following debts: \$800 due Sept. 1, \$900 due Oct. 10, \$1,000 due Oct. 30, and \$1,200 due Dec. 1. Ans. Oct. 23.

(31) A bill was to be paid as follows: $\frac{1}{2}$ in 20 days, $\frac{1}{3}$ in 30 days, $\frac{1}{6}$ in 60 days, and the remainder in 90 days. Find the equated time for paying the whole bill in one payment.

Ans. 47 days.

(32) Find the time when interest should begin on the sum required to balance the following account:

1898.				1898.			
Mar	1	Mdse., 30 days,	900	Apr	20	Cash,	400
	21	" 90 "	1,000	May	15	"	600
Apr	20	" 30 "	1,200	June	25	"	500
May	20	" 60 "	800	July	1	"	1,000

Ans. May 17, 1898.

(33) When may the following account be settled by cash payment without loss of interest to either party?

1899.				1899.			
June	8	Mdse., 10 days,	1,200	July	1	Cash,	1,000
July	10	" 30 "	2,000		25	Draft, 30 days,	500
Aug	20	" 30 "	3,000	Sept	30	Cash,	800
Sept	25	" 90 "	2,400	Oct	10	Real Estate,	6,000
Dec	1	" 30 "	5,000	Nov.	20	Cash,	900

Ans. Jan. 14, 1900.

(34) A mixture is composed of 12 lb. of tea worth 25 cents a pound, 12 lb. worth 35 cents a pound, and 48 lb. worth 45 cents a pound. Find the value of the mixture per pound.

Ans. 40 cents.

(35) A farmer sold at one time 84 bu. of potatoes at \$1.20 per bushel, at another time he sold 48 bu. at \$.90, and at a third time 48 bu. at \$.75. What was the average price per bushel?

Ans. \$1.00.

(36) A speculator bought lots: 24 at \$1,080 each, 36 at \$2,160 each, and 48 at \$3,240 each. He sold them all so as to average a gain of \$100 a lot. Find the average price at which he sold them.

Ans. \$2,500.

(37) If certain multiples of 29, 37, 41, and 47 be added together, the sum is exactly divisible by 40. What is the least sum that will answer these conditions?

Ans. 880.

(38) In what proportion must flour worth \$5, \$6½, and \$8 per barrel, respectively, be sold to average \$7 a barrel?

Ans. 2, 2, 5.

(39) A coal merchant sold coal at \$2½, \$3½, \$4½, and \$7½ per ton. During a certain month his average selling price per ton was \$5. In what proportion were the different grades sold?

Ans. 5, 5, 5, 9.

(40) A miller paid \$960 for 1,280 bu. of grain as follows: wheat at \$1.00, rye at \$.80, corn at \$.60, and oats at \$.40. How many bushels were there of each?

Ans. $\left\{ \begin{array}{l} 560, 240, 80, 400, \text{ or} \\ 240, 560, 400, 80, \text{ etc.} \end{array} \right.$



SPELLING.

LESSON 1.

INSTRUCTIONS FOR STUDYING.

The Instruction Paper in Spelling is apportioned into four divisions. The First Division contains thirty-four exercises and extends from page 1 to exercise 35 on page 7.

In studying the exercises in the Instruction Paper, look closely and carefully at each word, and form a correct mental picture of the word before attempting to write it. Form the habit of training the eye to *see* the word forms correctly before *writing* them.

Write all the words in each exercise as many times as it is necessary to write them before you can spell them correctly. If you can spell a word, it will not be necessary to write it more than once, but in case you cannot spell it, write it five, ten, or fifteen times, or until you *can* spell it. This may be done on paper or on a slate.

In order that the words in each exercise may be fully understood, we suggest that you consult a dictionary for the meaning of all words which are new.

Each exercise should be written in the order in which it occurs in the Instruction Paper. Number each exercise and write the words in columns, using capitals only where used in the Instruction Paper. Do not crowd your work.

In the exercises marked "Homonyms," write the definition after each word. Write, also, the sentences, putting in the right words, and underlining the words inserted.

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INSTRUCTIONS FOR SENDING IN WORK.

After you have written the words in the exercises in a division, write correctly the list of words in the Question Paper for that division, and send both in at the same time. All words given in the Question Papers should be corrected without referring to the Instruction Paper.

For Lesson 1, send to the School for correction, the words in the First Division and the words in the following list, written correctly:

- | | | | |
|-----------------|---------------|----------------|----------------|
| 1. cloke, | 26. libray, | 51. basen, | 76. minet, |
| 2. aporn, | 27. dence, | 52. boul, | 77. cousen, |
| 3. collar, | 28. city, | 53. seive, | 78. buzy, |
| 4. forehead, | 29. contries, | 54. saisor, | 79. waggen, |
| 5. juce, | 30. pigen, | 55. portrate, | 80. hearth, |
| 6. cheek, | 31. conder, | 56. sissors, | 81. schollars, |
| 7. clais, | 32. purch, | 57. mattress, | 82. remedy, |
| 8. chearful, | 33. iland, | 58. bureau, | 83. ment, |
| 9. pendilum, | 34. crumb, | 59. mirrer, | 84. consert, |
| 10. niece, | 35. dresst, | 60. bolester, | 85. parrot, |
| 11. nefew, | 36. charcole, | 61. curten, | 86. eagel, |
| 12. granfather, | 37. serface, | 62. musick, | 87. fethers, |
| 13. brekfast, | 38. hoing, | 63. climer, | 88. pinyon, |
| 14. shugar, | 39. greatful, | 64. croka, | 89. channell, |
| 15. sausage, | 40. strate, | 65. seder, | 90. middel, |
| 16. venzen, | 41. prarie, | 66. chestnut, | 91. longe, |
| 17. pudden, | 42. plateau, | 67. leaver, | 92. severel, |
| 18. practis, | 43. presant, | 68. pinsers, | 93. hickry, |
| 19. pallace, | 44. shovles, | 69. blosom, | 94. adz, |
| 20. trayle, | 45. ismus, | 70. sipress, | 95. mollases, |
| 21. cubboard, | 46. quarrel, | 71. bruze, | 96. carriges, |
| 22. sellar, | 47. mery, | 72. neither, | 97. shaul, |
| 23. fernice, | 48. reck, | 73. necessary, | 98. wrench, |
| 24. atick, | 49. speach, | 74. gaters, | 99. dumbly, |
| 25. transom, | 50. milatary, | 75. parosol, | 100. dettor. |

LESSON 2.

INSTRUCTIONS FOR STUDYING.

The Second Division of the Instruction Paper contains forty-three exercises and extends from exercise 35, page 7, to exercise 78, page 17.

Read carefully the instructions in Question Paper No. 1, and follow them in all the Question Papers in Spelling.

Write all the words in the exercises in the Second Division and send them to the School, together with the following list of words written correctly.

Correct the following list of words:

- | | | |
|-------------------|----------------|----------------|
| 1. skane, | 21. deceatful, | 41. extreme, |
| 2. habbit, | 22. tobbacko, | 42. discribe, |
| 3. dungon, | 23. rabit, | 43. decent, |
| 4. comfert, | 24. buffaloe, | 44. fredom, |
| 5. seive, | 25. patient, | 45. treeson, |
| 6. tong, | 26. hurrycane, | 46. sceme, |
| 7. lettice, | 27. vessle, | 47. sicze, |
| 8. Wensday, | 28. frigget, | 48. collum, |
| 9. orchard, | 29. screal, | 49. sourse, |
| 10. medow, | 30. rought, | 50. circle, |
| 11. turnop, | 31. pruen, | 51. daly, |
| 12. pumkin, | 32. currant, | 52. savige, |
| 13. sirpent, | 33. rasberry, | 53. paralel, |
| 14. radish, | 34. bannana, | 54. diviser, |
| 15. transparrent, | 35. mellon, | 55. chocalate, |
| 16. gorrilla, | 36. lilacks, | 56. vanila, |
| 17. cival, | 37. dasies, | 57. vinegar, |
| 18. obligeing, | 38. geraneums, | 58. journey, |
| 19. courteous, | 39. atention, | 59. atorney, |
| 20. onest, | 40. secreat, | 60. commic, |

- | | | |
|----------------|------------------|----------------|
| 61. galop, | 75. seeling, | 89. celary, |
| 62. guinea, | 76. surtan, | 90. chrismas, |
| 63. gorgous, | 77. cemetary, | 91. erand, |
| 64. common, | 78. pedler, | 92. ernest, |
| 65. biscut, | 79. engine, | 93. comit, |
| 66. surloin, | 80. compleat, | 94. jellous, |
| 67. crevaes, | 81. complection, | 95. intrest, |
| 68. promis, | 82. caben, | 96. silinder, |
| 69. croud, | 83. dessimal, | 97. poisen, |
| 70. scourge, | 84. musketo, | 98. sipher, |
| 71. carpentar, | 85. favorite, | 99. celibrate, |
| 72. crokay, | 86. ancent, | 100. cinamon, |
| 73. nectar, | 87. fatel, | |
| 74. legue, | 88. artic, | |

LESSON 3.

INSTRUCTIONS FOR STUDYING.

The Third Division of the Instruction Paper contains thirty-eight exercises and extends from exercise 78, page 17, to exercise 116, page 26.

When all the words in each exercise in the Third Division have been written, send them to the School, together with the following list of words written correctly.

Correct the following list of words:

- | | |
|-----------------------|----------------|
| 1. migreat, | 22. disperce, |
| 2. neather, | 23. recieve, |
| 3. prarie, | 24. present, |
| 4. omolet, | 25. emmigrate, |
| 5. cndeaver, | 26. resite, |
| 6. ridicule, | 27. isical, |
| 7. freckel, | 28. comprize, |
| 8. liquer, | 29. guilty, |
| 9. decied, | 30. excursion, |
| 10. determin, | 31. grannet, |
| 11. secret, | 32. metalic, |
| 12. temperit, | 33. sythe, |
| 13. remander, | 34. sterriil, |
| 14. batchilor, | 35. crulety, |
| 15. wate (heaviness), | 36. dubble, |
| 16. peice (a part), | 37. ceder, |
| 17. spicey, | 38. sphcar, |
| 18. crasy, | 39. tumer, |
| 19. axcede, | 40. scenery, |
| 20. adieu, | 41. colander, |
| 21. councel, | 42. elipse, |

- | | |
|--------------------|-------------------------------|
| 43. dispair, | 72. capatel (building), |
| 44. commerce, | 73. thear (in that place), |
| 45. deposite, | 74. dilicious, |
| 46. courtesey, | 75. noisy, |
| 47. villan, | 76. hight, |
| 48. occured, | 77. busely, |
| 49. truly, | 78. usualy, |
| 50. cisturn, | 79. mussels, |
| 51. transum, | 80. hesitate, |
| 52. ludicrous, | 81. surprize, |
| 53. corse (rough), | 82. campaign, |
| 54. creture, | 83. vegitable, |
| 55. rane (rule), | 84. salery, |
| 56. linen, | 85. seperate, |
| 57. callico, | 86. keresene, |
| 58. chints, | 87. meddicine, |
| 59. cashmear, | 88. believe, |
| 60. alpacca, | 89. warior, |
| 61. parsle, | 90. crule (unkind), |
| 62. solem, | 91. steake (a slice of meat), |
| 63. siringe, | 92. miricle, |
| 64. calandar, | 93. sergicle, |
| 65. splendor, | 94. phisicile, |
| 66. eraser, | 95. feirceness, |
| 67. musilage, | 96. fausett, |
| 68. mackeral, | 97. burgler, |
| 69. anualy, | 98. victals, |
| 70. porus, | 99. callous, |
| 71. studyous, | 100. boquet. |

LESSON 4.

INSTRUCTIONS FOR STUDYING.

The Fourth Division of the Instruction Paper contains thirty-five exercises; also, the names of the states and territories, and a table of common abbreviations. This division extends from exercise 116, page 26, to the end, on page 39.

Learn to spell the names of the states and territories and their abbreviations; study, also, the table of common abbreviations.

Send to the School the words in each exercise in the Fourth Division, together with the following list of words written correctly.

Correct the following list of words:

- | | |
|--------------------------------|---------------|
| 1. absense, | 17. conciet, |
| 2. bargin, | 18. ncibor, |
| 3. fatigue, | 19. exorcise, |
| 4. machien, | 20. idleize, |
| 5. notise, | 21. civalise, |
| 6. courige, | 22. varitey, |
| 7. marridge, | 23. rezemble, |
| 8. leisure, | 24. pacify, |
| 9. nusance, | 25. discuss, |
| 10. nostrels, | 26. reathe, |
| 11. tortise, | 27. mispend, |
| 12. kurnel (an army officer), | 28. similiar, |
| 13. shariad, | 29. imortal, |
| 14. celler (underground room), | 30. legicy, |
| 15. gravey, | 31. herecy, |
| 16. mischief, | 32. secrecy, |

- | | |
|------------------|-------------------|
| 33. abundance, | 67. proprietor, |
| 34. eligant, | 68. editer, |
| 35. temperence, | 69. corespondent, |
| 36. quincy, | 70. entry, |
| 37. dilagent, | 71. teligrams, |
| 38. feaver, | 72. amuzements, |
| 39. typhiod, | 73. miscelaneous, |
| 40. canser, | 74. warior, |
| 41. collick, | 75. procedes, |
| 42. nausea, | 76. sinical, |
| 43. dispepsey, | 77. promisary, |
| 44. neuralgia, | 78. anticedant, |
| 45. rhumatism, | 79. partecipal, |
| 46. parylasis, | 80. captain, |
| 47. pleurisy, | 81. precede, |
| 48. dilireum, | 82. decease, |
| 49. absess, | 83. pallisade, |
| 50. prohibbit, | 84. linement, |
| 51. independant, | 85. Cincinatti, |
| 52. unyun, | 86. penshion, |
| 53. decliration, | 87. scollop, |
| 54. cartrige, | 88. gitar, |
| 55. ofical, | 89. sinnegogue, |
| 56. redout, | 90. glicerin, |
| 57. tirany, | 91. nesesary, |
| 58. reprisent, | 92. guarantee, |
| 59. goverment, | 93. simton, |
| 60. cavelry, | 94. buisness, |
| 61. artilery, | 95. catipiler, |
| 62. bayonet, | 96. perpus, |
| 63. recrute, | 97. agreable, |
| 64. hospitle, | 98. explanation, |
| 65. furlow, | 99. suitible, |
| 66. pistle, | 100. curiosity. |

Write the abbreviations for each of the following words:

- | | |
|---------------|--------------|
| 1. Louisiana, | 3. Maryland, |
| 2. Missouri. | 4. Colorado. |

- | | |
|--------------------------|------------------|
| 5. Nebraska, | 16. Utah, |
| 6. Maine, | 17. California, |
| 7. Indian Territory, | 18. Wyoming, |
| 8. District of Columbia, | 19. Vermont, |
| 9. Georgia, | 20. Illinois, |
| 10. Kentucky, | 21. Iowa, |
| 11. Oklahoma, | 22. Idaho, |
| 12. Indiana, | 23. Alabama, |
| 13. Virginia, | 24. Alaska, |
| 14. Florida, | 25. Connecticut. |
| 15. Oregon, | |

Write the meaning of each of the following abbreviations:

- | | | |
|-----------------------------|---------------|--------------|
| 1. Col., | 10. ult., | 18. Atty., |
| 2. inst., | 11. i. e., | 19. e. g., |
| 3. Esq., | 12. viz., | 20. etc., |
| 4. MS., | 13. P. M., | 21. N. B., |
| 5. acct. or $\frac{a}{c}$, | 14. pro tem., | 22. P. S., |
| 6. D. D., | 15. Gen., | 23. R. R., |
| 7. Admr., | 16. doz., | 24. prox., |
| 8. Dec., | 17. vs., | 25. U. S. M. |
| 9. A. D., | | |



A KEY
TO ALL THE
QUESTIONS AND EXAMPLES
INCLUDED IN THE
QUESTION PAPERS ON ARITHMETIC.

It will be noticed that the Key is divided into sections which correspond to the sections of the Question Papers on Arithmetic. The answers and solutions are so numbered as to be similar to the numbers before the questions to which they refer.



1



1

ARITHMETIC.

- (1) See Art. 1.
- (2) See Art. 9.
- (3) See Arts. 2 and 3.
- (4) See Arts. 5 and 6.
- (5) The order of a figure is its position in the number relative to units place (see Art. 20).
- (6) See Art. 21.
- (7) Seven thousand five hundred three is expressed in Arabic notation by 7,503; in Roman notation by $\overline{\text{V}}\text{MMDIII}$ (see Arts. 12 and 22).
- (8) See Art. 19.
- (9) See Art. 12.
- (10) (a) Nine hundred eighty.
(b) Six hundred five.
(c) Twenty-eight thousand two hundred eighty-four.
(d) Nine million six thousand forty-two.
(e) Eight hundred fifty million three hundred seventeen thousand two.
(f) Seven hundred thousand four.
- (11) In the Roman notation,
 - (a) 76 is written LXXVI.
 - (b) 353 " " CCCLIII.
 - (c) 1,732 " " MDCCXXXII.
 - (d) 1,496 " " MCCCCXCVI.
 - (e) 1,888 " " MDCCCLXXXVIII.
- (12) See Art. 25.
- (13) (a) Seven thousand six hundred is written 7,600.
(b) Eighty-one thousand four hundred two is written 81,402.

- (c) Five million four thousand seven is written 5,004,007.
 (d) One hundred eighty million ten thousand one is written 108,010,001.
 (e) Ten million six is written 10,000,006.
 (f) Thirty thousand ten is written 80,010.

(14) In adding, place the numbers to be added directly under each other so that the figures of the same order will stand in the same column. Add the first column of figures at the extreme right; this gives $7 + 1 + 4 + 3 + 4 = 19$ units, or 1 ten and 9 units. Place 9 units under the units column and reserve 1 ten for the column of tens. $1 + 8 + 7 + 9 = 25$ tens, or 2 hundreds and 5 tens. Place 5 tens under the tens column, and reserve 2 hundreds for the hundreds column. $2 + 4 + 5 + 2 = 13$ hundreds, or 1 thousand and 3 hundreds. Place 3 hundreds under the hundreds column, and reserve the 1 thousand for the thousands column. $1 + 2 + 5 + 3 = 11$ thousands, or 1 ten-thousand and 1 thousand. Place the 1 thousand in the column of thousands, and reserve the 1 ten-thousand for the column of ten-thousands. $1 + 6 = 7$ ten-thousands. Place this 7 ten thousands in the ten thousands column. There is but one figure, 8 in the hundreds-of-thousands column in the numbers to be added, so it is placed in the hundreds-of-thousands column of the sum.

$$\begin{array}{r}
 (15) \quad 83027 \\
 46928 \\
 4769 \\
 81987 \\
 46729 \\
 479807 \\
 627 \\
 14896 \\
 \hline
 987649 \\
 \hline
 1746509 \quad \text{Ans.}
 \end{array}$$

(16) As both multiplicand and multiplier end in ciphers, place the right-hand digits under each other, as shown. Multiplying the first digit at the right of the multiplicand, or 8, by the multiplier 9, the result is 72 units, 7 tens and 2 units. Write the 2 units in units place in the product, and reserve the tens to add to the product of tens. Multiplying the second digit of the multiplicand by the multiplier 9, we have 81 tens; 81 tens + 7 tens reserved = 88 tens, or 8 hundreds and 8 tens. Write the 8 tens and reserve the 8 hundreds. 9×2 hundreds = 18 hundreds; 18 hundreds + 8 hundreds = 26 hundreds, or 2 thousands and 6 hundreds. Then multiply 298 by 3 and

$$\begin{array}{r}
 29800 \\
 390 \\
 \hline
 2682 \\
 894 \\
 \hline
 11622000 \quad \text{Ans.}
 \end{array}$$

write the product, 894, under the first partial product, as shown, with the right-hand digit, 4, under the multiplier 3. Add the two partial products; their sum gives the entire product of the numbers 298 and 39. Now annex the three ciphers (the number of ciphers to the right of the right-hand digits of both multiplier and multiplicand) to the product, as shown, and we have the entire product, 11,622,000.

$$\begin{array}{rcl}
 (17) & \$ 329 & = \text{amount paid the 1st yr.} \\
 & \$ 438 & = \text{amount paid the 2d yr.} \\
 & \underline{\$ 767} & = \text{whole amount paid.} \\
 & \$ 1000 & = \text{amount of debt.} \\
 & \$ \underline{767} & = \text{amount paid.} \\
 & \$ 233 & = \text{amount he still owes. Ans.}
 \end{array}$$

(18) The numbers connected by the plus (+) sign must first be added. Performing these operations, we have

$$\begin{array}{rcl}
 & 23896 & 31627 \\
 & 4982 & 54892 \\
 & 96875 & 6925 \\
 & 59674 & 8976 \\
 \hline
 & 185427 \text{ sum.} & 102420 \text{ sum.}
 \end{array}$$

Subtracting the less number (102,420) from the greater (185,427), we have

$$\begin{array}{r}
 185427 \\
 102420 \\
 \hline
 83007 \text{ difference. Ans.}
 \end{array}$$

$$\begin{array}{rcl}
 (19) & 8765 & 4695 \\
 & 987 & 823 \\
 \hline
 & 61355 & 14085 \\
 & 70120 & 9390 \\
 & 78885 & 37560 \\
 \hline
 & 8651055 & 3863985
 \end{array}$$

Subtracting the second product from the first, we have

$$\begin{array}{r}
 8651055 \\
 3863985 \\
 \hline
 4787070 \text{ Ans.}
 \end{array}$$

(20) We have given the minuend, or greater number (1,004), and the difference, or remainder (49). Placing these in the usual form of subtraction, we have

$$\begin{array}{r}
 1004 \\
 \hline
 49
 \end{array}$$

in which the dash (—) represents the number sought. This number

9,692 is contained in 57,607 five times. Multiplying 9,692 by 5 and subtracting the product, 48,460, from 57,607, we have 9,147, to which we annex the next figure of the dividend, making 91,478; 9,692 is contained in 91,478 about 9 times. Multiply the divisor, 9,692, by 9 and subtract the product, 87,228, from 91,478. The remainder is 4,250. We write this after the quotient with the divisor underneath. The exact quotient is then $859\frac{250}{9692}$.

(23) We first multiply 986 by 729, obtaining 718,794.

$$\begin{array}{r} 986 \\ 729 \\ \hline 8874 \\ 1972 \\ 6902 \\ \hline 718794 \end{array}$$

Dividing the product, 718,794, by 469, the quotient is $1,532\frac{286}{469}$. Ans.

$$\begin{array}{r} 469 \overline{) 718794} \quad (1532\frac{286}{469} \\ \underline{469} \\ 2497 \\ \underline{2345} \\ 1529 \\ \underline{1407} \\ 1224 \\ \underline{938} \\ 286 \end{array}$$

(24) The process of dividing 87,964,201 by 849 is shown below:

$$\begin{array}{r} 849 \overline{) 87964201} \quad (103609\frac{160}{849} \quad \text{Ans.} \\ \underline{849} \\ 3064 \\ \underline{2547} \\ 5172 \\ \underline{5094} \\ 7801 \\ \underline{7641} \\ 160 \end{array}$$

(25) 36 oxen at \$24 each would cost $36 \times \$24 = \864 .

$$\begin{array}{r} 24 \\ 36 \\ \hline 144 \\ 72 \\ \hline 864 \end{array}$$

23 cows at \$96 each would cost $23 \times \$96 = \$2,208$.

$$\begin{array}{r} 96 \\ 23 \\ \hline 288 \\ 192 \\ \hline 2208 \end{array}$$

\$2208 = cost of cows.

\$ 864 = cost of oxen.

\$3072 = total cost. Ans.

(26) $938 \times 672 \times 497 = 313,276,992$ Ans.

$$\begin{array}{r} 938 \\ 672 \\ \hline 1876 \\ 6566 \\ 5628 \\ \hline 630836 \\ 497 \\ \hline 4412352 \\ 5673024 \\ 2521844 \\ \hline 313276992 \end{array}$$

The product of 938 and 672 is 630,836; multiplying this result by 497, we obtain 313,276,992, the final product.

(27) The product of 989 and 47 is 46,483.

$$\begin{array}{r} 989 \\ 47 \\ \hline 6923 \\ 3956 \\ \hline 46483 \end{array}$$

Subtracting 46,483 from 839,672, the remainder is 793,189. Ans.

$$\begin{array}{r} 839672 \text{ minuend} \\ 46483 \text{ subtrahend} \\ \hline 793189 \text{ remainder.} \end{array}$$

Using the addition method of subtraction shown in Art. 49, add to 3 a number that will make 12. This number is 9 since 3 and 9 are 12. Write the 9 and carry the 1 of the 12, as in addition then, considering 7 to be 17, find what number added to 9 (carrying the 1 and adding it to the 9) will make 17; this is 8. Write the 8 and carry the 1, next, 5 and 1 are 6; write the 1; 6 and 3 are 9. Write the 3; next, 9 added

to 4 will make 13; write the 9 and carry 1. We have 1 to carry, but as we have no figure in the subtrahend to add it to, we say 1 and 7 make 8 and write the 7 in the remainder. The remainder is therefore 793,189.

$$(28) \quad (a) \quad \frac{72 \times 48 \times 28 \times 5}{96 \times 15 \times 7 \times 6} = \frac{\overset{2}{\cancel{72}} \times \overset{8}{\cancel{48}} \times \overset{4}{\cancel{28}} \times \cancel{5}}{\overset{8}{\cancel{96}} \times \overset{3}{\cancel{15}} \times \cancel{7} \times \overset{3}{\cancel{6}}} = \frac{8}{1} = 8. \text{ Ans.}$$

The 5 in the dividend and 15 in the divisor are both divisible by 5, since 5 divided by 5 gives a quotient of 1, and 15 divided by 5 gives a quotient of 3. Cancel the 5 and the 15, and write the 3 under the 15. 72 in the dividend and 96 in the divisor are each divisible by 12, since 72 divided by 12 gives 6, and 96 divided by 12 gives 8. Cancel the 72 and write 6 over it; also, cancel 96 and write 8 under it. Again, 28 in the dividend and 7 in the divisor are each divisible by 7, since 28 divided by 7 gives 4, and 7 divided by 7 gives 1. Cancel the 28 and write the 4 over it, also cancel the 7. Again, 48 in the dividend and 6 in the divisor are each divisible by 6, since 48 divided by 6 gives 8, and 6 divided by 6 gives 1. Cancel the 48 and write 8 over it; also, cancel the 6. Again, 3 in the divisor and 6 in the dividend are both divisible by 3, since 6 divided by 3 gives 2, and 3 divided by 3 gives 1. Cancel the 6 and write 2 over it, also cancel the 3. Finally, cancel the 8 in the dividend and 8 in the divisor. It is impossible to cancel further since there are no two numbers remaining (one in the dividend and one in the divisor) exactly divisible by any number except 1. Multiplying all the uncanceled numbers in the dividend together and dividing their product by the product of all the uncanceled numbers in the divisor, the result is the quotient, 8.

$$(b) \quad \frac{80 \times 60 \times 50 \times 16 \times 14}{70 \times 50 \times 24 \times 20} = \frac{\overset{8}{\cancel{80}} \times \overset{6}{\cancel{60}} \times \overset{2}{\cancel{50}} \times \overset{2}{\cancel{16}} \times \overset{2}{\cancel{14}}}{\overset{7}{\cancel{70}} \times \overset{5}{\cancel{50}} \times \overset{3}{\cancel{24}} \times \overset{2}{\cancel{20}}} = 32. \text{ Ans.}$$

$$(29) \quad 2,937 \times 864 \div 923 = 2,749 \frac{241}{923}. \text{ Ans.}$$

$$\begin{array}{r} 2937 \\ \times 864 \\ \hline 11748 \\ 17622 \\ 23496 \\ \hline 2537568 \end{array}$$

$$\begin{array}{r} 923 \overline{) 2537568} (2749 \frac{241}{923} \\ \underline{1846} \\ 6915 \\ \underline{6461} \\ 4546 \\ \underline{3692} \\ 8548 \\ \underline{8307} \\ 241 \end{array}$$

(30) We have given the quotient ($1,201\frac{1}{3}$) and the divisor (349) to find the dividend. Since we divide the dividend by the divisor to obtain the quotient, we must multiply the quotient by the divisor to obtain the dividend. Hence the dividend is $1,201\frac{1}{3} \times 349 = 419,328$. Now, 419,328 is the product of 468 and some other number; therefore, to find the other number, we divide 419,328 by 468. The number is 896. Ans.

$$\begin{array}{r}
 1201\frac{1}{3} \\
 \times 349 \\
 \hline
 10809 \\
 4804 \\
 3603 \\
 \hline
 179 \\
 419328
 \end{array}$$

$$\begin{array}{r}
 468 \overline{) 419328} \\
 \underline{3744} \\
 4492 \\
 \underline{4212} \\
 2808 \\
 \underline{2808}
 \end{array}$$

(31) $8,976 \times 4,298 = 38,578,848$.

$$\begin{array}{r}
 8976 \\
 \times 4298 \\
 \hline
 71808 \\
 80784 \\
 17952 \\
 \hline
 385904
 \end{array}$$

$$\begin{array}{r}
 38578848 \text{ minuend.} \\
 \underline{98765 \text{ subtrahend.}} \\
 38480083 \text{ remainder. Ans.}
 \end{array}$$

38578848 product.

(32) $[8,269 + 86,955 + 42,301 + 287,695] \div 1,492$.

$$\begin{array}{r}
 8269 \\
 86955 \\
 42301 \\
 287695 \\
 \hline
 1492 \overline{) 425220} \text{ (285 Ans.} \\
 \underline{2984} \\
 12682 \\
 \underline{11936} \\
 7460 \\
 \underline{7460}
 \end{array}$$

(33) (a) If an engine and boiler are worth \$3,246, and the building is worth 3 times as much, plus \$1,200, then the building is worth $\$3,246 \times 3 + \$1,200 = \$10,938$. If the tools are worth twice as much as the building, plus \$1,875, then the tools are worth $\$10,938 \times 2 + \$1,875 = \$23,751$. Ans.

$$\begin{array}{r}
 \$3246 \\
 \times 3 \\
 \hline
 \$9738 \\
 + \$1200 \\
 \hline
 \$10938 \\
 \times 2 \\
 \hline
 \$21876 \\
 + \$1875 \\
 \hline
 \$23751
 \end{array}$$

(b) $\$3,246 =$ value of engine and boiler.
 $\$10,938 =$ value of building and tools.
 $\$23,751 =$ value of whole plant. Ans.

(34)

$$\begin{array}{r}
 (a) \\
 60095001 \\
 43259867 \\
 \hline
 16835134 \text{ Ans.}
 \end{array}$$

$$\begin{array}{r}
 (b) \\
 842196340 \\
 557351092 \\
 \hline
 284845248 \text{ Ans.}
 \end{array}$$

$$\begin{array}{r}
 (c) \\
 10009876542 \\
 9877166431 \\
 \hline
 132710111 \text{ Ans.}
 \end{array}$$

$$\begin{array}{r}
 (d) \\
 43982176 \\
 969798 \\
 \hline
 42962378 \text{ Ans.}
 \end{array}$$

- (35) We have given the divisor (1,389) and the quotient (748), and are required to find the dividend. Placing these in the usual form of division, we have $1,389 \overline{) 748}$ — (748, in which the dash (—) represents the dividend or number sought. Leaving the remainder out of account, the dividend is evidently the product of 1,389 and 748, which is 1,038,972. But there is a remainder of 1,263, so that the actual dividend or number is
- $$1,038,972 + 1,263 = 1,040,235. \text{ Ans.}$$

- (36) Placing these numbers in the usual form of multiplication, we have

$$\begin{array}{r}
 4896 \\
 \hline
 3862944
 \end{array}$$

in which the dash (—) represents the multiplier or number we wish to find. 4,896 multiplied by this number gives the product, or 3,862,944. Then, if we divide 3,862,944 by 4,896, the quotient will be the multiplier required.

$$3,862,944 \div 4,896 = 789. \text{ Ans.}$$

$$\begin{array}{r}
 4896 \overline{) 3862944} (789 \\
 34272 \\
 \hline
 43574 \\
 39168 \\
 \hline
 44064 \\
 44064 \\
 \hline
 \end{array}$$

(37) (a)

$$\begin{array}{r}
 54085 \\
 72342 \\
 88700 \\
 11643 \\
 29872 \\
 \hline
 \text{balance } 24469 \\
 281111
 \end{array}$$

$$\begin{array}{r}
 38509 \\
 19714 \\
 38887 \\
 53141 \\
 25167 \\
 76905 \\
 28788 \\
 \hline
 281111 \\
 \text{balance } 24469 \text{ Ans.}
 \end{array}$$

(b)

847593	668721
288792	967842
989341	476976
540623	887674
249623	694523
474954	3695806
313671	balance 8791
3704597	3704597
balance 8791	Ans.

In example (a) it is clear that the sum of the right-hand set of numbers is the greater. Add both sets of figures and subtract the less sum, or that of the left-hand set, from the greater sum. Write the difference in the column that contains the less sum, thus writing 24,460 in the left-hand column, and also write it under the column containing the greater sum. Thus write 24,460 under 281,111. This difference is indicated by the word balance. In example (b) the same process is observed. Here the left-hand column is the greater.

(38)

\$ 25375 = amount left to his widow.
 \$ 12450 = amount left to one child.
 7
 \$ 87150 = amount left to 7 children.
 \$ 25375
 \$ 112525 = amount all received. Ans.

(39) If the farm is sold for \$375 an acre instead of \$246 an acre,
 129) 31734 (246 I gain \$375 - \$246 = \$129 on 1 acre. If the
 258 whole amount gained is \$31,734, then dividing
 593 the total gain on the farm by the amount
 510 gained on 1 acre gives the number of acres.
 774 31,734 ÷ 129 = 246. Hence the number of
 774 acres is 246. Ans.

(40) See Art. 94.

$$(a) \quad \frac{21 \times 93 \times 192 \times 43}{11 \times 24 \times 38 \times 18 \times 7} = \frac{3 \times 16 \times 3}{1} = 144. \text{ Ans.}$$

$$(b) \quad \frac{32 \times 64 \times 128 \times 96 \times 144}{11 \times 18 \times 25 \times 12 \times 36 \times 20} = \frac{3 \times 4 \times 2}{1} = 24. \text{ Ans.}$$

ARITHMETIC.

(1) The denominators are not alike, so before adding them we must find a common denominator for the denominators of all the fractions. Find the least common denominator as in Art. 26.

Place the denominators in a row, separated by commas.

Divide by some prime number that will divide at least two of the denominators without a remainder, bringing down to the row below those denominators that will not contain the divisor without a remainder. Thus using 2 as a divisor the second row becomes 3, 2, 3, 4, 6, since 2 will not divide 3 without a remainder. Using 3 as a divisor the result is 1, 2, 1, 4, 2. Again using 2 as a divisor for the third row, the result is 1, 1, 1, 2, 1. The product of these numbers and the divisors is $2 \times 3 \times 2 \times 2 = 24$, the least common denominator. We must reduce all the fractions to 24ths, and then add their numerators. $\frac{2}{3} = \frac{16}{24}$, $\frac{3}{4} = \frac{18}{24}$, $\frac{5}{6} = \frac{20}{24}$, $\frac{7}{8} = \frac{21}{24}$, $\frac{11}{12} = \frac{22}{24}$.

To reduce a fraction to a fraction having 24 for the denominator, multiply both terms of the fraction by some number that will make the denominator 24. Thus, $\frac{2}{3} \times 8 = \frac{16}{24}$; $\frac{3}{4} \times 6 = \frac{18}{24}$; $\frac{5}{6} \times 4 = \frac{20}{24}$; $\frac{7}{8} \times 3 = \frac{21}{24}$; $\frac{11}{12} \times 2 = \frac{22}{24}$.

Or divide the least common denominator by the denominator of the given fraction, and multiply both terms by the quotient.

The fractions now have a common denominator, 24; hence, we find their sum by adding the numerators and placing their sum over the common denominator; thus,

$$\frac{16}{24} + \frac{18}{24} + \frac{20}{24} + \frac{21}{24} + \frac{22}{24} = \frac{16 + 18 + 20 + 21 + 22}{24} = \frac{97}{24} = 4\frac{1}{24}. \text{ Ans.}$$

- (2) The common denominator is readily seen to be 24.

$$3\frac{1}{2} = 3\frac{12}{24}$$

$$6\frac{1}{3} = 6\frac{8}{24}$$

$$9 + 3\frac{12}{24} = 9 + 1\frac{12}{24} = 10\frac{12}{24}.$$

$$4\frac{1}{2} = 4\frac{12}{24}; 10\frac{12}{24} - 4\frac{12}{24} = 5\frac{12}{24}. \text{ Ans. See Art. 42.}$$

- (3)

$$\text{Bought } 12\frac{1}{2} = 12\frac{16}{32}$$

$$\text{Burned } 5\frac{1}{4} = \underline{5\frac{8}{32}}$$

$$\text{Remainder} = 6\frac{16}{32}.$$

Reducing the fractions to a common denominator, we find that $\frac{1}{8}$ in the subtrahend is greater than $\frac{1}{8}$. Borrow 1 from 12 (as in subtraction of whole numbers) calling the 1, $\frac{2}{8}$. $3\frac{2}{8} + \frac{1}{8} = 3\frac{3}{8}$. $3\frac{3}{8} - \frac{1}{8} = 3\frac{2}{8}$. We borrowed 1 from 12, hence, 5 from 11 is 6. The remainder is $6\frac{16}{32}$ tons. Ans.

(4) If 1 dress contains $6\frac{1}{2}$ yards, or $\frac{13}{2}$ yards, from $53\frac{1}{2}$ yards, or $\frac{107}{2}$ yards, you can make as many dresses as $\frac{13}{2}$ is contained in $\frac{107}{2}$. The fraction $\frac{13}{2}$ is contained in $\frac{107}{2}$, 8 times, for the denominators are the same and $160 \div 20 = 8$. Therefore, 8 dresses can be made. Ans.

$$(5) 83\frac{1}{2} \times 67\frac{1}{2} = \frac{167}{2} \times \frac{135}{2} = \frac{22545}{4} = 5.672\frac{1}{2}. \text{ Ans.}$$

First reduce the mixed numbers (see Art. 11) to improper fractions (see Art. 10) as $83\frac{1}{2} = \frac{167}{2}$ and $67\frac{1}{2} = \frac{135}{2}$ (see Art. 10). Then divide the product of the numerators by the product of the denominators, canceling if possible.

$$(6) 4\frac{1}{2} \times (2\frac{1}{2} + 1\frac{1}{2}) = \frac{9}{2} \times \frac{27}{4} + \frac{9}{2} = \frac{9}{2} \times \frac{27}{4} + \frac{5}{2} = \frac{135}{8} + \frac{10}{8} = 16\frac{1}{2}. \text{ Ans.}$$

Reducing the mixed numbers to improper fractions, inverting the divisor (see Art. 67), and multiplying, the result is $16\frac{1}{2}$.

(7) If 1 load consists of $1\frac{1}{2}$ cords, in 56 cords there are as many loads as $1\frac{1}{2}$ is contained in 56, or 32 loads. Ans.

$$56 \div 1\frac{1}{2} = \frac{56}{1} \times \frac{2}{3} = 32.$$

(8) If 1 yard of velvet costs \$3\frac{1}{2}, 24 yards will cost $\$3\frac{1}{2} \times 24 = \84 . Ans.

First multiply $\frac{1}{2}$ by 24; thus, $\frac{1}{2} \times \frac{24}{1} = 12$. Multiply 3 by 24 in the usual way.

$$\begin{array}{r} 3\frac{1}{2} \\ \times 24 \\ \hline 72 \\ 24 \\ \hline 84 \end{array}$$

$$(9) \quad \frac{4\frac{7}{8} \times 3\frac{2}{3} \times 3\frac{1}{2}}{1\frac{5}{8} \times 4\frac{2}{3}} = ?$$

Reducing the mixed numbers to fractions, the expression becomes

$$\frac{\frac{39}{8} \times \frac{11}{3} \times \frac{16}{5}}{\frac{13}{8} \times \frac{22}{3}}.$$

Regard the fractions above the long line as the numerator of a fraction whose denominator are the fractions below the line. Transferring the denominators of the fractions, or in reality inverting the divisor and multiplying (see Art. 67),

$$\frac{\overset{13}{39} \times \overset{2}{11} \times \overset{8}{16} \times 8 \times 3}{8 \times 3 \times 5 \times \underset{11}{13} \times \underset{22}{22}} = 8. \quad \text{Ans.}$$

(10) Reducing $8\frac{1}{10}$ to an improper fraction,

$$8\frac{1}{10} = \frac{81}{10}. \quad \text{Multiplying, } \frac{81}{10} \times \frac{1}{40} = \frac{81}{400}. \quad \text{Ans.}$$

(11) $43\frac{5}{7} \div 5\frac{2}{7} = ?$ Reducing to improper fractions, $43\frac{5}{7} = \frac{306}{7}$; $5\frac{2}{7} = \frac{37}{7}$. By Art. 67,

$$\frac{306}{7} \div \frac{37}{7} = \frac{306}{7} \times \frac{7}{37} = \frac{306}{37} = 8\frac{10}{37}. \quad \text{Ans.}$$

(12) First add $8\frac{3}{4}$, $7\frac{5}{8}$, and $10\frac{1}{2}$. The common denominator is 12. Then $8\frac{3}{4} = 8\frac{9}{12}$; $7\frac{5}{8} = 7\frac{9}{12}$; $10\frac{1}{2} = 10\frac{6}{12}$.

$$\begin{array}{r} 8 \frac{9}{12} \\ 7 \frac{9}{12} \\ 10 \frac{6}{12} \\ \hline 25 \frac{24}{12} = 25 + 2\frac{1}{2} = 27\frac{1}{2}. \end{array} \qquad \begin{array}{r} 100 \\ - 27\frac{1}{2} \\ \hline 72\frac{1}{2} \end{array}$$

Subtracting $27\frac{1}{2}$ from 100, the result is $72\frac{1}{2}$. Ans.

(13) If in 1 hour $3\frac{3}{4}$ gallons leak away, it will take as many hours for 40 gallons to leak away as $3\frac{3}{4}$ is contained in 40.

$$3\frac{3}{4} = \frac{15}{4}. \quad \frac{40}{1} \div \frac{15}{4} = \frac{40}{1} \times \frac{4}{15} = \frac{32}{3} = 10\frac{2}{3}.$$

Hence the time required is $10\frac{2}{3}$ hr. Ans.

$$(14) \quad \frac{65}{100} \times 4\frac{2}{3} \times 6\frac{1}{2} = \frac{13}{20} \times \frac{14}{3} \times \frac{13}{2} = \frac{13 \times 7 \times 13}{20 \times 3} = \frac{1,183}{60} = 19\frac{43}{60}. \quad \text{Ans.}$$

(15) If the diameter of a wheel is $8\frac{2}{3}$ feet and the circumference is $3\frac{5}{13}$ times the diameter, the circumference is

$$\frac{26}{3} \times \frac{355}{113} = \frac{9,230}{339} = 27\frac{77}{339} \text{ feet. Ans.}$$

(16) If a man earns \$97 $\frac{1}{2}$ in 26 days, in 1 day he will earn $\frac{1}{26}$ of \$97 $\frac{1}{2}$.

$$97\frac{1}{2} = 195; 195 \times \frac{1}{26} = \frac{195}{26} = \frac{15}{2} = 7\frac{1}{2}.$$

But $\frac{1}{2}$ of a dollar = 50 cents; hence, \$7 $\frac{1}{2}$ = \$7.50. Ans.

(17) If 1 soldier consumes $1\frac{5}{8}$ pounds, as many soldiers will be required to consume 663 pounds as $1\frac{5}{8}$ is contained in 663. $663 \div 1\frac{5}{8} = \frac{663}{1\frac{5}{8}}; 1\frac{5}{8} = \frac{13}{8}$. Dividing,

$$\frac{663}{1} \div \frac{13}{8} = \frac{663}{1} \times \frac{8}{13} = 102.$$

Hence there are 102 soldiers in the company. Ans.

(18) Since the son's weight is $\frac{5}{8}$ of the father's weight, it is evident that the weight of both is $1\frac{5}{8}$ times the weight of the father. $1\frac{5}{8} = \frac{13}{8}$; hence, $\frac{13}{8}$ of the father's weight is 312 pounds. Evidently, then, $\frac{1}{8}$ of the father's weight is $\frac{1}{13}$ of 312 pounds or 24 pounds, since $\frac{1}{8} = \frac{1}{13}$ of $\frac{13}{8}$; and if $\frac{1}{8}$ of his weight is 24 pounds, his weight is 24 pounds $\times 8 = 192$ pounds. Ans.

The son's weight is 192 pounds $\times \frac{5}{8} = 120$ pounds. Ans.

$$(19) \quad 8\frac{1}{2} \times 1\frac{1}{4} = \frac{17}{2} \times \frac{5}{4} = 11. \text{ Ans.}$$

(20) In example 15, it is stated that the circumference of a wheel is $3\frac{5}{13}$ times the diameter. Therefore, dividing the circumference, 281 feet, by $3\frac{5}{13}$, we obtain the diameter.

$$281 \div 3\frac{5}{13} = \frac{281}{1} \div \frac{355}{113} = \frac{281}{1} \times \frac{113}{355} = \frac{31753}{355} = 90\frac{113}{355}.$$

Hence the diameter is 90 $\frac{113}{355}$ feet. Ans.

(21) If 4 $\frac{1}{2}$ acres produce 96 bushels of wheat, 1 acre will produce as many bushels as 4 $\frac{1}{2}$, or $\frac{9}{2}$, is contained in 96.

$$\frac{96}{1} \div \frac{9}{2} = \frac{96}{1} \times \frac{2}{9} = 20.$$

The yield is therefore 20 bushels per acre. Ans.

(22) If 1 yard of cloth cost \$7, for \$91 you can buy as many yards as $\frac{7}{8}$ is contained in 91, or 104 yards. Ans.

$$\frac{91}{1} \div \frac{7}{8} = \frac{91}{1} \times \frac{8}{7} = 104.$$

$$(23) \quad \frac{2}{3} \text{ of } 6\frac{1}{2} = \frac{2}{3} \times \frac{13}{2} = \frac{13}{3}; \quad 4\frac{3}{4} \times \frac{13}{3} = \frac{19}{4} \times \frac{13}{3} = \frac{247}{12} = 20\frac{7}{12}. \quad \text{Ans.}$$

$$(24) \quad \text{One ton cost } \$23\frac{3}{4}, 24\frac{3}{4} \text{ tons will cost } 24\frac{3}{4} \times \$23\frac{3}{4} = \$587\frac{1}{8}. \quad \text{Ans.}$$

This may be solved in two ways: either by reducing the mixed numbers to improper fractions or by multiplying the mixed numbers as follows:

$$\begin{array}{r} 23\frac{3}{4} \\ 24\frac{3}{4} \\ \hline 4)69 \\ 4)72 \\ \hline 17\frac{1}{4} \\ 18 \\ 92 \\ \hline 46\frac{3}{4} \\ 587\frac{1}{8} \end{array}$$

First multiply 23 by $\frac{3}{4}$; multiply 23 by 3 and divide by 4, but in order to save space the division is merely indicated for the present. $\frac{3}{4}$ is multiplied by 24 in the same manner, multiplying 24 by 3, and indicating the division by 4. 69 (i. e., 23×3) is now divided by 4, obtaining $17\frac{1}{4}$; 72 (i. e., 24×3) is divided by 4, obtaining 18, which is written under the $17\frac{1}{4}$. Then multiply 23 by 24 in the usual manner, placing the unit figure under the unit figures of the two mixed numbers. Finally multiply $\frac{3}{4}$ by $\frac{3}{4}$. Adding the separate products, the result is $587\frac{1}{8}$.

(25) The first one gets $\frac{1}{3}$ of the amount or $\$3,696 \times \frac{1}{3} = \$1,232$. Ans. The second gets $\frac{2}{3}$ of the remainder or $(\$3,696 - \$1,232) \times \frac{2}{3} = \$2,464 \times \frac{2}{3} = \$985\frac{2}{3}$. Ans. The third gets what still remains, or $\$3,696 - (\$1,232 + \$985\frac{2}{3}) = \$3,696 - \$2,217\frac{2}{3} = \$1,478\frac{2}{3}$. Ans.

(26) If 25 turkeys cost $\$18\frac{3}{4}$, 1 turkey will cost as much as 25 is contained in $\$18\frac{3}{4}$.

$$18\frac{3}{4} = \frac{75}{4}; \quad \frac{75}{4} \div 25 = \frac{75 \div 25}{4} = \frac{3}{4}.$$

Hence 1 turkey costs $\$3\frac{3}{4}$. Ans.

(27) If $9\frac{3}{4}$ acres cost $\$357\frac{1}{2}$, 1 acre will cost as many dollars as $9\frac{3}{4}$ is contained in $\$357\frac{1}{2}$. $357\frac{1}{2} = \frac{715}{2}$; $9\frac{3}{4} = \frac{39}{4}$;

$$\frac{715}{2} \div \frac{39}{4} = \frac{715}{2} \times \frac{4}{39} = \frac{1,430}{39} = 36\frac{2}{3} = 36\frac{4}{6}.$$

The price per acre is therefore $\$36\frac{2}{3}$. Ans.

(28) If the sum of two numbers multiplied by $18\frac{2}{3}$ is $296\frac{2}{3}$, then $296\frac{2}{3}$, or $\frac{890}{3}$, divided by $18\frac{2}{3}$, or $\frac{56}{3}$, is equal to the sum of the numbers.

$$\frac{890}{3} \div \frac{56}{3} = \frac{890}{3} \times \frac{3}{56} = \frac{445}{28} = 15\frac{5}{8}.$$

$15\frac{5}{8}$ is the sum of the numbers; if one of the numbers is $12\frac{3}{8}$, then the other is $15\frac{5}{8} - 12\frac{3}{8} = 15\frac{5}{8} - 12\frac{3}{8} = 3\frac{2}{8} = 3\frac{1}{4}$. Ans.

(29) The difference between $28\frac{1}{2}$ and $14\frac{5}{8}$ is $13\frac{1}{4}$.

$$\begin{array}{r} 28\frac{1}{2} = 28\frac{2}{4} = 27\frac{4}{4} \\ 14\frac{5}{8} = 14\frac{5}{8} = 14\frac{5}{8} \\ \hline 13\frac{1}{4} = 13\frac{1}{4} \text{ difference.} \end{array}$$

If this difference, $13\frac{1}{2}$, be multiplied by a certain number, the product is $49\frac{1}{2}$, or if $49\frac{1}{2}$ be the product of two numbers one of which is $13\frac{1}{2}$, then the other must be $49\frac{1}{2}$ divided by $13\frac{1}{2}$. $49\frac{1}{2} \div 13\frac{1}{2} = 3\frac{1}{2}$;

$$\frac{246}{5} \div \frac{41}{3} = \frac{246}{5} \times \frac{3}{41} = \frac{18}{5} = 3\frac{1}{2}. \text{ Ans.}$$

(30) We have given the dividend ($114\frac{1}{2}$) and the quotient ($3\frac{1}{2}$), and are required to find the divisor. The divisor is equal to the dividend divided by the quotient.

$$114\frac{1}{2} \div 3\frac{1}{2} = 31\frac{1}{2};$$

$$\frac{915}{8} \div \frac{15}{4} = \frac{915}{8} \times \frac{4}{15} = \frac{61}{2} = 30\frac{1}{2}.$$

To give a quotient of $3\frac{1}{2}$, $114\frac{1}{2}$ must be divided by $30\frac{1}{2}$; hence, $20\frac{1}{2}$ must be increased by $30\frac{1}{2} - 20\frac{1}{2} = 9\frac{1}{2}$. Ans.

(31) If a man cuts $3\frac{1}{4}$ cords of wood per day, to cut 45 cords it will take as many days as $3\frac{1}{4}$, or $\frac{13}{4}$, is contained in 45.

$$\frac{45}{1} \div \frac{13}{4} = \frac{45}{1} \times \frac{4}{13} = 12.$$

Therefore, 12 days are required. Ans.

(32) If 1 barrel costs \$6 $\frac{1}{2}$, for \$81 you can buy as many barrels as $6\frac{1}{2}$ is contained in 81, or 12 barrels. Ans.

$$6\frac{1}{2} = \frac{27}{4}; \frac{81}{1} \div \frac{27}{4} = \frac{81}{1} \times \frac{4}{27} = 12.$$

(33) The difference between the numbers is the quotient of $41\frac{1}{2}$ divided by $12\frac{1}{2}$.

$$41\frac{1}{2} \div 12\frac{1}{2} = 3\frac{1}{2};$$

$$\frac{206}{5} \div \frac{103}{8} = \frac{206}{5} \times \frac{8}{103} = \frac{16}{5} = 3\frac{1}{5}.$$

Since $3\frac{1}{5}$ is the difference between two numbers, and 21 is the larger of the two numbers, the smaller must be $21 - 3\frac{1}{5} = 17\frac{4}{5}$. Ans.

(34) If the wheel goes 108 feet in turning $5\frac{1}{2}$ times, in turning once it will go as many feet as $5\frac{1}{2}$, or $\frac{11}{2}$, is contained in 108.

$$108 \div 5\frac{1}{2} = 108 \div \frac{11}{2} = \frac{216}{11} = 19\frac{6}{11}.$$

Hence the circumference is $19\frac{6}{11}$ feet. Ans.

$$\begin{aligned}
 (35) \quad & \frac{4\frac{1}{2} \times 5\frac{1}{2} \times 9\frac{5}{8}}{3\frac{1}{2} \times 8\frac{5}{14}} = \frac{\frac{14}{2} \times \frac{25}{2} \times \frac{77}{8}}{\frac{7}{2} \times \frac{117}{14}} \\
 & = \frac{\overset{2}{14} \times \overset{12}{33} \times \overset{4}{77} \times \overset{4}{12} \times 14}{\underset{2}{3} \times \underset{2}{7} \times \underset{2}{8} \times \underset{13}{47} \times \underset{13}{117}} = \frac{4,312}{611} = 7\frac{25}{11}. \text{ Ans.}
 \end{aligned}$$

First reduce the mixed numbers to fractions; then divide the fractions in the numerator by the fractions in the denominator, inverting the fractions in the denominator, and canceling wherever possible.

$$\begin{array}{r}
 (36) \quad \begin{array}{r} 19\frac{1}{2} \\ 42\frac{3}{16} \\ \hline 16)57 \\ 2)42 \\ \hline 3\frac{9}{16} \\ 21 \\ 38 \\ 76\frac{3}{2} \\ \hline 822\frac{21}{2} \end{array} \quad \begin{array}{l} 19\frac{1}{2} = \frac{39}{2}; \\ 42\frac{3}{16} = \frac{675}{16}; \\ \frac{39}{2} \times \frac{675}{16} = \frac{26325}{32} = 822\frac{21}{2}. \end{array}
 \end{array}$$

Proceed as in example 24; multiply 19 by $\frac{3}{16}$, then $\frac{1}{2}$ by 42, then $\frac{1}{2}$ by $\frac{3}{16}$, then 19 by 42. Adding the partial products, the result is $822\frac{21}{2}$. Ans.

$$\begin{aligned}
 (37) \quad & \frac{1}{2} + \frac{1}{4} + \frac{2}{5} + \frac{5}{6} = \frac{20}{60} + \frac{15}{60} + \frac{24}{60} + \frac{50}{60} = \frac{20 + 15 + 24 + 50}{60} \\
 & = \frac{109}{60} = 1\frac{49}{60}. \text{ Ans.}
 \end{aligned}$$

$$\begin{aligned}
 (38) \quad & \begin{array}{r} 67\frac{5}{8} = 67\frac{15}{24} \\ 48\frac{2}{3} = 48\frac{16}{24} \\ \hline 18\frac{29}{24} \text{ difference.} \end{array} \text{ Ans.}
 \end{aligned}$$

Reduce $\frac{5}{8}$ and $\frac{2}{3}$ to a common denominator, 24. $\frac{15}{24}$ is greater than $\frac{16}{24}$, so we borrow 1 from 7, calling it $\frac{24}{24}$. $\frac{15}{24} + \frac{24}{24} = \frac{39}{24}$; $\frac{39}{24} - \frac{16}{24} = \frac{23}{24}$. Then 48 from 66 leaves 18.

(39) If a man travels $85\frac{5}{12}$ miles in one day, $78\frac{9}{15}$ miles the next day, and $125\frac{17}{15}$ miles the third day; in the three days he will travel the sum of the three distances.

$$\begin{aligned}
 85\frac{5}{12} &= 85\frac{175}{120} \\
 78\frac{9}{15} &= 78\frac{528}{120} \\
 125\frac{17}{15} &= 125\frac{296}{120} \\
 \hline
 \text{Sum} &= 288\frac{999}{120} = 288 + 14\frac{11}{10} = 289\frac{11}{10}.
 \end{aligned}$$

Hence the distance traveled is $289\frac{11}{10}$ miles. Ans.

(40) $211\frac{1}{4} = 84^{\frac{1}{2}}$; $1\frac{7}{8} = 1^{\frac{7}{8}}$; $84^{\frac{1}{2}} \times 1^{\frac{7}{8}} = 126^{\frac{7}{8}}$; hence the cost of the lead was $126^{\frac{7}{8}}$ cents.

If by selling it at $2\frac{1}{2}$ cents per pound I received the same amount as I paid for the whole, or $126^{\frac{7}{8}}$ cents, I sold as many pounds as $2\frac{1}{2}$, or $\frac{5}{2}$, is contained in $126^{\frac{7}{8}}$.

$$\frac{12,675}{32} + \frac{5}{2} = \frac{2,535}{16} \times \frac{2}{5} = \frac{2,535}{16} = 158\frac{7}{8}.$$

I therefore sold $158\frac{7}{8}$ pounds and have left $211\frac{1}{4} - 158\frac{7}{8} = 52\frac{1}{8}$ pounds. Ans.

$$\begin{array}{r} 211\frac{1}{4} = 211\frac{2}{8} = 210\frac{10}{8} \\ 158\frac{7}{8} = 158\frac{7}{8} = 158\frac{7}{8} \\ \hline 52\frac{1}{8}. \end{array}$$

ARITHMETIC.

(1) To reduce a fraction to a decimal, annex ciphers to the numerator and divide by the denominator. Point off as many places in the quotient as there are ciphers annexed.

$$\begin{array}{r}
 32 \overline{) 17.00000} (.53125 \\
 \underline{160} \\
 100 \\
 \underline{96} \\
 40 \\
 \underline{32} \\
 80 \\
 \underline{64} \\
 160 \\
 \underline{160} \\
 0
 \end{array}$$

Hence, $\frac{17}{32} = .53125$. Ans.

$$\begin{array}{r}
 24 \overline{) 25.00000} (1.04166 \\
 \underline{24} \\
 100 \\
 \underline{96} \\
 40 \\
 \underline{24} \\
 160 \\
 \underline{144} \\
 160 \\
 \underline{144} \\
 16
 \end{array}$$

$$(2) \quad 2\frac{1}{2} \times 3\frac{1}{3} \div 8 = \frac{5}{2} \times \frac{10}{3} \times \frac{1}{8} = \frac{5}{24}.$$

Reduce $\frac{5}{24}$ to a decimal as in Art. 33.

Since this is an unending decimal (see Art. 30) and the fifth figure is 6, we increase the fourth figure by 1, obtaining as the result 1.0417—. Ans.

(8) By the dots placed over the 2 and the 4 in the decimal $\dot{2}8571\dot{4}$ we understand that the figures $\dot{2}8571\dot{4}$ repeat. Applying the rule in Art. 52, we have

$$\begin{array}{r}
 \dot{2}8571\dot{4} = \frac{285,714}{999,999} \\
 \S 3
 \end{array}$$

placing the decimal point in the sum directly under the decimal points above.

(10) If the diameter of the earth is 7,912 miles and the circumference is 3.1416 times the diameter, the circumference is $7,912 \times 3.1416 = 24,856.3392$ miles. Ans.

Point off four decimal places in the product as there are four decimal places in the multiplier.

$$\begin{array}{r} 7912 \\ 3.1416 \\ \hline 47472 \\ 7912 \\ 31648 \\ 7912 \\ \hline 23736 \\ \hline 24856.3392 \end{array}$$

(11) Since $.37\frac{1}{2} = \frac{3}{8}$, $\$2.37\frac{1}{2} = \$2\frac{3}{8}$, or $\$1\frac{1}{2}$.

If 1 yard of cloth cost $\$1\frac{1}{2}$, for $\$133$ you can buy as many yards as $\frac{1}{2}$ is contained in 133, or 56 yards. Ans.

$$\frac{133}{1} \div \frac{19}{8} = \frac{133}{1} \times \frac{8}{19} = 56.$$

(12) $\frac{3}{8}$ of $\frac{3}{4}$ of .0168 = .00672. Ans.

$\frac{3}{8} \times \frac{3}{4} = \frac{9}{32}$. $\frac{9}{32}$ reduced to a decimal equals .4.

The number of decimal places in the product must be five, since there are four in the multiplicand and one in the multiplier (see Art. 15). Hence, we prefix two ciphers to 672, to make five places in the product.

$$\begin{array}{r} .0168 \\ .4 \\ \hline .00672 \end{array}$$

(13) If $\frac{3}{4}$ of a ton of hay cost $\$14\frac{1}{4}$, or $\$17$, $\frac{1}{4}$ of a ton would cost only

$\frac{1}{3}$ as much, or $\frac{1}{3} \times \frac{19}{4} = \$\frac{19}{4}$, and $\frac{1}{4}$, or a whole ton, would cost $\frac{4}{1} \times \frac{19}{4} = \19 . Ans.

(14) .37 of a number + .23 of a number is 33.6.

If .6 of the number is 33.6, the number must be $33.6 \div .6 = 56$. Ans.

In dividing decimals observe Art. 22. In this example there is one decimal place in the divisor and one decimal place in the dividend; therefore, there are no decimal places in the quotient, or in other words, the quotient is a whole number.

$$\begin{array}{r} .37 \\ .23 \\ \hline .60 \\ .6 \overline{) 33.6} \\ \hline 56 \end{array}$$

(15) Expressing .8 in a fractional form, $.8 = \frac{8}{10}$, or $\frac{4}{5}$. Subtracting

$\frac{2}{3}$ from $\frac{4}{5}$,

$$\frac{4}{5} - \frac{2}{3} = \frac{12}{15} - \frac{10}{15} = \frac{2}{15}.$$

87.8 is $\frac{2}{15}$ of the number required; hence, $\frac{1}{15}$ of the number is $\frac{1}{2}$ of 87.8, or 43.9, and the number is $15 \times 43.9 = 658.5$. Ans.

(16) $14\frac{1}{2} - 9\frac{1}{2} = 5\frac{1}{2} = 5.125$. Since 5.125 yards of cloth are bought for \$8.20, the price per yard = $\$8.20 \div 5.125 = \1.60 . Ans.

$$\begin{array}{r} 5.125 \overline{) 8.20000} (1.60 \\ \underline{5125} \\ 30750 \\ \underline{30750} \\ 0 \end{array}$$

(17) In .5625 of a mile there are $5,280 \times .5625 = 2,970$ feet. Ans.

$$\begin{array}{r} 5280 \\ \times .5625 \\ \hline 26400 \\ 10560 \\ 31680 \\ \hline 2970.0000 \end{array}$$

(18) In 1 mile there are 5,280 feet; therefore, 4,620 feet = $4,620 \div 5,280 = .875$ mile.

$$\begin{array}{r} 5280 \overline{) 4620.000} (.875 \\ \underline{42240} \\ 39600 \\ \underline{36960} \\ 26400 \\ \underline{26400} \\ 0 \end{array}$$

Reducing $\frac{4620}{5280}$ to a decimal, 4,620 ft. = .875 mile. Ans.

$$\begin{array}{r} 16.083\frac{1}{3} \\ \times 36 \\ \hline 96498 \\ 48249 \\ \hline 579.000 \end{array}$$

(19) If the body falls $16.083\frac{1}{3}$ feet in 1 second, in 6 seconds it will fall 6^2 , or 36, times $16.083\frac{1}{3}$ feet, or 579 feet. Ans.

(20) In 1 day he earns $\$3.87\frac{1}{2}$, or \$3.875; therefore, to earn \$155 will take 40 days. Ans.

$$\begin{array}{r} 3.875 \overline{) 155.000} (40 \\ \underline{15500} \\ 0 \end{array}$$

(21) In 1 revolution the wheel travels $18\frac{1}{3}$, or $\frac{55}{3}$, feet. In going 1 mile, or 5,280 feet, it will make as many turns as $\frac{55}{3}$ is contained in 5,280, or 288 turns. Ans.

$$\frac{5,280}{1} \div \frac{55}{3} = \frac{5,280}{1} \times \frac{3}{55} = 288.$$

(22) If \$4,230 is .45 of the price of a farm, then \$4,230 divided by .45 gives the price of the farm.

Hence the price is \$9,400. Ans.

$$\begin{array}{r} .45 \overline{) 4230.00} (9400 \\ \underline{405} \\ 180 \\ \underline{180} \\ 0 \end{array}$$

(23) Since \$.18 $\frac{1}{4}$, or \$.1875, is the cost of 1 yard, 18.75 yards will cost $18.75 \times $.1875 = \$3.52$. Ans.

As there are four decimal places in the multiplicand and two in the multiplier, there are $4 + 2$, or 6, decimal places in the product. The result being expressed in dollars, it is unnecessary to use all the decimal places, so we call it \$3.52.

$$\begin{array}{r} .1875 \\ 18.75 \\ \hline 9375 \\ 13125 \\ 15000 \\ 1875 \\ \hline 3.515625, \text{ or } 3.52 \end{array}$$

(24) 6 barrels of flour at \$6.375 per barrel cost \$38.25. 28 bushels of potatoes at \$.875 per bushel cost \$24.50. 120 pounds of sugar at \$.03 $\frac{1}{4}$ a pound cost \$3.75.

$$\begin{array}{r} 6.375 \\ 6 \\ \hline 38.250 \end{array}$$

$$\begin{array}{r} .875 \\ 28 \\ \hline 7000 \\ 1750 \\ \hline 24.500 \end{array}$$

$$\begin{array}{r} .03\frac{1}{4} = .03125 \\ .03125 \\ \hline 120 \\ \hline 3.75000 \end{array}$$

Cost of flour,	\$ 38.25	
Cost of potatoes,	\$ 24.50	
Cost of sugar,	\$ 3.75	
Entire cost,	\$ 66.50	Ans.

(25) If the lot cost \$3,300, and this is .165 of the cost of the house, then the house cost $\$3,300 \div .165 = \$20,000$. Ans.

$$\begin{array}{r} .165 \overline{) 3300.000} \\ \underline{20000} \end{array}$$

(26) The increase per day is $\$2.25 - \1.875 , or \$.375; hence, the increase for 300 days is $$.375 \times 300 = \112.50 . Ans.

$$\begin{array}{r} \$2.250 \\ \$1.875 \\ \hline \$.375 \end{array} \qquad \begin{array}{r} \$375 \\ 300 \\ \hline \$112.500 \end{array}$$

(27)

Paid to butcher,	.143
Paid to grocer,	.347
Paid for clothing,	.256
Other expenses,	.154
Total expenses,	.900

Since he spent .9 of his salary, he saved .1 of it. If his butcher receives \$327.47, which is .143 of his entire salary, the salary must be $\$327.47 \div .143 = \$2,290$.

He therefore saves $\$2,290 \times .1 = \229 .
Ans.

$$\begin{array}{r} .143 \overline{) 327.470} \quad (2290 \\ \underline{286} \\ 414 \\ \underline{286} \\ 1287 \\ \underline{1287} \end{array}$$

(36) Reducing $\frac{5}{13}$ to a decimal, $\frac{5}{13} = .\dot{3}8461\dot{5}$. Ans.

$$\begin{array}{r}
 13 \overline{) 5.000000} \quad (. \dot{3}8461\dot{5} + \\
 \underline{39} \\
 110 \\
 \underline{104} \\
 60 \\
 \underline{52} \\
 80 \\
 \underline{78} \\
 20 \\
 \underline{13} \\
 70 \\
 \underline{65} \\
 50
 \end{array}$$

It will be noticed that after obtaining the decimal .384615, if we continue dividing, the number 384615 will repeat, as .384615 384615, and the process will never terminate. Now instead of writing such a result, it may be written $.\dot{3}8461\dot{5}$, the dots over the 3 and 5 indicating that the figures 384615 repeat.

(37) By Art. 52, $.35\dot{1} = \frac{351}{1000} = \frac{117}{333} = \frac{39}{111} = \frac{13}{37}$. Ans.

$$\begin{array}{r}
 .30208\dot{3}\dot{1} \\
 \underline{24} \\
 8 \\
 1208332 \\
 \underline{604166} \\
 7.250000
 \end{array}$$

(38) By Art. 53, $.30208\dot{3}\dot{1} = .302083\dot{1}_3 = .302083\dot{1}_3$. By Art. 41, $\frac{.302083\dot{1}_3 \times 24}{24} = \frac{7.25}{24}$, or $\frac{71}{24}$. Ans.

$$\begin{array}{r}
 .7292 \\
 \underline{40} \\
 29.1680
 \end{array}$$

(39) By Art. 41, $.7292 \times \frac{40}{10} = 29.168$, say $\frac{73}{25}$. Ans.

$$\begin{array}{r}
 (40) \quad .0357 \\
 .0714 \\
 37.13 \quad .7854 \quad 6.5 \quad 6.5 \quad 35 \\
 \hline
 74.26 \times 3.1416 \times 19.5 \times 19.5 \times 350 = \frac{37.13 \times .0357 \times 6.5 \times 6.5 \times 35}{100} \\
 \hline
 \frac{33000 \times 12 \times 4}{33000 \times 4} = \frac{33000 \times 2}{100} \\
 \hline
 = \frac{1,960.14}{100} = 19.6014. \text{ Ans.}
 \end{array}$$

Decimals may be canceled just as whole numbers, but care must be taken to properly locate the decimal point. After canceling wherever possible to shorten the work of multiplication, divide the product of the numbers in the numerator by the denominator.

ARITHMETIC.

(1) Since there are 3 ft. in 1 yd., in 83.75 yd. there are $3 \times 83.75 = 251.25$ ft. There are 12 in. in 1 ft.; therefore, in 251.25 ft. there are $12 \times 251.25 = 3,015$ in.

Ans.

$$\begin{array}{r}
 83.75 \\
 \underline{3} \\
 251.25 \\
 \underline{12} \\
 50250 \\
 25125 \\
 \hline
 3015.00
 \end{array}$$

$$\begin{array}{r}
 .67 \\
 320 \\
 \hline
 134 \\
 201 \\
 \hline
 214.40 \text{ rd.}
 \end{array}$$

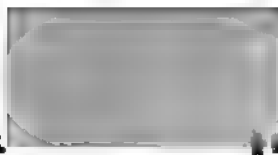
$$\begin{array}{r}
 .4 \text{ rd.} \\
 5\frac{1}{2} \\
 \hline
 2 \\
 20 \\
 \hline
 2.2 \text{ yd.}
 \end{array}$$

$$\begin{array}{r}
 .2 \\
 3 \\
 \hline
 .6 \text{ ft.} \\
 12 \\
 \hline
 7.2 \text{ in.}
 \end{array}$$

(2) Multiply .67 by 320, the number of rods in a mile, and the product, 214.4, is the number of rods in .67 of a mile. Multiply now the decimal part, .4, by $5\frac{1}{2}$, the number of yards in a rod, and the product, 2.2, is the number of yards in .4 of a rod. Again multiplying the decimal part, .2, by 3, we reduce the .2 of a yard to .6 of a foot. Multiplying by 12 to reduce .6 ft. to inches, we have 7.2 in. Collecting, the final result is 214 rd. 2 yd. 7.2 in. Ans.

(3) In 1 mi. there are 5,280 ft.; hence, in 15 mi. there are $5,280 \times 15 = 79,200$ ft. If the poles are 120 ft. apart, it will take as many poles as 120 is contained in 79,200, or 660 poles. Ans.

$$\begin{array}{r}
 5280 \\
 \underline{15} \\
 26400 \\
 5280 \\
 \hline
 120 \overline{)79200} \\
 \underline{660}
 \end{array}$$



(4) As one place is in *west longitude* and the other in *east longitude*, we must add to find the difference. $23^{\circ} 23' + 5^{\circ} 23' = 28^{\circ} 50'$; $50' = \frac{5}{6}$, or $\frac{1}{6}$. If in 1° there are 69.16 mi. (see example 16), in $28\frac{5}{6}$ there are $69.16 \times 28\frac{5}{6} = 1,994.11$ mi. Ans.

$$\begin{array}{r}
 23^{\circ} 23' \\
 5^{\circ} 23' \\
 \hline
 28^{\circ} 50' = 28\frac{5}{6}
 \end{array}
 \qquad
 \begin{array}{r}
 69.16 \\
 \times 28\frac{5}{6} \\
 \hline
 6) 34580 \\
 5768\frac{1}{6} \\
 55828 \\
 \hline
 13888 \\
 199411\frac{1}{6}, \text{ or say } 1,994.11
 \end{array}$$

$$(5) \quad 28 \times \frac{3}{4} + \frac{7}{8} = \frac{4}{28} \times \frac{3}{4} \times \frac{2}{7} = 24.$$

The surface to be lined is $28 \times \frac{3}{4}$, or 21 sq. yd. A piece of lining a yard long has an area of $\frac{3}{4}$ square yard; hence, as many yards will be required as $\frac{3}{4}$ is contained in 21. $21 \div \frac{3}{4} = \frac{4}{3} \times 21 = 28$. Hence 28 yd. are required. Ans.

$$\begin{array}{r}
 (6) \quad 3 \times \frac{9}{2} \times \frac{3}{8} = 81. \qquad \begin{array}{r} 1375 \\ 81 \\ \hline 1875 \\ 11000 \\ \hline 111375 \end{array} \qquad \begin{array}{l} \text{The dimensions of the block} \\ \text{being } 3 \text{ ft.} \times 4\frac{1}{2} \text{ ft.} \times 8 \text{ ft., it} \\ \text{contains } 81 \text{ cu. ft. If 1 cu. ft.} \\ \text{costs } \$1.375, 81 \text{ cu. ft. will cost} \\ 81 \times \$1.375 = \$111.375. \quad \text{Ans.} \end{array}
 \end{array}$$

$$\begin{aligned}
 (7) \quad \text{Perimeter of the room} &= 2(20 \text{ ft.} + 30 \text{ ft.}) = 100 \text{ ft.} \\
 \text{Area of the walls} &= (12 \times 100) \text{ sq. ft.} = 1,200 \text{ sq. ft.} \\
 \text{Area of the ceiling} &= (20 \times 30) \text{ sq. ft.} = 600 \text{ sq. ft.} \\
 \text{Total area} &= 1,200 \text{ sq. ft.} + 600 \text{ sq. ft.} = 1,800 \text{ sq. ft.} \\
 \text{Area in square yards} &= (1,800 \div 9) \text{ sq. yd.} = 200 \text{ sq. yd.}
 \end{aligned}$$

Allowing 1 bunch of laths for 3 sq. yd. of surface, for 200 sq. yd. $2\frac{2}{3}$ bunches are required. If 1 bunch costs \$.37 $\frac{1}{2}$, or $3\frac{1}{4}$, $2\frac{2}{3}$ bunches will cost $3\frac{1}{4} \times 2\frac{2}{3} = \25 . Ans.

(8) If the diameter is 12 ft. the circumference is $12 \times 3.1416 = 37.6992$ ft. 2 ft. 8 in. = $2\frac{2}{3}$ ft., or $\frac{8}{3}$ ft. To give 37.6992 ft. $\frac{8}{3}$ ft. must be repeated as many times as $\frac{8}{3}$ is contained in 37.6992, or

$$37.6992 \div \frac{8}{3} = \frac{4.7124}{1} \times \frac{3}{8} = 14.1372 \text{ times. Ans.}$$

(9)	da.	hr.	min.	sec.
	365	5	48	49.7
				5
	8) 1,825	25	240	248.5
	228	6	37	61.0625 (1 min. 1.0625 sec.)
			1	
	228	6	38	1.0625

Multiplying the units of each denomination by 5, we have 1,825 da. 25 hr. 240 min. 248.5 sec. Dividing, 1,825 contains 8,228 times with 1 da. remaining; 1 da. = 24 hr., and, adding to the 25 hr. in the dividend, $24 + 25 = 49$ hr., $49 \div 8 = 6$ hr. and 1 hr. remaining; 1 hr. = 60 min., and $60 + 240 = 300$ min.; $300 \div 8 = 37$ min. and 4 min. remaining. Reducing 4 min. to seconds, $4 \times 60 = 240$ sec.; $240 + 248.5 = 488.5$ sec. $488.5 \div 8 = 61.0625$ sec. But 61.0625 sec. is equal to 1 min. 1.0625 sec. Adding 1 min. to the 37 min., the result is 38 min. The final result is 228 da. 6 hr. 38 min. 1.0625 sec. Ans.

(10) $48 \times 72 = 3,456$; therefore, a rectangular field, 48 rd. \times 72 rd. contains 3,456 sq. rd.

In 1 A. there are 160 sq. rd., and since $3,456 \div 160 = 21.6$, in 3,456 sq. rd. there are 21.6 A. Ans.

$$\begin{array}{r}
 72 \\
 48 \\
 \hline
 576 \\
 288 \\
 \hline
 160 \overline{) 3456} \quad (21.6 \\
 320 \\
 \hline
 256 \\
 160 \\
 \hline
 960 \\
 960 \\
 \hline
 \end{array}$$

(11) Since there are 10 sq. ch. in 1 A., in 12.625 A. there are $10 \times 12.625 = 126.25$ sq. ch. Ans.

(12) 5 T. 875 lb. = $5\frac{875}{2240}$ T. = $5\frac{25}{64}$ T. = $5\frac{45}{64}$ T.

If 5 T. 875 lb., or $5\frac{45}{64}$ T., cost \$9.135, 1 T. will cost as many dollars as $5\frac{45}{64}$ is contained in \$9.135, or \$1.69. Ans.

$$9.135 \div \frac{345}{64} = \frac{1.827}{1} \times \frac{64}{345} = \frac{116.928}{69} = 1.69$$

(18) 5 lb. 12 oz. = $5\frac{1}{2}$ lb. = 5.75 lb.
 If 5.75 lb. cost \$1.15, 1 lb. will cost \$1.15 divided
 by 5.75, or \$.20. Ans.

$$\begin{array}{r} 5.75 \overline{) 1.1500} (.20 \\ \underline{1150} \\ 0 \end{array}$$

$$\begin{array}{r} (14) \ 8.1416 \\ \quad 28 \\ \hline 251828 \\ 62882 \\ \hline 87.9648 \end{array}$$

$$\begin{array}{r} 5280 \\ \quad 8.5 \\ \hline 26400 \\ 15840 \\ \hline 18480.0 \\ \quad 12 \\ \hline 87.9648 \overline{) 221760.00000000} (2521.008+ \\ \underline{1759296} \\ 4583040 \\ \underline{4398240} \\ 1848000 \\ \underline{1759296} \\ 887040 \\ 879648 \\ \hline 7892000 \\ \underline{7087184} \end{array}$$

3.5 mi. = 18,480 ft. = 221,760 in. If the diameter is 28 in., the circumference is $28 \times 3.1416 = 87.9648$ in. The wheel will turn as many times as 87.9648 is contained in 221,760 in., or 2,521.008+ times. Ans.

(15) An acre contains 160 sq. rd. 8 in. = $\frac{8}{12}$ ft. = $\frac{1}{15}$ rd. = $\frac{4}{99}$ rd.

$\frac{4}{99} \times 40 = \frac{160}{99}$; hence, the area of a furrow 8 in. wide and 40 rd. long is $\frac{160}{99}$ sq. rd. $160 \div \frac{160}{99} = 160 \times \frac{99}{160} = 99$; therefore, 99 of these furrows are equivalent to 1 A. Ans.

(16) $20' \ 30'' = 20.5' = \frac{20.5^\circ}{60}$
 $= .341\frac{2}{3}^\circ$. $42^\circ 20' 30'' = 42.341\frac{2}{3}^\circ$.
 Since 1° is equal to 69.16 mi.,
 $42.341\frac{2}{3}^\circ$ is equal to $69.16 \times 42.341\frac{2}{3}$
 $= 2,928.3496+$ mi. Ans.

$$\begin{array}{r} 42.341\frac{2}{3} \\ \quad 69.16 \\ \hline 4610\frac{2}{3} \\ 254046 \\ 42341 \\ 381069 \\ \hline 254046 \\ \hline 2928.34966\frac{2}{3}, \text{ or } 2928.3497- \end{array}$$

(17) 8 ft. 6 in. = 102 in.
We first find the volume of the cube, which is 1,061,208 cu. in. Dividing this by 231 (see Art. 21), the number of cubic inches in a gallon, we obtain 4,594— gal. as the contents of the cistern. Ans.

$$\begin{array}{r}
 102 \\
 102 \\
 \hline
 204 \\
 102 \\
 \hline
 10404 \\
 102 \\
 \hline
 20808 \\
 10404 \\
 \hline
 231 \overline{) 1061208} (4593.9, \text{ say } 4594- \\
 924 \\
 \hline
 1372 \\
 1155 \\
 \hline
 2170 \\
 2079 \\
 \hline
 918 \\
 693 \\
 \hline
 2250 \\
 2079 \\
 \hline
 \end{array}$$

(18) From Jan. 14, 1898, to Jan. 14, 1900, is 2 years.
From the table of Art. 68, the time from Jan. 14 to July 14, is 181 days. From July 14 to July 23, is 9 days. $181 + 9 = 190$; hence, from Jan. 14, 1898, to July 23, 1900, is 2 yr. 190 da. Ans.
February has only 28 da. in 1900; this being a secular year not divisible by 400 (see Art. 28).

(19) To reduce $60^\circ 40' 30''$ to the decimal part of a circle:

Reducing $30''$ to a decimal of a minute, $30'' = \frac{30}{60}' = .5'$. $40' + .5' = 40.5'$. Reducing $40.5'$ to a decimal of a degree, $40.5' = \frac{40.5}{60}^\circ = .675^\circ$. $60^\circ + .675^\circ = 60.675^\circ$. In circle there are 360° ; then, 60.675° is $\frac{60.675}{360}$ or .1685+ of a circle. Ans.

$$\begin{array}{r}
 60 \overline{) 30.0} \\
 .5 \\
 60 \overline{) 40.5} \\
 .675 \\
 360 \overline{) 60.675} (.1685+ \\
 360 \\
 \hline
 2467 \\
 2160 \\
 \hline
 3075 \\
 2880 \\
 \hline
 1950 \\
 1800 \\
 \hline
 1500
 \end{array}$$

(20) $6 \times 6\frac{1}{2} \times 8 = 812.$

$$\begin{array}{r} 1728 \\ 812 \\ \hline 8456 \\ 1728 \\ \hline 5184 \\ \hline 589186 \end{array}$$

$$\begin{array}{r} 2150.42)589186.000(250.7+ \\ 480084 \\ \hline 1090520 \\ 1075210 \\ \hline 1581000 \\ 1505204 \\ \hline \end{array}$$

Multiplying together the linear dimensions of the bin, its volume is found to be 812 cu. ft. Since in 1 cu. ft. there are 1,728 cu. in., in 812 cu. ft. there are $812 \times 1,728 = 589,186$ cu. in. In 1 bu. there are 2,150.42 cu. in. (see Art. 24), and in 589,186 cu. in. there are as many bushels as 2,150.42 is contained in 589,186, or 250.7+ bu. Ans.

(21) An avoirdupois pound contains 7,000 gr. In one dose there are $8\frac{1}{2}$ gr.; therefore, in 7,000 gr. there are $7,000 \div 8\frac{1}{2} = \frac{1000}{7} \times \frac{2}{1} = 2,000$ doses. Ans.

(22)

lb.	3	3	9	gr.
8	9	5	2	15
				9

$$27 \quad 81 \quad 45 \quad 18 \quad 135 = 34 \text{ lb. } 38 \text{ } 35 \text{ gr. } 15. \text{ Ans.}$$

Consult Table IX, Art. 19.

$15 \text{ gr.} \times 9 = 135 \text{ gr.} = 96 \text{ gr. } 15$; write the gr. 15 and reserve the 96. $9 \times 92 + 96 = 924 = 38$; reserve this and add to the product of the drams (3). $345 + 38 = 383 = 36 \text{ } 35$. Write the 35 and add the 36 to the product of the ounces (3). $381 + 36 = 417 = 7 \text{ lb. } 33$. The entire result is 34 lb. 33 35 gr. 15.

(23)

$$\begin{array}{r} 8\frac{1}{4} \\ 80 \\ \hline 4)240 \\ \hline 60 \\ 640 \\ \hline 700 \end{array}$$

$$\begin{array}{r} 16)700(43\frac{1}{4} \\ 64 \\ \hline 60 \\ 48 \\ \hline 12 \\ 16 = \frac{1}{4} \end{array}$$

If they used $8\frac{1}{4}$ oz. per day, in 80 da. they would use $80 \times 8\frac{1}{4} = 700$ oz. In 1 lb. there are 16 oz. (see Table VI) and in 700 oz. there are as many pounds as 16 is contained in 700, or $43\frac{1}{4}$ lb. Ans.

(24) Dividing 75 rd. by 36, the quotient is 2 rd. and the remainder is 3 rd. Reducing the remainder to yards and adding the 4 rd. of the dividend, $3 \times 5\frac{1}{2} + 4 = 20\frac{1}{2}$ yd.

rd.	yd.	ft.	in.
36)75	4	2	8
<hr/>			
2 rd.	0 yd.	1 ft.	$9\frac{1}{2}$ in.

We cannot divide $20\frac{1}{2}$ yd. by 36, so we reduce to feet. $20\frac{1}{2} \times 3 + 2$

$63\frac{1}{2} + 36 = 1$ ft. and $27\frac{1}{2}$ ft. remaining. Reducing to inches
 ing the 8 in. of the dividend, $27\frac{1}{2} \times 12 + 8 = 338$ in.; $338 \div 36$
 The final quotient is 2 rd. 0 yd. 1 ft. $9\frac{7}{8}$ in. Ans.

25 ft. square is, an area 25 ft. long and 25 ft. wide, or (25×25)
 sq. ft. $625 - 75 = 550$ sq. ft. Ans.

Since the volume is the product of the length, width, and thick-
 ness, it follows that the width is equal to the volume divided by the
 product of the length and thickness. 10 in. $= \frac{5}{8}$ ft. $40 \div (36 \times \frac{5}{8})$
 $= 30 = \frac{4}{3}$. The width is therefore $\frac{4}{3}$ ft. $= \frac{4}{3} \times 12$ in. $= 16$ in. Ans.

$$\begin{array}{r}
 150.42 \quad 231 \overline{) 215042.000} \quad (930.917, \text{ or say } 930.92- \\
 \underline{100} \qquad \qquad \underline{2079} \\
 15042.00 \qquad \qquad 714 \\
 \qquad \qquad \underline{693} \\
 \qquad \qquad \qquad 2120 \\
 \qquad \qquad \underline{2079} \\
 \qquad \qquad \qquad \qquad 410 \\
 \qquad \qquad \qquad \underline{281} \\
 \qquad \qquad \qquad \qquad 1790 \\
 \qquad \qquad \qquad \underline{1617} \\
 \qquad \qquad \qquad \qquad \qquad 173
 \end{array}$$

There are 2,150.42 cu. in. in 1 bu. (see Art. 24) and in 100 bu. there
 are $100 \times 2,150.42 = 215,042$ cu. in. In 1 gal. there are 231 cu. in.;
 therefore, in 215,042 cu. in. there are as many gallons as 231 is contained
 in 215,042, or 930.92— gal. Ans.

(28) April of the minuend is the 4th month and November of the
 subtrahend is the 11th month. Now we cannot
 subtract 9 da. from 2 da., so we borrow 1 mo., yr. mo. da.
 or 30 da.; $30 + 2 = 32$, $32 - 9 = 23$. Since 1889 4 2
 1 mo. was borrowed from the 4 mo., only 3 mo. 1842 11 9
 remain; we cannot subtract 11 mo. from 3 mo., 46 4 23 Ans.
 so 1 yr. is borrowed from the column of years,
 reduced to months and added to the months. $12 + 3 = 15$; $15 - 11 = 4$,
 and $1888 - 1842 = 46$. The final result is therefore 46 yr. 4 mo. 23 da.

(29) (a) 18.75 ft. $= 18\frac{3}{4}$, or $\frac{75}{4}$ ft.
 2 ft. 10 in. $= 2\frac{5}{6}$ ft. $= 2\frac{5}{6}$ ft. $= \frac{17}{6}$ ft.

$$\begin{array}{r}
 25 \\
 75 \\
 \frac{75}{4} \times \frac{17}{6} = \frac{425}{8} = 53.125. \\
 2
 \end{array}$$

Hence the area is 53.125 sq. ft. Ans.

(b) $53.125 \times \$0.08 = \4.25 . Ans.

- (30) (a) 1 pt. = $\frac{1}{4}$ qt. 8 qt. 1 pt. = $8\frac{1}{4}$ qt. = $\frac{31}{4}$ gal. = $\frac{7}{2}$ gal. 18 gal.
 8 qt. 1 pt. = $18\frac{1}{4}$ gal. $18\frac{1}{4}$ gal. is $\frac{18\frac{1}{4}}{68} = \frac{111}{68}$ of 68 gal. Ans.
 (b) $\frac{111}{68} = .2996$. Ans.

$$\begin{array}{r}
 504 \overline{) 151.0000} (.2996+ \\
 \underline{1008} \\
 5020 \\
 \underline{4586} \\
 4340 \\
 \underline{4586} \\
 8040 \\
 \underline{8024}
 \end{array}$$

- (31) A solar year is 365 da. 5 hr. 48 min. 49.7 sec. (see Art. 28).

da.	hr.	min.	sec.
7) 365	5	48	49.7
<hr/>			
52 da.	4 hr.	15 min.	32.81+ sec. Ans.

- (32) In 1 bu. there are 32 qt. and in 6 bu. there are $6 \times 32 = 192$ qt. If 8 qt. sell for \$.25, for 1 qt. he receives $\frac{1}{8}$ of \$.25, or \$.08 $\frac{1}{4}$, and for 192 qt. he receives $192 \times $.08 $\frac{1}{4}$ = $16. But this is $4 more than they cost; therefore, the cost of the 6 bu. was $16 - $4 = $12. Therefore, 1 bu. cost $\frac{1}{6}$ of $12, or $2. Ans.$

- (33) 8 hr. 40 min. = $8\frac{40}{60}$ hr. = $8\frac{2}{3}$ hr. $8\frac{2}{3}$ hr. a day for 5 da. a week for 40 wk. per year and for 4 yr. is $8\frac{2}{3} \times 5 \times 40 \times 4 = 6,933\frac{1}{3}$ hr. Ans.

$$\begin{array}{r}
 8\frac{2}{3} \\
 5 \\
 \hline
 40 \\
 3\frac{1}{3} \\
 \hline
 43\frac{1}{3} \\
 40 \\
 \hline
 1720 \\
 13\frac{1}{3} \\
 \hline
 1733\frac{1}{3} \\
 4 \\
 \hline
 6933\frac{1}{3}
 \end{array}$$

- (34) Write the numbers as for addition, with units of like denomination under each other. Since 6 d. cannot be subtracted from 8 d., 1 s. (= 12 d.) is borrowed from the column of shillings. $12 + 8 = 20$ d. Then, $20 - 6 = 14$ d. Since 1 shilling was borrowed from 5 s., only 4 s. remain; and as 9 s. cannot be subtracted from 4 s., £1 (= 20 s.) is borrowed from the £ column, reduced to shillings and added to 4 s., giving 24 s.; then,
- | £ | s. | d. |
|-------|-------|------|
| 21 | 5 | 3 |
| 13 | 9 | 6 |
| <hr/> | | |
| £7 | 15 s. | 9 d. |

4 s. - 9 s. = 15 s. Finally, £20 - £13 = £7; and the final remainder s £7 15 s. 9 d. Ans.

(35) The sum of the units of the first column is 122 sq. li. Write 122 under the right hand column. The sum of the next column is 20 sq. rd., or 1 sq. ch. 4 sq. rd. (See Table IV.) Writing the 4 sq. rd. and carrying the 1 sq. ch. to the third column and adding, the result is 324 sq. ch. = 32 A. 4 sq. ch. Writing the 4 sq. ch. and carrying the 32 A. to the fourth column, the sum is 255 A. The final result is 255 A. 4 sq. ch. 4 sq. rd. 122 sq. li. Ans.

(36) There are 640 A. in 1 sq. mi. and in 16 sq. mi. there are $16 \times 640 = 10,240$ A. 10,240 A. is to be divided into 62 farms. Dividing 10,240 A. by 62 gives 165 A., with a remainder of 10 A., which = 10×160 or 1,600 sq. rd. $1,600 \div 62 = 25$ sq. rd., with a remainder of 50 sq. rd. = $50 \times 30\frac{1}{2} = 1,512\frac{1}{2}$ sq. yd.; $1,512\frac{1}{2} \div 62 = 24$ sq. yd., with a remainder of $24\frac{1}{2}$ sq. yd. = $24\frac{1}{2} \times 9 = 220\frac{1}{2}$ sq. ft. $220\frac{1}{2} \div 62 = 3$ sq. ft. and a remainder of $34\frac{1}{2}$ sq. ft. = 4,968 sq. in. $4,968 \div 62 = 80+$ sq. in. The result is 165 A. 25 sq. rd. 24 sq. yd. 3 sq. ft. 80+ sq. in. Ans.

$$\begin{array}{r}
 640 \text{ A.} \\
 \underline{16} \\
 8840 \\
 \underline{640} \\
 62) 10240 \text{ A. (165 A.} \\
 \underline{62} \\
 404 \\
 \underline{372} \\
 320 \\
 \underline{310} \\
 10 \text{ A.} \\
 \underline{160} \\
 62) 1600 \text{ sq. rd. (25 sq. rd.} \\
 \underline{124} \\
 360 \\
 \underline{310} \\
 50 \text{ sq. rd.} \\
 \underline{30\frac{1}{2}} \\
 12\frac{1}{2} \\
 \underline{1500} \\
 1512\frac{1}{2} \text{ sq. yd.}
 \end{array}$$

$$\begin{array}{r}
 62) 1512\frac{1}{2} \text{ sq. yd. (24 sq. yd.} \\
 \underline{124} \\
 272 \\
 \underline{248} \\
 24\frac{1}{2} \text{ sq. yd.} \\
 \underline{9} \\
 4\frac{1}{2} \\
 \underline{216} \\
 62) 220\frac{1}{2} \text{ sq. ft. (3 sq. ft.} \\
 \underline{186} \\
 34\frac{1}{2} \text{ sq. ft.} \\
 \underline{144} \\
 72 \\
 \underline{136} \\
 136 \\
 \underline{34} \\
 62) 4968 \text{ sq. in. (80+ sq. in.} \\
 \underline{496} \\
 8
 \end{array}$$

(87) First reduce 2 bu. 3 pk. 6 qt. to bushels and decimals.

$$6 \text{ qt.} = \frac{1}{2} \text{ pk.} = \frac{1}{4} \text{ pk.}$$

$$3 \text{ pk. 6 qt.} = 3\frac{1}{2} \text{ pk.} = \frac{8\frac{1}{2}}{4} \text{ bu.} = 2\frac{1}{8} \text{ bu.}$$

$$2 \text{ bu. 3 pk. 6 qt.} = 2\frac{1}{8} \text{ bu.} = 2.9375 \text{ bu.}$$

If in 1 bbl. there are 2.9375 bu., in 9 bbl. there are $9 \times 2.9375 = 26.4375$ bu. Ans.

(88) The sum of the first column is $107''$, or $1' 47''$. 11° $16'$ $12''$
 Write the $47''$ and carry the $1'$ to the next column. 18° $19'$ $30''$
 The sum of the second column is $79' = 1^\circ 19'$. 20° $0'$ $25''$
 Write the $19'$ and carry 1° to the third column. 0° $26'$ $29''$
 The sum of the third column is 55° . The final 10° $17'$ $11''$
 result is $55^\circ 19' 47''$. Ans. $\hline 55^\circ \quad 19' \quad 47''$

(89) See Table II.

$$\begin{array}{rcl} 1 \text{ rd.} & = & 25 \text{ li.} \\ 1 \text{ ch.} & = & 4 \text{ rd.} \\ 80 \text{ ch.} & = & 1 \text{ mi.} \end{array} \qquad \begin{array}{r} 25 \overline{) 4768254 \text{ li.}} \\ 4 \overline{) \quad 190580 \text{ rd.} + 4 \text{ li.}} \\ 80 \overline{) \quad \quad 47682 \text{ ch.} + 2 \text{ rd.}} \\ \hline 595 \text{ mi.} + 32 \text{ ch.} \end{array}$$

Hence, $4,768,254 \text{ li.} = 595 \text{ mi. } 32 \text{ ch. } 2 \text{ rd. } 4 \text{ li.}$ Ans.

ARITHMETIC.

(1) If the tank is $3 \text{ m.} \times 2 \text{ m.} \times 3 \text{ m.}$, its volume is 18 cu. m. 1 cu. m. = 1,000 cu. dm.; therefore, $18 \text{ cu. m.} = 18 \times 1,000 \text{ cu. dm.} = 18,000 \text{ cu. dm.}$ (See Table III, Art. 15.) Since 1 cu. dm. of water weighs 1 Kg. (see Art. 19), 18,000 cu. dm. weigh 18,000 Kg. Ans.

(2) The area of the field is 60,000 sq. m. To reduce 60,000 sq. m. to hectares, begin at the right, point off two places to the left and say 600.00 sq. Dm. (see Table II, Art. 12), then two more places and say 6.0000, or 6 Ha. Ans.

(3) One metric ton is equal to $1.1023 \times 2,000 = 2,204.6 \text{ lb.}$ (see Table V). 1.5 metric tons = $1.5 \times 2,204.6 = 3,306.9 \text{ lb.}$ At $2\frac{1}{2}$ cents a pound, 3,306.9 lb. will cost $3,306.9 \times \$0.025 = \82.6725 , or \$82.67+. Ans.

(4) $25.875 \text{ Kg.} = 258.75 \text{ Hg.} = 2,587.5 \text{ Dg.} = 25,875 \text{ g.}$ (see Table V). 1 g. = 15.432 gr.; therefore, $25,875 \text{ g.} = 25,875 \times 15.432 = 399,303 \text{ gr.}$ At 10 cents a grain, 399,303 gr. will cost $399,303 \times \$0.10 = \$39,930.30$.
Ans.

(5) The area of the ceiling is $(12 \times 10) \text{ sq. m.} = 120 \text{ sq. m.}$ 1 sq. m. = 1.196 sq. yd. (see Table II), and $120 \text{ sq. m.} = 120 \times 1.196 = 143.52 \text{ sq. yd.}$ At 25 cents a square yard, 143.52 sq. yd. will cost $143.52 \times \$0.25 = \35.88 . Ans.

(6) $50 \text{ m.} = 500 \text{ dm.} = 5,000 \text{ cm.}$ (see Table I). The row is 5,000 cm. long, and if the diameter of the 5-cent nickel coin is 2 cm., there are $5,000 \div 2 = 2,500$ nickel coins in the row. If 1 coin is worth 5 cents, 2,500 are worth $2,500 \times .05 = \$125$. Ans.

(7) In linear measure, the scale is 10; hence, in reducing 18.725 m., move the decimal point one place for each denomination (see Table I). $18.725 \text{ m.} = 187.25 \text{ dm.} = 1,872.5 \text{ cm.}$ Ans.

(8) One liter is equal to .9078 of a dry quart (see Table IV); therefore, $1,000 \text{ liters} = 1,000 \times .9078 = 907.8 \text{ qt.}$ At 12 cents a quart, 907.8 qt. will cost $907.8 \times .12 = \$108.94$. Ans.

(9) The volume of the block is obtained by multiplying the three dimensions, thus, $8 \times 2 \times 1.5 = 9$ cu. m. **Ans.**

(10) Since 1 m. = 3.28 ft. (see Table I), 56.8 m. = $56.8 \times 3.28 = 186.804$ ft. = $\frac{186.804}{3} = 62.1+$ yd. **Ans.**

(11) (a) To square a number, we must multiply the number by itself once; that is, use the number twice as a factor. Thus the square of 689 is $689 \times 689 = 474,721$.

$$\begin{array}{r} 689 \\ 689 \\ \hline 6201 \\ 5512 \\ \hline \text{Ans. } 4184 \\ \hline 474721 \end{array}$$

$$\begin{array}{r} 689 \\ 689 \\ \hline 6201 \\ 5512 \\ \hline 4184 \\ 474721 \\ 689 \\ \hline 4272489 \\ 3797768 \\ 2848326 \\ \hline 327082769 \end{array}$$

(b) The cube of 689 is the number obtained by using 689 as a factor three times. Thus, the cube of 689 is $689 \times 689 \times 689 = 327,082,769$. **Ans.**

(12) To find the cube root of 63.86 to four decimal places, annex sufficient ciphers and point off into periods of three figures each, beginning at the decimal point; this gives 63.360'000'000'000. The first period is 63 and the nearest perfect cube is 64; therefore, $\sqrt[3]{64} = 4$ is taken as the first trial root. The first period is divided by three times the square of the trial root, and to the quotient is added two-thirds of the trial root. Thus,

$$\frac{63}{3 \times 4^2} + \frac{2}{3} \times 4 = 1.312 + 2.666 = 3.978.$$

Retaining two figures and neglecting the decimal point gives 40 (the third figure being greater than 5) as the second trial root. Use this trial root and the first two periods as a trial dividend, thus,

$$\frac{63'360}{3 \times 40^2} + \frac{2}{3} \times 40 = 13.2 + 26.67 = 39.87.$$

Retaining three figures, the third trial root is 399. Repeat the process, using three periods. Thus,

$$\frac{63'360'000}{3 \times 399^2} + \frac{2}{3} \times 399 = 132.66 + 266 = 398.66.$$

Since there is one period, 63, in the whole-number part of the given number, there must be one figure in the whole-number part of the root. Therefore, the required root is 3.9866. Ans.

$$(13) \quad \sqrt{32^2 + 255^2} = \sqrt{32 \times 32 + 255 \times 255} = \sqrt{66,049} = 257. \quad \text{Ans.}$$

$$\begin{array}{r} 32 \\ 32 \\ \hline 64 \\ 96 \\ \hline 1024 \end{array} \quad \begin{array}{r} 255 \\ 255 \\ \hline 1275 \\ 1275 \\ \hline 510 \\ 65025 \\ 1024 \\ \hline 66049 \end{array}$$

To extract the square root of 66,049, first point off the number into periods of two figures each, beginning at the right; this gives 6'60'49. 2 is the number whose square most nearly equals the first period, since $2^2 = 4$, and is taken as the trial root. Divide the first period by twice the trial root, and to the quotient add one-half the trial root. Thus,

$$\frac{6}{2 \times 2} + \frac{2}{2} = 1.5 + 1 = 2.5.$$

Neglect the decimal point and call this result the new trial root. Repeat the process, using the first and second periods for a trial dividend, and 25 for a trial root. Thus,

$$\frac{6'60}{2 \times 25} + \frac{25}{2} = 13.2 + 12.5 = 25.7.$$

Use three periods and 257 for the trial root. Thus,

$$\frac{6'60'49}{2 \times 257} + \frac{257}{2} = 128.5 + 128.5 = 257.0.$$

In 66,049 there are three periods; hence, there must be three figures in the whole-number part of the root; hence, $\sqrt{66,049} = 257$.

$$(14) \quad \sqrt{120^2 + 209^2} = \sqrt{120 \times 120 + 209 \times 209} = \sqrt{58,081} = 241. \quad \text{Ans.}$$

$$\begin{array}{r} 120 \\ 120 \\ \hline 14400 \\ 43681 \\ \hline 58081 \end{array} \quad \begin{array}{r} 209 \\ 209 \\ \hline 1881 \\ 418 \\ \hline 43681 \end{array}$$

Pointing 58,081 off into periods of two figures each, we have 5'80'81. The number whose square is nearest 5 is 2.

Substituting 2 in the formula, Art. 48, the first approximate root is

$$\frac{5}{2 \times 2} + \frac{2}{2} = 1.25 + 1 = 2.25.$$

Substituting 23 for R ,

$$\frac{5'80}{2 \times 23} + \frac{23}{2} = 12.6 + 11.5 = 24.1.$$

Substituting 241,

$$\frac{5'80'81}{2 \times 241} + \frac{241}{2} = 120.5 + 120.5 = 241.0.$$

As there are three periods in the number, there are three figures in the whole-number part of the root, or $\sqrt{5'80'81} = 241$.

(15) Substituting the values of the letters,

$$\frac{5(xyz - abc)}{bc - ax} = \frac{5(10 \times 4 \times 6 - 3 \times 5 \times 8)}{5 \times 8 - 3 \times 10} = \frac{5(240 - 120)}{40 - 30} = \frac{600}{10} = 60. \quad \text{Ans.}$$

$$(16) \quad \frac{(a+b)(b+c)(c+a)}{a^2 + b^2 + c^2} = \frac{(3+5)(5+8)(8+3)}{3^2 + 5^2 + 8^2} = \frac{8 \times 13 \times 3}{9 + 25 + 64} = \frac{312}{98} = 3.18 \approx 16. \quad \text{Ans.}$$

$$(17) \quad \begin{array}{r} 289 \\ 289 \\ \hline 2601 \\ 2312 \\ 578 \\ \hline 83521 \end{array} \quad \begin{array}{r} 161 \\ 161 \\ \hline 161 \\ 966 \\ 161 \\ \hline 25921 \end{array} \quad \begin{array}{r} 83521 \\ 25921 \\ \hline 57600 \end{array}$$

$$\sqrt[4]{57600} = 240. \quad \text{Ans.}$$

$$\frac{5}{2 \times 2} + \frac{2}{2} = 1.25 + 1 = 2.25, \text{ 1st approximation.}$$

$$\frac{576}{2 \times 23} + \frac{23}{2} = 12.5 + 11.5 = 24.0, \text{ 2d approximation.}$$

$$\frac{57600}{2 \times 240} + \frac{240}{2} = 120 + 120 = 240, \text{ true root.}$$

If the square root of the sum is 289, the sum is 289^2 , or $289 \times 289 = 83,521$. If 83,521 is the sum of the squares of two numbers, one of which is 161^2 , or 25,921, then the square of the other number must be $83,521 - 25,921 = 57,600$. Since this is the square of the number, the number itself is $\sqrt{57,600} = 240$. Ans.

(18) If the field is 10 rd. square, its area is 1,600 sq. rd. 1 A. = 160 sq. rd.; hence, in 1,600 sq. rd. there are $1,600 \div 160 = 10$ A. 1 Ha. = 2.471 A. (Table II); hence, in 10 A. there are $\frac{10}{2.471} = 4.0469$ Ha. Ans.

$$(19) \quad \sqrt[3]{87.51} \approx \sqrt[3]{12.1} \approx 1.2765. \quad \text{Ans.}$$

Find the trial root by Art. 55. Thus, $4^3 = 64$; $5^3 = 125$; $(125 - 64) \div 3 = 64 \div 3 = 21.33$. Hence, 5 will be used for the first trial root.

The successive approximations are as follows:

$$\frac{87}{3 \times 5^2} + \frac{2 \times 5}{3} = 1.16 + 3.33 = 4.49.$$

$$\frac{87510}{3 \times 45^2} + \frac{2 \times 45}{3} = 11.4 + 30 = 41.4.$$

$$\frac{87540000}{3 \times 441^2} + \frac{2 \times 441}{3} = 118 + 296 = 414.0.$$

$$\frac{87540000000}{3 \times 4440^2} + \frac{2 \times 4440}{3} = 4440.2.$$

Since there is only one period in the whole-number part of the given number, there is only one figure in the whole-number part of the root; hence, $\sqrt[3]{87.54} = 4.4402$.

In obtaining the square root of 12, the first trial root is 3. The successive approximations are as follows:

$$\frac{12}{2 \times 3} + \frac{3}{2} = 2 + 1.5 = 3.5.$$

$$\frac{12'10}{2 \times 35} + \frac{35}{2} = 17.28 + 17.5 = 34.78.$$

$$\frac{12'10'00}{2 \times 348} + \frac{348}{2} = 173.85 + 174 = 347.85.$$

$$\frac{12'10'00'00}{2 \times 3,479} + \frac{3,479}{2} = 1,739 + 1,739.5 = 3,478.5.$$

Locating the decimal point, we have $\sqrt{12.1} = 3.4785$. Then, $4.4402 \div 3.4785 = 1.2765$ —.

(20) Evidently the length of an edge is $\sqrt[3]{3.788}$ inches. Dividing the number into periods of three figures, we have 3'788. The first period is 3; therefore, 2 is taken as the trial root (see Art. 60).

$$\frac{3}{3 \times 2^2} + \frac{2 \times 2}{3} = .25 + 1.33 = 1.58.$$

$$\frac{3'788}{3 \times 16^2} + \frac{2 \times 16}{3} = 4.93 + 10.67 = 15.60.$$

$$\frac{3'788'000}{3 \times 156^2} + \frac{2 \times 156}{3} = 51.88 + 104 = 155.88.$$

Since there are two periods in the whole number, there must be two figures in the whole-number part of the root; hence, using two decimal places only, $\sqrt[3]{3.788} = 15.59$, and the length of the edge is 15.59 in. Ans.

(21) One liter is equal in volume to 1 cu. dm. (see Art. 17); therefore, 4,913 liters = 4,913 cu. dm. The length of an edge is $\sqrt[3]{4,913}$ dm. = 17 dm. 1 dm. = 3.937 in. (see Table I); hence, 17 dm. = 17×3.937 = 66.929 in. Ans.

The extraction of the cube root is as follows:

$$\frac{4}{3 \times 2^2} + \frac{2 \times 2}{3} = .33 + 1.33 = 1.66.$$

$$\frac{4'913}{3 \times 17^2} + \frac{2 \times 17}{3} = 5.667 + 11.333 = 17.$$

(22) One Ha. = 2.471 A. (Table II); therefore, 40 A. = $40 \div 2.471$ = 16.1878 Ha. Reduce 16.1878 Ha. to square meters by moving the decimal point two places to the right for each denomination (see Art. 13); thus, 16.1878 Ha. = 1,618.78 sq. Dm. = 161,878 sq. m. The

area of the square is 161,878 sq. m.; to find the length of a side, extract the square root of 161,878.

The first period is 16 and the first trial root is 4.

$$\frac{16}{2 \times 4} + \frac{4}{2} = 2 + 2 = 4.0.$$

$$\frac{16'18}{2 \times 40} + \frac{40}{2} = 20.2 + 20 = 40.2.$$

$$\frac{16'18'78}{2 \times 402} + \frac{402}{2} = 201.84 + 201 = 402.84—.$$

The length of the side is 402.84— m. **Ans.**

(23) To find any power of a mixed number, first reduce it to an improper fraction, and then raise the numerator and the denominator separately to the required power.

$$\begin{aligned} (48\frac{1}{2})^2 + (4\frac{1}{2})^2 &= (\frac{97}{2})^2 + (\frac{9}{2})^2 = \frac{87 \times 87}{2 \times 2} + \frac{14 \times 14 \times 14}{3 \times 3 \times 3} = \frac{7,569}{4} + \frac{2,744}{27} \\ &= \frac{204,863 + 10,976}{108} = \frac{215,839}{108} = 1,998.879+. \quad \text{Ans.} \end{aligned}$$

$$\begin{aligned} (24) \quad \sqrt[3]{72a^2bc^3} &= \sqrt[3]{72 \times 2 \times 2 \times 6 \times 3 \times 3 \times 3} = \sqrt[3]{46,656} = 36. \quad \text{Ans.} \\ \frac{46}{3 \times 4^2} + \frac{2 \times 4}{3} &= 3.62+. \quad \frac{46'656}{3 \times 36^2} + \frac{36 \times 2}{3} = 36. \end{aligned}$$

(25) Reducing the different quantities to the same denomination before adding (see Table V):

$$\begin{aligned} 2.75 \text{ Kg.} &= 2 \text{ 7.5 Hg.} \\ 429.17 \text{ Dg.} &= 4 \text{ 2.9 1 7 Hg.} \\ 38,295 \text{ g.} &= 3,829.5 \text{ Dg.} = \frac{3 \text{ 8 2.9 5}}{10} \text{ Hg.} \\ &= 4 \text{ 5 3.3 6 7 Hg.} \quad \text{Ans.} \end{aligned}$$

(26) One gallon contains 231 cu. in.; therefore, 100 gal. contains $100 \times 231 = 23,100$ cu. in. 1 liter = 61.023 cu. in. (see Table IV); hence, in 23,100 cu. in. there are as many liters as 61.023 is contained in 23,100, or $23,100 \div 61.023 = 378.546—$ liters. **Ans.**

(27) One gram = 15.432 gr. (see Art. 19). One kilogram, or kilo = 1,000 g. (see Art. 23) or 1 Kg. = $1,000 \times 15.432 = 15,432$ gr. One gold eagle weighs 258 gr., then 1 dollar weighs $\frac{1}{10}$ of 258 gr. = 25.8 gr. If 1 dollar weighs 25.8 gr. in 15,432 gr., there are as many dollars as 25.8 is contained in 15,432, or \$598.14—. **Ans.**

(28) Contents of the block are $(2.5 \times 2.8 \times 3.75)$ cu. m. = 26.25 cu. m. One cubic meter = 35.3145 cu. ft. (see Table III); then, 26.25 cu. m. = $26.25 \times 35.3145 = 927.005$ cu. ft. If 1 cu. ft. weighs 168.75 lb., 927.005 cu. ft. weigh $927.005 \times 168.75 = 156,432.09+$, or say 156,432 lb. **Ans.**

$$(29) \sqrt[4]{4'32'22'41} = 2,079. \text{ Ans.}$$

Since the first period, 4, is a perfect square, we can use the first two periods for the trial dividend; annex a cipher to the trial root, and proceed in the usual manner.

$$\frac{4'32}{2 \times 20} + \frac{20}{2} = 10.8 + 10 = 20.8.$$

$$\frac{4'32'22}{2 \times 208} + \frac{208}{2} = 103.9 + 104 = 207.9.$$

$$\frac{4'32'22'41}{2 \times 2,079} + \frac{2,079}{2} = 1,039.5 + 1,039.5 = 2,079.$$

$$(30) \sqrt[4]{7,921} \times \sqrt[4]{9,604} = 89 \times 98 = 8,722. \text{ Ans.}$$

$$\frac{79}{2 \times 9} + \frac{9}{2} = 8.9. \quad \frac{79'21}{2 \times 89} + \frac{89}{2} = 89.$$

$$\text{Hence, } \sqrt[4]{7,921} = 89.$$

$$\frac{96}{2 \times 10} + \frac{10}{2} = 9.8. \quad \frac{96'04}{2 \times 98} + \frac{98}{2} = 98.$$

$$\text{Hence, } \sqrt[4]{9,604} = 98.$$

$$(31) \sqrt[4]{19.00'00'00'00} = 4.3589. \text{ Ans.}$$

The first trial root is 4, and the successive approximations are as follows:

$$\frac{19}{2 \times 4} + \frac{4}{2} = 4.4.$$

$$\frac{19'00}{2 \times 44} + \frac{44}{2} = 43.6.$$

$$\frac{19'00'00}{2 \times 436} + \frac{436}{2} = 435.9.$$

$$\frac{19'00'00'00}{2 \times 4,359} + \frac{4,359}{2} = 4,358.9.$$

Since there is only one period in the whole-number part of the given number, there is only one figure in the whole-number part of the root; hence, $\sqrt[4]{19} = 4.3589$.

(32) If the cost of the farm is \$250 a hektare, 200 Ha. will cost $200 \times \$250 = \$50,000$. Since 1 Ha. = 2.471 A. (see Table II), 200 Ha. = $200 \times 2.471 = 494.2$ A. If sold at \$150 per acre, the amount received for 494.2 A. was $494.2 \times \$150 = \$74,130$. The farm cost \$50,000 and was sold for \$74,130; the gain was therefore $\$74,130 - \$50,000 = \$24,130$.
Ans.

(33) The dimensions of the wall are 23.5 m. \times .5 m. \times 2.4 m.; hence, the contents are 28.2 m³. At \$5.25 per m³, the wall will cost $28.2 \times \$5.25 = \148.05 . Ans.

(34) Reducing $\frac{7}{8}$ to a decimal, it becomes .875. In pointing off a decimal into periods, begin at the decimal point and point off to the right. The first period is .875, and cipher periods may now be annexed until the root has as many figures as desired. The result will be wholly decimal. The first trial root is 10 (see Art. 60), and the successive approximations are as follows.

$$\begin{array}{r}
 \begin{array}{r}
 875 \\
 3 \times 10^2 + 2 \times 10 \\
 3
 \end{array} = 9.58, \\
 \begin{array}{r}
 875'000 \\
 3 \times 90^2 + 2 \times 90 \\
 3
 \end{array} = 95.6, \\
 \begin{array}{r}
 875'000'000 \\
 3 \times 956^2 + 2 \times 956 \\
 3
 \end{array} = 956.46, \\
 \begin{array}{r}
 875'000'000'000 \\
 3 \times 9565^2 + 2 \times 9565 \\
 3
 \end{array} = 9564.65.
 \end{array}$$

Increasing the fifth figure by 1 and locating the decimal point, we have $\sqrt[3]{\frac{7}{8}} = .95647$. Ans.

(35) $\sqrt[3]{571,787} \times \sqrt[3]{15,625} = 83 \times 125 = 10,375$. Ans.

The cube root is extracted as follows

$$\begin{array}{r}
 571 \\
 3 \times 8^2 + 2 \times 8 \\
 3
 \end{array} = 8.3, \\
 \begin{array}{r}
 571'787 \\
 3 \times 83^2 + 2 \times 83 \\
 3
 \end{array} = 83.$$

Hence, $\sqrt[3]{571,787} = 83$.

The square root is extracted as follows

$$\begin{array}{r}
 156 \\
 2 \times 10^2 + 2 \times 10 \\
 2
 \end{array} = 12.8, \\
 \begin{array}{r}
 156'25 \\
 2 \times 128^2 + 2 \times 128 \\
 2
 \end{array} = 125.035, \\
 \begin{array}{r}
 156'25'00 \\
 2 \times 1,250^2 + 2 \times 1,250 \\
 2
 \end{array} = 1,250.$$

Locating the decimal point, $\sqrt[3]{15,625} = 125$. $83 \times 125 = 10,375$.

(36) The area of 1 lot is $(25 \times 100) = 2,500$ sq. ft. One sq. m. = 10.7637 sq. ft. (See Table II.) Since the scale in square measure is 100, the decimal point must be moved two places to the right for each denomination. 1 sq. Dm. = 1,076.37 sq. ft. hence 1 sq. 11m. = 107,637 sq. ft. If 1 city lot has an area of 2,500 sq. ft., in 107,637 sq. ft., or 1 Ha., there are $107,637 \div 2,500 = 43$ lots. Ans.

(37) Since 1 m. = 3.28 ft. (see Table I), 100 m. = $100 \times 3.28 = 328$ ft., the length of the field. In 1 rd. there are 16 $\frac{1}{2}$ ft., and in 328 ft. there are $\frac{328}{16\frac{1}{2}} = \frac{328 \times 2}{33} = \frac{656}{33}$ rd. The area of the field is 1 A.

or 160 sq. rd. Dividing the area by one dimension gives the other dimension:

$$\frac{160}{1} \div \frac{656}{33} = \frac{160}{1} \times \frac{33}{656} = \frac{330}{41} = 8.0488\text{—}.$$

Hence, the width of the field is 8.0488— rd. Ans.

(38) If 1 Kg. costs 30 cents, 50 Kg. will cost $50 \times \$.30 = \15.00 . Since 1 Kg. = 2.2046 lb. (see Table V), 50 Kg. = $50 \times 2.2046 = 110.23$ lb. At 18 cents a pound, the selling price of the fish is $110.23 \times .18 = \$19.84+$. The gain is $\$19.84 - \$15 = \$4.84+$. Ans.

(39) By a cube 2 in. each way, we mean a cube that is 2 in. long, 2 in. wide, and 2 in. thick. The volume of such a cube is $2 \times 2 \times 2 = 8$ cu. in. A 10-inch cube contains $10 \times 10 \times 10 = 1,000$ cu. in. It will take as many 2-inch cubes to equal a 10-inch cube as the volume of the 2-inch cube is contained in the volume of the 10-inch cube, $1,000 \div 8 = 125$. Hence, 125 two-inch cubes are required. Ans.

(40) As the longest side of a right triangle is equal to the square root of the sum of the squares of the other two sides, it is equal to

$$\sqrt{52^2 + 675^2} = \sqrt{52 \times 52 + 675 \times 675} = \sqrt{2,704 + 455,625} = \sqrt{45'83'29}.$$

Using 7 as the first trial root, the root is extracted by the following series of approximations:

$$\begin{aligned} \frac{45}{2 \times 7} + \frac{7}{2} &= 6.7. \\ \frac{45'83}{2 \times 67} + \frac{67}{2} &= 67.7. \\ \frac{45'83'29}{2 \times 677} + \frac{677}{2} &= 677. \end{aligned}$$

Hence, the length of the side is 677 ft. Ans.

ARITHMETIC.

(1) The area of a parallelogram = base \times altitude (see Art. 28).

12 ft. 8 in. = $12\frac{2}{3}$ ft. = $\frac{38}{3}$ ft.; 8 ft. 6 in. = $8\frac{1}{2}$ ft. = $\frac{17}{2}$ ft.; $\frac{38}{3} \times \frac{17}{2}$
 = $107\frac{2}{3}$; hence, the area is $107\frac{2}{3}$ sq. ft. Ans.

(2) Altitude = $\frac{\text{area}}{\text{base}}$. $3\frac{1}{2}$ A. = $(3\frac{1}{2} \times 160)$ sq. rd. = 560 sq. rd.; hence,
 the altitude is $\frac{560}{35} = 16$ rd. Ans.

(3) Area of the rectangle = base \times altitude. $56\frac{1}{2} = 113$; $16\frac{1}{2} = 33$;
 $113 \times 33 = 3729$; therefore, the area of the rectangle is 3729 sq. rd.
 As the area of the square is the same as the area of the rectangle, its
 area is also 3729 sq. rd. The length of a side of the square is $\sqrt{3729}$ rd.
 = 61 rd. = $30\frac{1}{2}$ rd. Ans.

To extract the square root of the fraction, extract the root of the
 numerator and of the denominator separately; thus, $\sqrt{8281} = 91$, and
 $\sqrt{9} = 3$; hence, $\sqrt{\frac{8281}{9}} = \frac{91}{3}$.

(4) Since the width is $\frac{3}{4}$ of the length, the sum of the length and
 width is $(\frac{1}{4} + \frac{3}{4})$, that is, $\frac{1}{1}$ of the length, and the perimeter, which is
 double the sum of the length and breadth (see Art. 31), is $\frac{1}{2}$ of the
 length. Since the perimeter is 140 ft., $\frac{1}{2}$ of the length = 140 ft.; hence,
 $\frac{1}{4}$ of the length = $\frac{1}{1}$ of 140 = 10 ft., and the length is $4 \times 10 = 40$ ft.
 The width is $\frac{3}{4}$ of 40 ft. = 30 ft.; hence, the room is 30 ft. \times 40 ft. Ans.

(5) The area is 1 A., or 43,560 sq. ft. (see Table III, § 4). The
 length of a side = $\sqrt{43,560} = 208.7+$ ft. Ans.

(6) Perimeter of room = $2(24\frac{1}{2} + 30\frac{1}{2}) = 110$ ft. (see Art. 31).
 Area of walls = $(110 \times 12) = 1,320$ sq. ft. = $147\frac{1}{2}$ sq. yd. Area of

ceiling = $24\frac{1}{2} \times 30\frac{1}{2} = \frac{49}{2} \times \frac{61}{2} = \frac{2989}{2}$ sq. ft. = $(\frac{2989}{2} \times \frac{1}{9}) = 165\frac{5}{6}$ sq. yd.

= 475,200 ft., it would make $475,200 \div 18.8496 = 25,210.08+$ revolutions. Ans.

(16) The area = $.7854 D^2$ (see Art. 71) = $.7854 \times 20^2 = 314.16$ sq. in. If the pressure is 60 lb. per square inch, the entire pressure is $314.16 \times 60 = 18,849.6$ lb. Ans.

(17) To find the volume of a cylinder, use the rule in Art. 88. Area of base = $.7854 D^2 = .7854 \times 2^2 = 3.1416$ sq. ft. (see Art. 71). Volume = $3.1416 \times 6 = 18.8496$ cu. ft. According to Art. 21, § 4, 1 cu. ft. = 7.481 gal.; therefore, 18.8496 cu. ft. = $18.8496 \times 7.481 = 141+$ gal. Ans.

(18) Using the rule of Art. 85, perimeter of base = $18 \times 3.1416 = 56.5488$ in.; $56.5488 \times 24 = 1,357.1712$; hence, the convex surface is 1,357.1712 sq. in. Ans.

(19) Perimeter of hexagon = $8 \text{ in.} \times 6 = 48 \text{ in.}$ $12 \times 48 = 576$; the convex surface is, therefore, 576 sq. in. Ans.

(20) Using the rule of Art. 119, area of upper base = $.7854 D^2 = .7854 \times 18^2 = 254.4696$ sq. in.; area of lower base = $.7854 \times 20^2 = 314.16$ sq. in.; the square root of the product of the areas of the two bases = $\sqrt{254.4696 \times 314.16} = 282.744$. Adding these three results and multiplying by one-third the altitude, $254.4696 + 314.16 + 282.744 = 851.3736$; 2 ft. = 24 in., $851.3736 \times \frac{24}{3} = 6,810.9888$. The volume is, therefore, 6,810.9888 cu. in. Ans.

(21) The cubical contents of the bin = $12 \times 5 \times 4 = 240$ cu. ft. = 414,720 cu. in. One bushel contains 2,150.42 cu. in. (Art. 24, § 4). Number of bushels = $414,720 \div 2,150.42 = 192.9-$. Ans.

(22) Using the rule of Art. 119, area of upper base = $4 \times 4 = 16$ sq. ft., area of lower base = $5 \times 5 = 25$ sq. ft., the square root of the product of the areas of the two bases = $\sqrt{16 \times 25} = 20$. Adding these three results, and multiplying by one-third of the altitude, $25 + 16 + 20 = 61$, and $61 \times \frac{6}{3} = 122$. Hence, the volume of the frustum is 122 cu. ft. Ans.

(23) A 7-foot square contains $7 \times 7 = 49$ sq. ft.; a 10-foot square contains $10 \times 10 = 100$ sq. ft. The area of the third square is $49 + 100 = 149$ sq. ft.; therefore, the length of a side = $\sqrt{149} = 12.207-$ ft. Ans.

(24) The volume of a 3-foot cube is $3 \times 3 \times 3 = 27$ cu. ft.; the volume of a 5-foot cube is $5 \times 5 \times 5 = 125$ cu. ft. Number of 3-foot cubes = $\frac{125}{27} = 4.63$, nearly. Ans.

(25) Using the rule in Art. 88, area of base = $.7854 \times 40^2 = 1,256.64$ sq. in.; altitude = 12 ft., or 144 in.; therefore, the volume of the cylinder = $1,256.64 \times 144 = 180,956.16$ cu. in. According to

Art. 21, § 4, 1 gal. contains 231 cu. in., hence, the tank holds $\frac{176000}{231}$ gal. = 763.36 gal. At 8 cents a gallon, the oil is worth $763.36 \times .08 = \$62.6688$, or \$62.67. Ans.

(26) Length of wall = $2 \times 150 + 2 \times 100 = 500$ ft. (see Art. 92). The actual length = $500 - 4 \times 1\frac{1}{2} = 494$ ft. Cubical contents = $494 \times 8 \times 1\frac{1}{2} = 5,928$ cu. ft. By the rule of Art. 91, the number of perches required for wall = $5,928 \div 24\frac{1}{2} = 239.52$ — Ans.

(27) Volume of wall = $90 \times 36 \times 4 = 12,960$ cu. ft. = $477\frac{2}{3}$ cu. yd. Number of bricks = $477\frac{2}{3} \times 574 = 275,520$. Ans.

(28) Use rule, Art. 99. Volume of bin = $12 \times 8 \times 6 = 576$ cu. ft.; or $576 \times 1.728 = 995,328$ cu. in. In 1 heaped bushel there are 2,747.71 cu. in.; therefore, in 995,328 cu. in. there are $995,328 \div 2,747.71 = 362.24$ — bu. Ans.

(29) Using rule, Art. 70, $D = \frac{C}{3.1416} = \frac{1,000}{3.1416}$. But (Art. 71) area = $7854 D^2 = 7854 \times \left(\frac{1,000}{3.1416}\right)^2 = \frac{7854 \times 1,000 \times 1,000}{3.1416 \times 3.1416} = \frac{1,000,000}{12.5664} = 79,577.285$ — Ans.

(30) The base is 26 ft. and the altitude is 23 in., or $1\frac{1}{4}$ ft. Area = base \times altitude = $26 \times \frac{23}{12} = 49\frac{1}{2}$ sq. ft. At 15 cents per square foot, the cost is $49\frac{1}{2} \times 15 = \7.48 — Ans.

(31) (a) The diameter is the diagonal of the square, and the hypotenuse of a right-angled triangle, the other two sides of which are equal, being the sides of a square. By Art. 58, the square of the hypotenuse is equal to the sum of the squares of the other two sides; then, 48^2 , or 2,304, is equal to double the square of one of the sides of the square. Hence, the side of the square is $\sqrt{\frac{2304}{2}} = 24$, and the area of the square is $(\sqrt{\frac{2304}{2}})^2 = 24^2 = 1,152$ sq. in. Ans.

(b) Area of circle = $7854 D^2$ (Art. 71) = $7854 \times 48^2 = 7854 \times 2,304 = 1,809,5616$ sq. in. Area of square = 1,152 sq. in., then the zinc cut off = $1,809.5616 - 1,152 = 657.5616$ sq. in. Ans.

(32) Area of the larger outside circle = $3.1416 R^2$ (Art. 71) = $3.1416 \times 24^2 = 1,809.5616$ sq. ft. Area of the inner circle = $3.1416 R^2 = 3.1416 \times 15^2 = 706.86$ sq. ft. Number of square feet of the larger circle outside of the inner circle = $1,809.5616 - 706.86 = 1,102.7016$ sq. ft. Ans.

(33) 8 ft. 6 in. = 102 in. Volume or contents = $102 \times 102 \times 102 = 1,061,208$ cu. in. According to Art. 21, § 4, 1 gal. contains

231 cu. in. Hence, the cask can hold $\frac{1,061,208}{231} = 4,593.974$ gal. If the faucet discharges 5 gal. per minute, it will take it $\frac{4,593.974}{5} = 918.795$ min., or 15 hr. 18.795 min. Ans.

(34) Since one bu. contains 2,150.42 cu. in., 1,000 bu. contain $1,000 \times 2,150.42 = 2,150,420$ cu. in. Since the volume or cubical contents is 2,150,420 cu. in., the length of an edge $= \sqrt[3]{2,150,420} = 129+$ in., or 10.75+ ft. Ans.

(35) See Art. 117. Area of the base $= 5^2 \times .7854 = 19.635$ sq. ft. Hence, the volume $= 19.635 \times \frac{8}{3} = 52.36$ cu. ft. If a cubic foot weighs 2,838 oz., 52.36 cu. ft. weigh $52.36 \times 2,838 = 148,597.68$ oz. $= 11,852.72$ lb. $= 9,287.355$ lb. Ans.

(36) See Art. 124. 12 in. = 1 ft. Volume of ball $= .5236 \times 1^3 = .5236$ cu. ft. Weight of ball $= .5236 \times 450 = 235.62$ lb. Ans.

(37) See Art. 127. Mean diameter $= \frac{50+56}{2} = 53$ in. Length $= 6$ ft. $= 72$ in. Capacity $= 53^2 \times 72 \times .0034 = 687.64+$ gal. Ans.

(38) First find the diagonal of the floor, which is evidently the hypotenuse of a right-angled triangle whose sides are 14 ft. By Art. 60, hypotenuse $= \sqrt{14^2 + 14^2} = 19.8$ ft. The required line is the hypotenuse of a right triangle, of which the height of the room, 14 ft., is one short side, and the floor diagonal, 19.8 ft., is the other short side; therefore, the length of the line is $\sqrt{19.8^2 + 14^2} = 24.25-$ ft. Ans.

(39) Since the sphere fits exactly inside a cubical box, its diameter is equal to the side or edge of the cube. Edge $= \sqrt[3]{8} = 2$ ft. = diameter of sphere. By Art. 124, volume of sphere $= .5236 \times 2^3 = 4.1888$ cu. ft. Ans.

(40) The diameter is the diagonal of the square, or the hypotenuse of a right-angled triangle whose short sides are equal, being sides of a square. By Art. 58, $20^2 = 400 =$ double the sum of the square of one side of the square. Hence, the side of the square is $\sqrt[4]{200}$ in. and the area of the square is $(\sqrt[4]{200})^2 = 200$ sq. in. $= \frac{200}{144}$ sq. ft. Volume of timber $=$ area of end multiplied by length $=$

$$\frac{200}{144} \times \frac{24}{1} = \frac{200}{6} = 33\frac{1}{3} \text{ cu. ft. Ans.}$$

(41) Contents of the car $= 40 \times 6\frac{1}{2} \times 4\frac{1}{2} = 1,170$ cu. ft. According to Art. 107, a ton of Schuylkill coal measures 35 cu. ft. Number of tons in car $= \frac{1,170}{35} = 33.4+$ T. Ans.

(42) Perimeter of first room = $12 \times 2 + 14 \times 2 = 52$ ft.
 Perimeter of second room = $15 \times 2 + 15 \times 2 = 60$ ft.
 Perimeter of third room = $16 \times 2 + 18 \times 2 = 68$ ft.
 Perimeter of fourth room = $15 \times 2 + 19 \times 2 = 68$ ft.
 Entire perimeter of rooms..... 248 ft.
 Area of ceiling of first room = $12 \times 14 = 168$ sq. ft.
 Area of ceiling of second room = $15 \times 15 = 225$ sq. ft.
 Area of ceiling of third room = $16 \times 18 = 288$ sq. ft.
 Area of ceiling of fourth room = $15 \times 19 = 285$ sq. ft.
 Area of ceilings..... 966 sq. ft.
 Area of walls = $248 \times 9\frac{1}{2} = 2,314\frac{1}{2}$ sq. ft.
 Area of ceilings = 966 sq. ft.
 Total area..... 3,280 $\frac{1}{2}$ sq. ft.
 Area in square yards = $3,280\frac{1}{2} \div 9 = 364.52$.
 Cost = $364.52 \times \$0.475 = \173.14 . Ans.

(43) See rule I, Art. 37. Perimeter of room = $2 \times 16 + 2 \times 20\frac{1}{2} = 73$ ft. Width of openings = $5 \times 3 + 3 \times 3 = 24$ ft. Perimeter, allowing for openings, = $73 - 24 = 49$ ft. = $16\frac{1}{2}$ yd. = $32\frac{1}{2}$ half yards, hence, 33 strips are required. Assuming that the strips extend the height of the baseboard above the bottom edge of the border, the length of a strip is (since 18 in. = $1\frac{1}{2}$ ft.) $11 - 1\frac{1}{2} = 9\frac{1}{2}$ ft. The number of strips in a double roll is therefore $16 \times 3 \div 9\frac{1}{2} = 5$, and the number of rolls required is $33 \div 5 = 6\frac{3}{5}$, or say 7 double rolls. Ans.

The number of strips in a single roll of 8 yd., or 24 ft. = $24 \div 9\frac{1}{2} = 2$ strips, and the number of single rolls required is $33 \div 2 = 16\frac{1}{2}$, or say 17. Ans.

Since the perimeter of the room is 73 ft. = $24\frac{1}{2}$ yd., the number of double rolls of border required is $24\frac{1}{2} \div 16 = 1.52$, say 2. Ans.

(44) The question does not state which way the strips are to run, but it is economical to have them run lengthwise. Width of room = 15 ft. = 180 in. Width of carpet = 27 in. The number of strips = $180 \div 27 = 6\frac{2}{3}$. Hence, 7 strips must be used. Allowing 1 ft. for matching, length of strip = $17 + 1 = 18$ ft. = 6 yd. Number of yards required = $7 \times 6 = 42$. Cost = $42 \times \$1.25 = \52.50 . Ans.

(45) Outside length of wall = $2 \times 145 + 2 \times 75 = 440$ ft.
 Net length of wall = $440 - 4 \times 1 = 436$ ft. (see Art. 92).
 Contents of wall = $436 \times 80 \times 1 = 34,880$ cu. ft.
 Deduction for windows = $7 \times 31 \times 1 \times 110 = 2,695$ cu. ft.
 Deduction for 4 doors = $8 \times 10 \times 1 \times 4 = 320$ cu. ft.
 Deduction for 2 doors = $6 \times 8 \times 1 \times 2 = 96$ cu. ft.
 Net contents of wall = $34,880 - (2,695 + 320 + 96) = 31,769$ cu. ft.
 Number of bricks = $31,769 \div 22\frac{1}{2} = 1,412.4$, or 1,412 M.

Cost of bricks = $232.005 \times \$5.60 = \$1,299.23$. Ans.

Cost of laying = $232.005 \times \$1.45 = \336.41 . Ans.

(46) Perimeter of room = $2 \times 75 + 2 \times 52 = 254$ ft.
 Area of walls = $254 \times 20 = 5,080$ sq. ft.
 Area of ceiling = $75 \times 52 = 3,900$ sq. ft.
 Total..... 8,980 sq. ft.

Area in square yards = $8,980 \div 9 = 997.778$ sq. yd.

At $12\frac{1}{2}$ cents, or $\frac{1}{8}$ dollar, per square yard, 997.778 sq. yd. will cost $997.778 \times \$\frac{1}{8} = \124.72 . Ans.

(47) See Art. 46. Shingles required on one side = $\frac{55 \times 19 \times 144}{4 \times 5}$
 = 7,524. Shingles required on other side = $\frac{55 \times 26 \times 144}{4 \times 5} = 10,296$.
 Total number of shingles required = $7,524 + 10,296 = 17,820$. Ans.
 According to Art. 45, number of bundles = $\frac{17,820}{250} = 71.3$ —, say 72. Ans.

(48) From example 42, the area of the rooms is 364.52 — yd. At 21 cents per square yard, 364.52 — yd., of plastering will cost 364.52 — $\times \$\frac{21}{100} = \76.55 . Ans.

(49) See Arts. 91 and 92.

Length of wall (outside) = $2 \times 80 + 2 \times 40 = 240$ ft.

Actual length = $240 - 4 \times 3 = 228$ ft.

Actual cubical contents = $228 \times 36 \times 3 = 24,624$ cu. ft.

Allowance for windows = $6 \times 12 \times 3 \times 12 = 2,592$ cu. ft.

Allowance for doors = $7 \times 10 \times 3 \times 3 = 630$ cu. ft.

Net contents = $24,624 - (2,592 + 630) = 21,402$ cu. ft.

(a) Perches required for wall = $21,402 \div 24\frac{1}{2} = 864.7$. Ans.

(b) Since in estimating the cost of the work, no allowance is made for corners, doors, and windows,

Cubical contents = $240 \times 36 \times 3 = 25,920$ cu. ft.

Perches of stonework = $25,920 \div 24\frac{1}{2} = 1,047\frac{3}{4}$.

Cost of laying walls = $1,047\frac{3}{4} \times \$3.75 = \$3,927.27$. Ans.

(50) See Art. 42. (a) Number of thousand feet = $\frac{18 \times 12 \times 1\frac{1}{2} \times 48}{1,000}$
 = 15.552. Ans.

(b) Value = $15.552 \times \$19.65 = \305.60 . Ans.



ARITHMETIC.

(1) (a) The ratio $24 : 18$ may also be expressed as $\frac{24}{18}$ (see Art. 2).

$$\frac{24}{18} \div 2 = \frac{\overset{2}{\cancel{24}}}{\underset{3}{\cancel{18}}} \times \frac{1}{2} = \frac{2}{3} \text{ Ans.}$$

(b) The ratio $\frac{3}{4} : \frac{4}{5} = \frac{\frac{3}{4}}{\frac{4}{5}} = \frac{3}{4} \times \frac{5}{4} = \frac{15}{16}$; $\frac{15}{16} \div 5 = \frac{\overset{3}{\cancel{15}}}{16} \times \frac{1}{\cancel{5}} = \frac{3}{16}$ Ans.

(c) The ratio $.2 : .05 = \frac{.2}{.05} = 4$; $4 \div 4 = 4 \times \frac{1}{4} = 1$. Ans.

(d) The inverse ratio of 1 to .2 is $.2 : 1 = \frac{.2}{1} = .2$; $.2 \div \frac{1}{2} = .4$.
Ans.

(e) The inverse ratio of 12.5 to 125 is

$$125 : 12.5 = \frac{125}{12.5} = 10; 10 \div \frac{3}{2} = \frac{10}{1} \times \frac{2}{3} = 6\frac{2}{3} \text{ Ans.}$$

(2) (a) The ratio $\frac{4}{3} : \frac{3}{4} = \frac{\frac{4}{3}}{\frac{3}{4}} = \frac{4}{3} \times \frac{4}{3} = \frac{16}{9}$; $\frac{16}{9} \div 3 = \frac{16}{9} \times \frac{1}{3} = \frac{16}{27}$ Ans.

(b) The ratio $1\frac{1}{2} : 2\frac{1}{2} = \frac{1\frac{1}{2}}{2\frac{1}{2}} = \frac{\frac{3}{2}}{\frac{5}{2}} = \frac{3}{5}$; $\frac{3}{5} \times \frac{10}{1} = 6$. Ans.

(c) The volume of a gallon is 231 cu. in., a cubic foot is 1,728 cu. in.

The ratio is $231 : 1,728$, or $\frac{231}{1,728}$; $\frac{231}{1,728} \times \frac{27}{1} = \frac{231}{64} = 3.6+$. Ans.

(d) One rod is equal to $5\frac{1}{2}$ yd. The direct ratio of a yard to a rod is $1 : 5\frac{1}{2}$; hence, the inverse ratio (see Art. 4) is

$$5\frac{1}{2} : 1 \text{ or } \frac{5\frac{1}{2}}{1} = 5\frac{1}{2}; 5\frac{1}{2} \times 4 = \frac{11}{2} \times 4 = 22. \text{ Ans.}$$

(c) The inverse ratio of $33\frac{1}{2}$ to $187\frac{1}{2} = 187\frac{1}{2} : 33\frac{1}{2} = \frac{187\frac{1}{2}}{33\frac{1}{2}} = \frac{272}{100}$
 $= \frac{15}{375} \times \frac{3}{100} = \frac{45}{8} \cdot \frac{45}{8} \times 4 = \frac{45}{2} = 22.5. \text{ Ans.}$

(3) (a) The ratio $\frac{2}{3} : \frac{3}{4} = \frac{\frac{2}{3}}{\frac{3}{4}} = \frac{2}{3} \times \frac{4}{3} = \frac{8}{9}$; the ratio $5 : 1\frac{1}{2} = \frac{5}{1\frac{1}{2}}$
 $= \frac{5}{1} \times \frac{2}{3} = \frac{10}{3} \cdot \frac{8}{9} + \frac{10}{3} = \frac{8+30}{9} = \frac{38}{9} = 4\frac{2}{9}. \text{ Ans.}$

(b) $.5 : 1.2 = \frac{.5}{1.2} = \frac{5}{12}$; $\frac{3}{4} : 1\frac{1}{2} = \frac{\frac{3}{4}}{1\frac{1}{2}} = \frac{3}{4} \times \frac{2}{15} = \frac{2}{5} \cdot \frac{5}{12} + \frac{2}{5} = \frac{25+24}{60}$
 $= \frac{49}{60}. \text{ Ans.}$

(c) $\frac{2}{3} : .8 = \frac{\frac{2}{3}}{\frac{8}{10}} = \frac{2}{3} \times \frac{10}{8} = \frac{10}{12} = \frac{5}{6}$; $1\frac{1}{2} : 4\frac{1}{2} = \frac{\frac{3}{2}}{\frac{9}{2}} = \frac{3}{2} \times \frac{2}{9} = \frac{1}{3}$
 $\frac{5}{6} + \frac{1}{3} = \frac{5}{6} + \frac{2}{6} = \frac{7}{6} = 1\frac{1}{6}. \text{ Ans.}$

(d) $2.3 : 11\frac{1}{2} = \frac{2.3}{11.5} = .2$. The inverse ratio of .8 to $1.2 = 1.2 : .8$
 $= \frac{1.2}{.8} = 1.5. .2 + 1.5 = 1.7. \text{ Ans.}$

(e) The inverse ratio of 4.2 to 21 $= 21 : 4.2 = \frac{21}{4.2} = 5$; $4 : 13\frac{1}{3}$
 $= \frac{4}{13\frac{1}{3}} = \frac{4}{1} \times \frac{3}{40} = \frac{3}{10}$, or .3; $5 + 3 = 5.3. \text{ Ans.}$

(4) (a) The inverse ratio of $7.2 : 5 = 5 : 7.2 = \frac{5}{7.2} = \frac{5}{72} = 5 \times \frac{10}{72} = \frac{50}{72}$.
 Since multiplying both terms of the fraction by the same number does not change the value of the fraction, $\frac{50}{72} = \frac{50 \times 2}{72 \times 2} = \frac{100}{144} \cdot \sqrt[4]{144} = \sqrt[4]{100}$
 $= \frac{10}{12} = \frac{5}{6}. \text{ Ans.}$

(b) The inverse ratio of 16 to 54 $= 54 : 16 = \frac{54}{16}$. Multiplying both terms of the fraction by 4, $\frac{54}{16} = \frac{216}{64} \cdot \sqrt[3]{64} = \sqrt[3]{216} = \frac{6}{4} = 1.5. \text{ Ans.}$

(5) (a) The inverse ratio of

$\frac{2}{3}$ to $\frac{3}{2} = \frac{3}{2} : \frac{2}{3} = \frac{3}{2} = \frac{3}{2} \times \frac{3}{2} = \frac{9}{4} \cdot (\frac{3}{2})^2 = \frac{9}{4} = 5.0625. \text{ Ans.}$

(b) $4.2 : .14 = \frac{4.2}{.14} = 30$; $(30)^2 = 27,000. \text{ Ans.}$

(6) (a) The inverse ratio of

$$5.2 \text{ to } 41.6 = 41.6 : 5.2 = \frac{41.6}{5.2} = 8. \quad \frac{3}{4} \text{ of } 8 = 6. \quad \text{Ans.}$$

(b) In a bushel there are 2,150.42 cu. in., and in a cubic foot there are 1,728 cu. in. The direct ratio of a bushel to a cubic foot = 2,150.42 : 1,728, and the inverse ratio is

$$1,728 : 2,150.42 = \frac{1,728}{2,150.42} = .804-. \quad \text{Ans.}$$

(7) (a) $20 : 23 = 39 : 45$. The products $20 \times 45 = 900$ and $23 \times 39 = 897$ are not equal, and the numbers cannot form a proportion, since in a proportion the product of the means must be equal to the product of the extremes (see Art. 25).

(b) $20 : x = 39 : 45$; $39x = 900$, $x = 23\frac{1}{3}$. Hence, 23 must be increased by $\frac{1}{3}$. Ans.

(8) If the proportion were direct it would be $3 : 5 = x : 8$. The inverse proportion is evidently $5 : 3 = x : 8$. From this, $3x = 40$, and $x = \frac{40}{3} = 13\frac{1}{3}$. Ans.

(9) 2 lb. 3 oz. = 35 oz.; 5 lb. $1\frac{1}{2}$ oz. = $81\frac{1}{2}$ oz.

The ratio is $35 : 81\frac{1}{2}$; multiplying both terms of the ratio by 2, it becomes $70 : 163$. Ans.

$$(10) \quad 3 : 8 = \frac{3}{8} \text{ and } 5 : 12 = \frac{5}{12}, \quad \frac{3}{8} = \frac{9}{24}, \quad \frac{5}{12} = \frac{10}{24}, \quad \frac{10}{24} - \frac{9}{24} = \frac{1}{24}.$$

Hence, the ratio $5 : 12$ is greater by $\frac{1}{24}$. Ans.

(11) Here the vertical line is used instead of the colon to express the ratio (see Art. 2). The product of all the numbers included between the vertical lines is equal to the product of all the numbers without them; hence,

$$\begin{array}{c|c} 12 & 3 \\ 15 & 5 \\ 7 & 10 \end{array} = x \quad \left| \begin{array}{c} 20 \end{array} \right.$$

$$\begin{aligned} \frac{12}{15} \times \frac{2}{3} \times 7 \times 20 &= \frac{3}{5} \times \frac{5}{9} \times 10 \times x, \text{ or } x = \frac{12 \times 2 \times 7 \times 20}{\frac{3}{5} \times \frac{5}{9} \times 10} \\ &= \frac{4}{5} \times \frac{2}{3} \times \frac{2}{3} \times \frac{3}{5} \times \frac{20 \times 5 \times 9}{10} = \frac{112}{5} = 22.4. \quad \text{Ans.} \end{aligned}$$

(12) (a) $3 : .5 = x : .9$. The product of the means is equal to the product of the extremes; hence,

$$.5x = 2.7 \text{ or } x = \frac{2.7}{.5} = 5.4 = 5\frac{1}{5} = 5\frac{2}{5}. \quad \text{Ans.}$$

$$(b) \quad 3 : x = 7 : 19. \quad x = \frac{19 \times 3}{7} = \frac{57}{7} = 8\frac{1}{7}. \quad \text{Ans.}$$

(19) According to Art. 50, the volume of a cylinder varies directly as its length and directly as the square of the diameter. Taking the dimensions for the causes and squaring the diameters,

$$\begin{array}{c|c} 8^2 & 5^2 \\ \hline 5 & 9 \end{array} = 100 \quad \text{or} \quad \begin{array}{c|c} 9 & 25 \\ \hline 5 & 9 \end{array} = 100 \quad x;$$

whence, $x = 500$. The second cylinder will therefore hold 500 gal. Ans.

(20) Since the cylinders have equal volumes but different diameters, the diameters are to each other inversely as the square roots of their lengths (see Art. 41). The direct proportion would be

$$15 : x = \sqrt{49} : \sqrt{36}.$$

Therefore the inverse proportion is

$$15 : x = \sqrt{36} : \sqrt{49}, \text{ or } 15 : x = 6 : 7; \text{ whence, } x = \frac{7 \times 15}{6} = 17.5.$$

The diameter of the second cylinder is, therefore, 17.5 in. Ans.

(21) Since the intensity of light varies inversely as the square of the distance from the source of light (see Art. 42, example 1), we have the proportion, if written direct, $20 : x = 4^2 : 5^2$. The inverse proportion is $20 : x = 5^2 : 4^2$, or, $20 : x = 25 : 16$. Whence, $x = \frac{16 \times 20}{25} = 12.8$.

Ans.

(22) As the volumes vary as the cubes of the diameter, we have the proportion

$$6^3 : 12^3 = 113.0976 \text{ cu. in.} : x, \text{ or } 216 : 1,728 = 113.0976 \text{ cu. in.} : x.$$

$$x = \frac{1,728 \times 113.0976}{216} = 904.7808 \text{ cu. in.} \quad \text{Ans.}$$

$$(23) \quad \begin{array}{c|c} 3 & 3 \\ \hline 4 & 8 \\ \hline 5 & 10 \end{array} = \begin{array}{c} 3,315 \\ 9,945 \text{ lb.} \end{array} \quad \left| \quad x, \text{ or } x = 2 \times 10 \times 3,315 \text{ lb.} = 66,300 \text{ lb.} \quad \text{Ans.} \right.$$

(24) Since $3 : 7 = 8 : 12$ is not a correct proportion, substituting x for the first term, the proportion is $x : 7 = 8 : 12$, from which $x = \frac{56}{12} = 4\frac{2}{3}$. Hence, $4\frac{2}{3} - 3 = 1\frac{2}{3}$ is to be added to the first term. Ans.

Substituting x for the second term, $3 : 7 = 8 : 12$, whence $x = \frac{24}{7} = 4\frac{1}{7}$. Hence, $7 - 4\frac{1}{7} = 2\frac{6}{7}$ must be subtracted from the second term to form a correct proportion. Ans.

(25) Since 5 men do something, 5 men is a cause, and since they take 9 da. of 10 hr. each to do the work, 9 da. and 10 hr. are each an element of the first cause. For the same reason 8 men working x da. of 9 hr. each constitute a second cause. The first effect, that which is done, is a wall 100 ft. long, 6 ft. high, and 18 in. thick, and the second effect is a

wall 120 ft. long, 8 ft. high, and 20 in. thick. Hence, we write the following proportion and cancel where possible:

$$\begin{array}{c|c|c} 5 & 8 & 20 \\ \cancel{9} & x = \frac{100}{9} & \frac{120}{2} \\ \cancel{10} & 9 & \cancel{20} \end{array}, \text{ or } x = \frac{20 \times 5}{9} = \frac{100}{9} = 11\frac{1}{9}.$$

The required time is $11\frac{1}{9}$ da., and since a day's work consists of 9 hr., $\frac{1}{9}$ da. = 1 hr.; hence, $11\frac{1}{9}$ da. = 11 da. 1 hr. Ans.

(26) Letting x = the length of the second pendulum, then the proportion is $\sqrt{39}$ in. : $\sqrt{x} = 1 : 2$. Squaring each term of the proportion, 39 in. : $x = 1 : 4$, or $x = 156$ in. Ans.

(27) See rule, Art. 56. Sum of proportional parts = $2 + 3 + 5 = 10$. The three proportions are therefore:

$$2 : 10 = x : \$4,750;$$

$$3 : 10 = x : \$4,750;$$

$$5 : 10 = x : \$4,750.$$

Hence, 1st man's share = $\frac{\$4,750 \times 2}{10} = \950 . Ans.

2d man's share = $\frac{\$4,750 \times 3}{10} = \$1,425$. Ans.

3d man's share = $\frac{\$4,750 \times 5}{10} = \$2,375$. Ans.

(28) By inspection $\frac{1}{2} \div \frac{2}{3} = \frac{3}{4}$; and $\frac{15}{28} \div \frac{5}{7} = \frac{3}{4}$. Since the ratios are equal, those numbers will form the proportion $\frac{1}{2} : \frac{2}{3} = \frac{15}{28} : \frac{5}{7}$. Reducing to common denominator, $\frac{12}{24} : \frac{16}{24} = \frac{45}{44} : \frac{50}{44}$.

Multiplying each term by 84, $42 : 56 = 45 : 60$. Ans.

PROOF.—The ratio of the first couplet is $\frac{42}{56} = \frac{3}{4}$, and that of the second is $\frac{45}{60} = \frac{3}{4}$. Hence, the proportion is correct.

$$(29) \begin{array}{c|c|c} 4^2 & 5^2 & 23 \\ 50 & 120 = 5,385.6 \text{ gal.} & 4 \\ 33 & 44 & 16 \\ & & 2 \\ & & 10 \\ & & 40 \\ & & 33 \\ & & 44 \\ & & 3 \\ & & 4 \end{array}, \text{ or } \begin{array}{c|c} 50 & 120 = 5,385.6 \text{ gal.} \\ 33 & 44 \\ 3 & 4 \end{array} x;$$

whence $x = 5 \times 5,385.6 \text{ gal.} = 26,928 \text{ gal.}$ Ans.

(30) $4 : 8 = 6 : 12$; increasing the antecedents by 10 and substituting x for the first consequent,

$$14 : x = 16 : 12,$$

$$16x = 14 \times 12,$$

$$16x = 168,$$

$$x = 10\frac{1}{2}.$$

Hence, $10\frac{1}{2} - 8 = 2\frac{1}{2}$ must be added to the first consequent. Ans.

(31) Reducing the fractions to the least common denominator and adding, $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} = \frac{6+8+9}{12} = \frac{23}{4}$. Multiplying all parts by 12, which does not change their relative value, $6+8+9 = 23$. Applying rule, Art. 56,

1st part, $6 : 23 = x : 100$, or $23x = 600$; $x = 26\frac{2}{3}$. Ans.

2d part, $8 : 23 = x : 100$, or $23x = 800$; $x = 34\frac{1}{3}$. Ans.

3d part, $9 : 23 = x : 100$, or $23x = 900$; $x = 39\frac{3}{23}$. Ans.

(32) Calling the load the effect, the proportion will be written

$$\begin{array}{c|c} 20 & 15 \\ 3 & 4 \\ 6^3 & 2^3 \end{array} = 6,000 \text{ lb.} \quad \left| \quad x.$$

According to the statement of the problem, however, the load varies inversely as the length; hence, the lengths 20 and 15 must be transposed, and the proportion becomes

$$\begin{array}{c|c} 15 & 20 \\ 3 & 4 \\ 36 & 25 \\ 9 & \end{array} = \begin{array}{c} 400 \text{ lb.} \\ 6,000 \text{ lb.} \end{array} \quad \left| \quad x;$$

whence, $x = \frac{20 \times 25 \times 400}{9 \times 3} = 7,407.4 \text{ lb.}$ Ans.

(33) Since the weights vary as the cubes of their lengths, the proportion is $20^3 : 30^3 = 10 : x$, or $8,000 : 27,000 = 10 : x$.

$$x = \frac{27 \times 10}{8} = 33\frac{3}{4} \text{ lb.} \text{ Ans.}$$

(34) See example 33; $5.5^3 : 6^3 = 140 : x$, or $166.375 : 216 = 140 : x$.

$$x = \frac{216 \times 140}{166.375} = 181.76 \text{— lb.} \text{ Ans.}$$

(35) Here taking \$250, 2 yr. 6 mo., and 8% as the first cause and \$450, 3 yr. 10 mo., and 6% as the second cause, \$50 for the first effect, x for the second effect, and reducing the time to months, we have

$$\begin{array}{c|c} 250 & 450 \\ 30 & 46 \\ 8 & 6 \end{array} = 50 \quad \left| \quad x.$$

Canceling where possible,

$$\begin{array}{c|c} 5 & 3 \\ 250 & 15 \\ 30 & 45 \\ 8 & 23 \\ 4 & 46 \\ 2 & 6 \\ & 3 \end{array} = 50 \quad \left| \quad x,$$

from which

$$x = \frac{3 \times 23 \times 3}{2} = \frac{207}{2} = 103.5.$$

Hence, the interest is \$103.50. Ans.

(36) Taking the dimensions as the causes and the prices paid for the effects, we have

$$\frac{9}{8} \left| \frac{20}{16} = \$100 \right| x; \text{ or } x = \frac{20 \times 2 \times \$100}{9} = \$444.44+. \quad \text{Ans.}$$

(37) Since the distances through which bodies will fall vary as the squares of the times, we have the proportion $144 : x = 3^2 : 5^2$. Squaring, $144 : x = 9 : 25$, or

$$x = \frac{25 \times 144}{9} = 400 \text{ ft.} \quad \text{Ans.}$$

(38) The radii of the spheres are respectively $\frac{12}{2} = 6$ in. and $\frac{30}{2} = 15$ in. Since the weight of spheres of similar material vary as the cubes of their radii, we have the proportion $6^3 : 15^3 = 371.75 : x$. Dividing both terms of the first couplet by 3^3 (see Art. 13), $2^3 : 5^3 = 371.75 : x$, or $8 : 125 = 371.75 : x$; whence,

$$x = \frac{125 \times 371.75}{8} = 5808.59+ \text{ lb.} \quad \text{Ans.}$$

(39) Since the times of oscillation of a pendulum vary directly as the square roots of the length of the pendulum, the proportion is

$$\sqrt{39} : \sqrt{20} = 1 : x.$$

Squaring the terms, $39 : 20 = 1 : x^2$, or $x^2 = \frac{20}{39} = .51282+$; whence, $x = \sqrt{.51282} = .7161 \text{ sec.} \quad \text{Ans.}$

(40) The capacities of the bins are proportional respectively to the products, $2 \times 3 \times 5 = 30$; $3 \times 4 \times 6 = 72$; and $4 \times 5 \times 6 = 120$. Using the rule of Art. 56, $30 + 72 + 120 = 222$. Hence, the proportions are

$$30 : 220 = x : 3,700, \text{ or } x = \frac{3,700 \times 30}{222} = 500 \text{ bu.} \quad \text{Ans.}$$

$$72 : 220 = x : 3,700, \text{ or } x = \frac{3,700 \times 72}{222} = 1,200 \text{ bu.} \quad \text{Ans.}$$

$$120 : 220 = x : 3,700, \text{ or } x = \frac{3,700 \times 120}{222} = 2,000 \text{ bu.} \quad \text{Ans.}$$

ARITHMETIC.

(1) See Art. 3. $\frac{1}{2}\% = \frac{\frac{1}{2}}{100} = \frac{1}{2} \times \frac{1}{100} = \frac{1}{200} = .005.$ Ans.

$\frac{3}{4}\% = \frac{\frac{3}{4}}{100} = \frac{3}{4} \times \frac{1}{100} = \frac{3}{400} = .0075.$ Ans.

$1\frac{3}{8}\% = \frac{1\frac{3}{8}}{100} = \frac{11}{8} \times \frac{1}{100} = \frac{11}{800} = .01375.$ Ans.

$\frac{5}{8}$ of $1\% = \frac{5}{8}$ of $\frac{1}{100} = \frac{5}{800} = .00625.$ Ans.

$\frac{2}{3}$ of $\frac{3}{4}\% = \frac{2}{3}$ of $\frac{3}{400} = \frac{2}{3} \times \frac{3}{400} = \frac{1}{200} = .005.$ Ans.

(2) The rate per cent. is $6\frac{1}{4}$; that is, $6\frac{1}{4}\%$ of the entire crop is potatoes. The rate is $.06\frac{1}{4} = .0625$. 1,575 bu. is the difference between the entire crop and the number of bushels of potatoes planted. The base is the number of bushels in the crop. Then, by Art. 30, base = difference \div (1 - rate); whence, base = $1,575 \div (1 - .0625) = 1,575 \div .9375 = 1,680$ bu. Ans.

(3) The base is \$448.64 and the rate is .375. Hence, by Art. 14, percentage = base \times rate = $\$448.64 \times .375 = \$168.24.$ Ans.

(4) $37\frac{1}{2}\% + 12\frac{1}{2}\% + 16\frac{2}{3}\% = 66\frac{2}{3}\%$, the total per cent. of the salary expended; hence, $100\% - 66\frac{2}{3}\% = 33\frac{1}{3}\% =$ per cent. saved. Hence, \$90 must be $33\frac{1}{3}\%$ of the month's salary. $33\frac{1}{3}\% = \frac{1}{3}$. Since \$90 is $\frac{1}{3}$ of the month's salary, $\$90 \times 3 = \270 must be the month's salary. The salary per year is therefore $12 \times \$270 = \$3,240.$ Ans.

(5) Since 23% of some number is 315 more than $12\frac{1}{2}\%$ of the number, 315 is evidently the difference between 23% and $12\frac{1}{2}\% = 10\frac{1}{2}\%$ of the number. The rate is .105, and the percentage is 315; by Art. 19, base = percentage \div rate = $315 \div .105 = 3,000.$ Ans.

(6) Water consists of $2 + 16 = 18$ parts of H and O . The proportions are:

$18 : 2 = 100\% : H$. Hence,

$$H = \frac{100\% \times 2}{18} = 11\frac{1}{9}\% \text{ Ans.}$$

$18 : 16 = 100\% : O$, or

$$O = \frac{100\% \times 16}{18} = 88\frac{8}{9}\% \text{ Ans.}$$

(7) If goods are bought at 20% below list price, they cost $100\% - 20\% = 80\%$ of list price. If the gain is 30%, then by Art. 40, selling price = cost $\times (1 + \text{rate of gain}) = 80 \times (1 + .30) = 80 \times 1.30 = 104\%$ of list price. 104% of list price is $104\% - 100\% = 4\%$ above list price. Ans.

(8) Since $\frac{1}{3} = .40 = 40\%$, $\frac{1}{3}$ of the distance is 40% of the distance. 30% of 40% = $.30 \times .40 = .12$, or 12%; hence, $10\frac{1}{2}$ mi. is 12% of the distance. Evidently $10\frac{1}{2}$ mi. is the percentage and .12 is the rate. By Art. 19, base = percentage \div rate = $10.8 \div .12 = 90$ mi. Ans.

(9) The selling price of each horse is \$120, the rate of gain on one horse is .25 and the rate of loss on the other is .25. To find the cost of each horse we use the rule of Art. 41. Cost of 1st horse = selling price $\div (1 + \text{rate of gain}) = \$120 \div 1.25 = \$96$. Cost of 2d horse = selling price $\div (1 - \text{rate of loss}) = \$120 \div .75 = \$160$. Gain on 1st horse, $\$120 - \$96 = \$24$. Loss on 2d horse, $\$160 - \$120 = \$40$. Net loss = $\$40 - \$24 = \$16$. Ans.

(10) Coal used $15 - 9\frac{1}{2} = 5\frac{1}{2}$ T. Here $5\frac{1}{2}$ T. is a certain percentage of 15 T, which must be considered as the base. Since rate = percentage \div base (see Art. 17), the rate is $5\frac{1}{2} \div 15 = 5.75 \div 15 = .38\bar{3}$. Hence, $5\frac{1}{2}$ T. is $38\frac{1}{3}\%$ of 15 T. Ans.

(11) If 65% of the cost of the farm is added to the cost of the farm, the amount obtained is \$13,691.70. Considering the cost of the farm the base, .65 is the rate, and \$13,691.70 is the amount obtained by adding the percentage to the base. To find the base, or cost of the farm, we use the rule of Art. 29. Cost of farm = $\$13,691.70 \div (1 + .65) = \$8,298$. Ans.

Cost of buildings = 65% of cost of farm = $\$8,298 \times .65 = \$5,393.70$, or cost of buildings = $\$13,691.70 - \$8,298 = \$5,393.70$. Ans.

(12) 28% of 6,375 = $6,375 \times .28 = 1,785$, the increase in 5 yr. Adding the increase to the original population, the number of inhabitants after the increase is $6,375 + 1,785 = 8,160$. Ans.

(13) From the statement of the question, B's buying price is 120% of A's buying price, and C's buying price is 125% of B's. $1.20 \times 1.25 = 1.50$;

hence, C's buying price is 150% of A's buying price, that is, C paid 50% more for the horse than A originally paid for it. But from the statement, C paid \$60 more than A; hence, \$60 must be 50% of what the horse cost A.

$$\text{Original cost} = \text{base} = \frac{\text{percentage}}{\text{rate}} = \frac{\$60}{.50} = \$120. \quad \text{Ans.}$$

$$\text{B's buying price} = 120\% \text{ of } \$120 = \$120 \times 1.20 = \$144. \quad \text{Ans.}$$

$$\text{C's buying price} = \text{original cost} + \$60 = \$120 + \$60 = \$180. \quad \text{Ans.}$$

(14) Since the same amount was invested in the two speculations, the net gain is $30\% - 25\% = 5\%$ of the investment. 5% of \$180 = $\$180 \times .05 = \9 , the gain. Ans.

(15) Of $206.95 + 32.06 = 239.01$ parts of ore there are 206.95 parts of lead; hence, the lead constitutes $\frac{206.95}{239.01} = .865863 = 86.5863\%$ of the ore. Ans. Likewise the sulphur constitutes $\frac{32.06}{239.01} = .134137 = 13.4137\%$ of the ore. Ans. Evidently in this example, the 239.01 is the base and the 206.95 and 32.06 are percentages; hence, to find the rates, we divide the percentages by the base.

(16) The flour cost $200 \times \$5.40 = \$1,080$. He sold 65% of it, or $.65 \times 200 = 130$ bbl. $130 \times \$5.90 = \767 . The remainder is $200 - 130 = 70$ bbl. At \$6.50 a barrel, 70 bbl. sell for $70 \times \$6.50 = \455 . Selling price = $\$767 + \$455 = \$1,222$. Gain = $\$1,222 - \$1,080 = \$142$. By Art. 39,

$$\text{rate of profit} = \frac{\text{profit}}{\text{cost}} = \frac{\$142}{\$1,080} = .1317 = 13\frac{1}{7}\%. \quad \text{Ans.}$$

(17) After losing 35% of his money he had $100\% - 35\% = 65\%$ of it left. Since he lost also 10% of the remainder = $.10 \times .65 = .065 = 6.5\%$ of his money, he had remaining $65\% - 6.5\% = 58\frac{1}{2}\%$ of it. \$17,550 is $58\frac{1}{2}\%$ of the original sum of money; hence, \$17,550 is the percentage, $58\frac{1}{2}\%$ is the rate per cent., and the original sum is the base.

$$\text{Base} = \frac{\text{percentage}}{\text{rate}} = \frac{\$17,550}{.58\frac{1}{2}} = \$30,000.$$

Therefore, he had originally \$30,000. Ans.

(18) See Art. 49. $(1 - .40) \times (1 - .20) \times (1 - .10) = .432$. After deducting discounts, the selling price is 43.2% of the list price. Since the selling price is \$216, the list price is $\$216 \div .432 = \500 . Ans.

(19) According to the statement of the question, the man earned the second year \$120 for every \$100 earned the first year; and for every \$120 earned the second year, he earned 25% more, or $\$120 \times (1 + .25) = \150 , the third year. The amounts earned in the three years are

proportional, therefore, to the numbers 100, 120, and 150. Using the rule of Art. 56, § 7, the sum of proportional parts = $100 + 120 + 150 = 370$, and the three proportions are:

$$100 : 370 = \text{earnings first year} : \$7,400.$$

$$120 : 370 = \text{earnings second year} : \$7,400.$$

$$150 : 370 = \text{earnings third year} : \$7,400.$$

From these proportions

$$\left. \begin{aligned} \text{Earnings first year} &= \frac{\$7,400 \times 100}{370} = \$2,000. \\ \text{Earnings second year} &= \frac{\$7,400 \times 120}{370} = \$2,400. \\ \text{Earnings third year} &= \frac{\$7,400 \times 150}{370} = \$3,000. \end{aligned} \right\} \text{Ans.}$$

(20) $20\% + 30\% + 40\% = 90\%$; using the rule for proportional parts, the proportions are:

$$90 : 20 :: \$9,459 : x; \text{ whence, } x = \frac{\$9,459 \times 20}{90} = \$2,102. \text{ Ans.}$$

$$90 : 30 :: \$9,459 : x; \text{ whence, } x = \frac{\$9,459 \times 30}{90} = \$3,153. \text{ Ans.}$$

$$90 : 40 :: \$9,459 : x; \text{ whence, } x = \frac{\$9,459 \times 40}{90} = \$4,204. \text{ Ans.}$$

(21) By Art. 41, the cost = selling price \div $(1 + \text{rate of gain})$ = $\$140 \div (1 + .12) = \$140 \div 1.12 = \$125$. To gain 28%, the selling price must be cost $\times (1 + \text{rate of gain}) = \$125 \times (1 + .28) = \$125 \times 1.28 = \160 . Ans.

(22) By Art. 83, 8 francs = $8 \times \$1.193 = \9.544 ; therefore, 18,600 yd. cost $18,600 \times \$1.544 = \$28,718.40$. Duty = $\$28,718 \times .22 = \$6,317.96$. Ans. The \$.40 is not considered, as it is less than \$.50 (see Art. 85).

(23) Commission and guarantee = $\$8,407.96 \times (.02 + .015) = \$8,407.96 \times .035 = \$294.28$. Total charges = $\$294.28 + \$275 = \$569.28$. Amount sent to principal = $\$8,407.96 - \$569.28 = \$7,838.68$. Ans.

(24) Tax on real estate = $\$7,800 \times .0125 = \97.50 . Tax on personal property = $\$5,640 \times .0125 = \70.50 . Poll tax = $\$.75 \times 3 = \2.25 . Total tax = $\$97.50 + \$70.50 + \$2.25 = \170.25 . Ans.

(25) The gain, \$32.40, is 18 per cent. of the cost; hence, the cost = $\$32.40 \div .18 = \180 . Selling price = $\$180 + \$32.40 = \$212.40$. Ans.

(26) By Art. 49, $1 - .20 = .80$; whence, $.80 \times .80 = .64$; single equivalent discount = $1 - .64 = .36 = 36\%$. Difference in favor of single discount = $40\% - 36\% = 4\%$. Gain by taking single discount = $\$987.50 \times .04 = \39.50 . Ans.

(27) In selling 3 oranges the boy receives the cost of 5 oranges. Hence, his profit on the 3 oranges is equal to the cost of $5 - 3 = 2$ oranges. Since in selling 3 oranges he gains the cost of 2 oranges, his gain is $\frac{2}{3}$ of the cost. $\frac{2}{3} = 66\frac{2}{3}\%$. Ans.

(28) By Art. 72, amount of insurance = premium + rate of premium = $\$267 + .015 = \$17,800$. Ans.

(29) By Art. 71, premium = $\$75,375 \times .0075 = \$565.31+$. Ans.

(30) $\pounds 378\ 15s. = \pounds 378\frac{3}{4}$, or $\pounds 378\frac{3}{4}$. $\pounds 1 = \$4.86\frac{1}{2}$ (see Table, Art. 83); hence, $\pounds 378\frac{3}{4} = 378.75 \times \$4.86\frac{1}{2} = \$1,843.25$. Rejecting the \$.25 according to Art. 85, duty = $\$1,843 \times .28 = \516.04 . Ans.

(31) Amount of assessment = $\$637.83 + .0285 = \$22,380$. Assessment on the personal property = $\$22,380 - \$18,600 = \$3,780$. Ans.

(32) Total expenses = $2\frac{1}{2}\% + \frac{1}{4}\% = 2\frac{3}{4}\%$ of gross proceeds. The net proceeds, \$6,613, is the difference, $2\frac{3}{4}\%$ is the rate per cent., and the gross proceeds is the base (see Art. 61); hence, gross proceeds = net proceeds $\div (1 - \text{rate}) = \$6,613 \div (1 - .02\frac{3}{4}) = \$6,613 \div .9725 = \$6,800$. Ans.

(33) See Art. 64. Deducting drayage and storage, the agent has $\$520.45 - (\$18.75 + \$8.50) = \493.20 to pay for peaches and his commission. Since his commission is computed on the prime cost as a base, \$493.20 must be the amount; hence, prime cost = $\$493.20 \div (1 + .0275) = \$493.20 \div 1.0275 = \$480$. At \$.75 a crate, he bought $480 \div .75 = 640$ crates. Ans.

(34) By Art. 72, amount of insurance = premium + rate of premium, or $\$372 + .015 = \$24,800$. Since \$24,800 is $\frac{4}{5}$, or 80%, of the value of the store and stock, $\$24,800 \div .80 = \$31,000$, is the total value of store and stock. Since the value of the store is to the contents as 9 is to 11, we have by the rule for proportional parts the following proportions:

Value of store : \$31,000 :: 9 : 20; whence, value of

$$\text{store} = \frac{\$31,000 \times 9}{20} = \$13,950. \text{ Ans.}$$

Contents : \$31,000 :: 11 : 20; whence,

$$\text{contents} = \frac{\$31,000 \times 11}{20} = \$17,050. \text{ Ans.}$$

(35) For the first series $(1 - .20)(1 - .15)(1 - .05) = .80 \times .85 \times .95 = .646$, and the equivalent single discount is $1.000 - .646 = .354 = 35.4\%$.

For the second series $(1 - .20)(1 - .10)(1 - .10) = .80 \times .90 \times .90 = .648$, and the equivalent single discount is $1.000 - .648 = 35.2\%$.

The difference is $35.4\% - 35.2\% = .2\%$; hence, \$25.48 is .2% of the amount of the bill, without discount. Since \$25.48 is the percent-

age and .2 is the rate per cent., the base or amount of the bill is $\$25.48 + .002 = \$12,740$. Ans.

(36) By Art. 71, premium = $\$25,000 \times .00875 = \218.75 . Premium for amount reinsured in second company = $\$25,000 \times .28 \times .005 = \35 . Premium for amount reinsured in the third company = $\$25,000 \times .48 \times .0075 = \80 . Amount paid out = $\$35 + \$80 = \$125$. The company reinsured $28\% + 48\% = 76\%$ of the risk. On this portion of the risk they received a premium of $\$25,000 \times .76 \times .007 = \166.25 , the company therefore gains $\$166.25 - \$125 = \$41.25$. Ans.

(37) The duty, $\$2,364.25$, is 35% of the value of the goods; hence, considering the value of goods as the base, $\$2,364.25$ is the percentage and 35 is the rate per cent.

$$\text{Invoice price} = \frac{\text{duty}}{\text{rate of duty}} = \frac{\$2,364.25}{.35} = \$6,755.$$

The price per yard is 40 francs $40 \times \$1.93 = \7.72 , the number of yards is therefore $6,755 \div 7.72 = 875$. Ans.

(38) A discount of 20% on $\frac{1}{4}$ of a bill is equivalent to a discount of $20\% \times \frac{1}{4} = 5\%$ on the whole bill. A discount of 15% on the remainder, or $\frac{3}{4}$ of the bill, is equivalent to a discount of $15 \times \frac{3}{4} = 3.75\%$ on the whole. The two discounts are therefore equivalent to a single discount of $5\% + 3.75\% = 8.75\%$ on the whole bill, which is 2.75% greater than a discount of 6% . From the statement of the question, $\$324.50$ is 2.75% of the whole bill, hence, the amount of the bill is $\$324.50 \div .0275 = \$11,800$. Ans.

(39) Allowing the first discount series, the dealer then receives $(1 - .20)(1 - .10)(1 - .10) = .80 \times .90 \times .90 = .648 = 64.8\%$ of his list price. With the second series he receives $(1 - .25)(1 - .15)(1 - .05) = .75 \times .85 \times .95 = .605625 = 60.5625\%$ of the list price. Difference, $64.8\% - 60.5625\% = 4.2375\%$. Since $\$33.90$ is 4.2375% of the list price, this price is $\$33.90 \div .042375 = \800 . Ans.

(40) A gain of 20% on 20% of the stock is equivalent to a gain of $20 \times 20 = .04 = 4\%$ on the whole stock. After selling 20% of the stock there is 80% of it remaining. 30% of this remainder is $.80 \times .30 = .24 = 24\%$ of the original stock. Since this was sold for 1.25 of the cost, the gain was 25% . A gain of 25% on 24% of the stock is equivalent to a gain of $.24 \times .25 = .06 = 6\%$ on the original stock. The remainder of the stock, after selling 20% and 24% of it, is 56% ($100 - 20 - 24$) of it. By selling this for $\frac{3}{4}$ of its cost, he loses $\frac{1}{4}$ or 12% . A loss of 12% on 56% of the stock, is equivalent to a loss of $.56 \times .125 = .07 = 7\%$. The total gain is $4\% + 6\% = 10\%$, and the loss is 7% ; hence, the net gain is $10\% - 7\% = 3\%$ of the cost. Ans.

ARITHMETIC.

(1) (a) Changing the time to years, 24 da. = $2\frac{4}{3}$ mo. = $\frac{4}{3}$ mo., 5 $\frac{1}{3}$ mo. = $2\frac{2}{3}$ mo. = $2\frac{2}{3}$ yr., $3\frac{2}{3}$ yr. = $2\frac{2}{3}$ yr. Then, $\$875 \times .05 \times 2\frac{2}{3} = \152.396 , or \$152.40. Ans.

(b) 20 da. = $\frac{2}{3}$ mo., 11 $\frac{2}{3}$ mo. = $2\frac{1}{3}$ mo. = $\frac{35}{3 \times 12}$ yr. = $2\frac{1}{6}$ yr., 2 $\frac{1}{6}$ yr. = $2\frac{1}{6}$ yr. $\$1,468.50 \times .045 \times 2\frac{1}{6} = \196.41 . Ans.

(2) (a) $3\frac{1}{2}\% \times 2 = 7\%$.

\$ 4 3 9.4 8
 .0 7

\$ 3 0.7 6 3 6 = interest for 2 yr.

7.6 9 0 9 = interest for 6 mo. = 2 yr. $\div 4$.

3.8 4 5 5 = interest for 3 mo. = 6 mo. $\div 2$.

.7 6 9 1 = interest for 18 da. = 6 mo. $\div 10$.

.1 2 8 2 = interest for 3 da. = 18 da. $\div 6$.

\$ 4 3.1 9 7 3, or \$43.20 = interest for 2 yr. 9 mo. 21 da. Ans.

(b) At $3\frac{1}{2}\%$ per year the rate for 5 yr. = $5 \times 3\frac{1}{2}\% = 19\%$.

\$ 5,6 2 8.4 0
 .1 9

\$ 1,0 6 9.3 9 6 0 = interest for 5 yr.

1 0 6.9 3 9 6 = interest for 6 mo. = 5 yr. $\div 10$.

1 7.8 2 3 3 = interest for 1 mo. = 6 mo. $\div 6$.

8.9 1 1 6 = interest for 15 da. = 1 mo. $\div 2$.

2.9 7 0 5 = interest for 5 da. = 15 da. $\div 3$.

5.3 4 7 0 = interest for 9 da. = 1 mo. $\times .3$.

\$ 1,2 1 1.3 8 8 0, or \$1,211.39 = interest for 5 yr. 7 mo. 29 da. Ans.

(3) (a) See Art. 15.

$$\text{Interest of \$1 at 6\% for 3 yr.} = .06 \times 3 = \$1.8$$

$$\text{Interest of \$1 at 6\% for 3 mo.} = .005 \times 3 = .015$$

$$\text{Interest of \$1 at 6\% for 3 da.} = .000\frac{1}{2} \times 3 = .0005$$

$$\text{Interest of \$1 at 6\% for 3 yr. 3 mo. 3 da.} = \$1.955$$

$$\text{Interest of \$4,280} = 4,280 \times \$1.955 = \$836.74. \text{ Ans.}$$

$$(b) \quad \text{Interest of \$1 at 6\% for 1 yr.} = \$0.6$$

$$\text{Interest of \$1 at 6\% for 11 mo.} = .055$$

$$\text{Interest of \$1 at 6\% for 27 da.} = .0045$$

$$\text{Interest of \$1 at 6\% for 1 yr. 11 mo. 27 da.} = \$1.195$$

$$\text{Interest at 6\%} = \$1.195 \times 5,670.80 = \$677.6606.$$

$$\text{Interest at 1\%} = \$677.6606 \div 6 = \$112.9434.$$

$$\text{Interest at } 4\frac{1}{2}\% = \$112.9434 \times 4\frac{1}{2} = \$508.2453+, \text{ or } \$508.25. \text{ Ans.}$$

(4) (a) The interest at 9% for half of a given time is equal to the interest at $4\frac{1}{2}\%$ for the whole of the time. Since the interest at 6% is \$834.72, the interest at $4\frac{1}{2}\%$ is $\frac{4\frac{1}{2}}{6}$, or $\frac{3}{4}$, of \$834.72. $\$834.72 \times \frac{3}{4} = \$626.04.$ Ans.

(b) If at 6% a certain principal in a certain time gives \$834.72 interest, $\frac{2}{3}$ of the same principal gives $\frac{2}{3}$ as much interest, or $\frac{2}{3}$ of \$834.72, in the same time; therefore, in $\frac{1}{2}$ of the time the interest is $\frac{2}{3} \times \frac{1}{2} \times \$834.72 = \$278.24.$ Ans.

(5) (a) If the annual income at 5% is \$4,060, at $4\frac{1}{2}\%$ it is $\frac{4\frac{1}{2}}{5} \left(= \frac{9}{10} \right)$ of \$4,060 = \$3,654. Ans.

(b) At $4\frac{2}{3}\%$ the interest is $\frac{4\frac{2}{3}}{5} \left(= \frac{14}{15} \right)$ of \$4,060 = \$3,789.33+. Ans.

(6) By Art. 20,

$$\text{Interest for 60 da.} = \$46.895$$

$$\text{Interest for 20 da.} = \frac{1}{3} \text{ of 60 da.} = \$15.6317$$

$$\text{Interest for 6 da.} = \frac{1}{10} \text{ of 60 da.} = \$4.6895$$

$$\text{Interest for 1 da.} = \frac{1}{6} \text{ of 6 da.} = \$0.7816$$

$$\text{Interest for 87 da.} = \$67.9978, \text{ or } \$68.$$

Ans.

$$(7) (a) \quad \text{Interest of \$1 at 6\% for 2 yr.} = \$1.2$$

$$\text{Interest of \$1 at 6\% for 9 mo.} = \$0.45$$

$$\text{Interest of \$1 at 6\% for 18 da.} = \$0.03$$

$$\text{Int. for 2 yr. 9 mo. 18 da.} = \$1.68$$

Interest at 1% = $\$1.68 \div 6 = \$0.28.$ Interest at 5% = $\$1.68 - \$0.28 = \$1.40 = 14\%$ of \$1.00; hence, the principal must be multiplied by .14. Ans.

(b) Interest at 6% = \$.168, at 1% = \$.028, and at $4\frac{1}{2}\%$ = $4\frac{1}{2} \times $.028 = $.126; hence, the multiplier is .126. Ans.$

(8) 3 yr. 8 mo. 15 da. = $\frac{89}{24}$ yr. $5\frac{1}{2}\% - 4\frac{3}{4}\% = \frac{5}{8}\% = .00\frac{5}{8}$.
 $\$6,600 \times .00\frac{5}{8} \times \frac{89}{24} = \203.96 . Ans.

(9) By the formula, Art. 37, $t = \frac{100 I}{Pr} = \frac{100 \times .2365}{1 \times 6} = \frac{23.65}{6}$
 = $3.94\frac{1}{8}$ yr. $.96\frac{1}{8} \times 12 = 11.3$ mo.; $.3 \times 30 = 9$ da. Therefore, the time is 3 yr. 11 mo. 9 da. Ans.

(10) Number of days in January, February, March, and April = $31 + 28 + 31 + 30 = 120$ da. By Art. 20, interest for 60 da. = \$240, and for 120 da. = $2 \times \$240 = \480 . By Art. 26, the exact interest is $\$480 - (\frac{1}{3} \text{ of } \$480) = \$480 - \$6.58 = \$473.42$. Ans.

(11) In Feb., 1896, there were 29 da., it being a leap year; 29 da. = $\frac{29}{366}$ yr. Exact interest = $\$100,000 \times .06 \times \frac{29}{366} = \475.41 . Feb., 1897, had 28 da.; and 28 da. = $\frac{28}{366}$ yr. The exact interest = $\$100,000 \times .06 \times \frac{28}{366} = \460.27 . Difference, $\$475.41 - \$460.27 = \$15.14$. Ans.

(12) From July 1, 1898, to July 1, 1899 = 1 yr. = 365 da. From July 1, 1899, to April 25, 1900 = $30 + 31 + 30 + 31 + 30 + 31 + 31 + 28 + 31 + 25 = 298$ da. $365 + 298 = 663$ da. The exact interest = $\$14,600 \times .06 \times \frac{663}{366} = \$1,591.20$. Ans.

(13) (a) 5 yr. 3 mo. = $5\frac{1}{4}$ yr. Interest = $\$4,780 \times .08 \times 5\frac{1}{4} = \$2,007.60$.
 Ans.

(b) 4 yr. 9 mo. 20 da. = $4\frac{17}{8}$ yr. Interest = $\$8,690 \times .075 \times 4\frac{17}{8} = \$3,132.02$. Ans.

(14) By Art. 15, the interest on \$1 for 3 yr. 5 mo. 18 da. at 6% = \$.208. At $4\frac{1}{2}\%$ the interest is $$.208 \times \frac{4\frac{1}{2}}{6} = $.208 \times $\frac{3}{4}$ = $.156. Since $.156 is .156 of $1.00, the interest is .156 of the principal. Ans.$

(15) (a) 2 yr. 234 da. = $2\frac{334}{366}$ yr. = $\frac{769}{915}$ yr. Int. = $\$2,375 \times .045 \times \frac{769}{915} = \282.27 . Ans.

(b) $\$468.90 \times .05 \times \frac{110}{366} = \$8.968+$, or \$8.97. Ans.

(16) From April 8, 1899, to Oct. 17, 1899 = 192 da., or $\frac{192}{366}$ yr. Exact interest = $\$928.60 \times .07 \times \frac{192}{366} = \34.19 . Ans.

(17) Since the interest is compounded semiannually, in 3 yr. 7 mo. 20 da. there are seven periods of 6 mo. each, and 1 mo. 20 da. in addition. Since the annual rate is 6%, for 6 mo. the rate is 3%. Referring to the compound-interest table, Art. 54, the amount of \$1 for 7 yr. at 3% is \$1.229874. The simple interest for 1 mo. 20 da., or 50 da., at 6% is $\$1.229874 \times .008\frac{1}{3} = $.01024895. The total amount is $\$1.229874 + $.01024895 = \$1.24012295$, say $1.240123. $3,690 \times \$1.240123 = \$4,576.05+$. $\$4,576.05 - \$3,690 = \$886.05$. Ans.$

(18) 270 da. = $\frac{11}{4}$ yr. = $\frac{3}{4}$ yr., considering 30 da. as a month.
 Ordinary interest = $\$3,600 \times .06 \times \frac{3}{4} = \162 .

According to Art. 23, the difference is $\frac{1}{73}$ of the ordinary interest;
 $\$162 \div 73 = \2.22 . Ans.

(19) See Art. 29.

$$\begin{array}{r}
 \text{Interest for 1 yr.} = \$8,000 \times .06 = \underline{\$480} \\
 \text{Interest for } 4\frac{3}{4} \text{ yr.} = \$480 \times 4\frac{3}{4} = \underline{\$2,280} \\
 \text{Interest of } \$480 \text{ for } 3\frac{3}{4} \text{ yr.} + 2\frac{3}{4} \text{ yr.} + 1\frac{3}{4} \text{ yr.} + \frac{3}{4} \\
 \text{yr.} = \text{interest for 9 yr.} = \$480 \times .06 \times 9 = \underline{\$259.20} \\
 \text{Total interest} \dots\dots\dots = \underline{\$2,539.20} \text{ Ans}
 \end{array}$$

(20) 2 yr. 3 mo. 18 da. = 2.3 yr. By Art. 40,

$$\text{Principal} = \frac{100 \times 535.20}{100 + 5 \times 2.3} = \frac{53,520}{111.5} = \$480. \text{ Ans.}$$

(21) 3 yr. 5 mo. 20 da. = $3\frac{5}{6}$ yr. By Art. 32,

$$\text{Principal} = \frac{100 \times \$150}{6 \times \frac{125}{36}} = \frac{\$15,000}{\frac{125}{6}} = \$15,000 \times \frac{6}{125} = \$720. \text{ Ans}$$

(22) 6 mo. 24 da. = $\frac{17}{6}$ yr. By Art. 46,

$$\begin{aligned}
 \text{Present worth} &= \frac{100 \times \$1,260}{100 + 5 \times \frac{17}{6}} = \frac{\$126,000}{100 + \frac{17}{6}} = \$126,000 \times \frac{6}{617} \\
 &= \$1,225.28. \text{ Ans.}
 \end{aligned}$$

(23) See Art. 52.

$$\begin{array}{rcl}
 \$240 & = & \text{prin. 1st yr.} \\
 \$12 & = & \text{int. 1st yr.} = \$240 \times .05. \\
 \hline
 \$252 & = & \text{prin. 2d yr.} \\
 \$12.60 & = & \text{int. 2d yr.} = \$252 \times .05. \\
 \hline
 \$264.60 & = & \text{prin. 3d yr.} \\
 \$13.23 & = & \text{int. 3d yr.} = \$264.60 \times .05. \\
 \hline
 \$277.83 & = & \text{prin. 4th yr.} \\
 \$13.89 & = & \text{int. 4th yr.} = \$277.83 \times .05. \\
 \hline
 \$291.72 & = & \text{prin. for 5 mo. 20 da.} \\
 \$6.89 & = & \text{int. for 5 mo. 20 da.} = \$291.72 \times .05 \times \frac{17}{6}. \\
 \hline
 \$298.61 & = & \text{amt. for 4 yr. 5 mo. 20 da.} \\
 \$240.00 & = & \text{original principal.} \\
 \hline
 \$58.61 & = & \text{comp. int. for 4 yr. 5 mo. 20 da.} \text{ Ans}
 \end{array}$$

(24) See Art. 87. At 6%, interest on \$1 for

$$\begin{array}{r}
 90 \text{ da.} = \$.015 \\
 3 \text{ da.} = \underline{\$.0005} \\
 93 \text{ da.} = \$.0155
 \end{array}$$

Proceeds of \$1 for 90 + 3 da. = $\$1 - \$.0155 = \$.9845$. Face of the note = $\$1,200 + .9845 = \$1,218.89$. Ans.

(25) Interest of \$1 at 6% for 60 + 3 da., or 63 da. = \$.0105. Bank discount = $\$2,400 \times .0105 = \25.20 . Ans.

(26)

$\$6,078\frac{51}{100}$

Newark, N. J., Dec. 6, 1898.

Ninety days after date, for value received, I promise to pay Charles Allen, or order, Six Thousand Seventy-Eight and

$\frac{51}{100}$ *Dollars, at the Ninth National Bank.*

John Clark.

Proceeds of \$1 at 5% for 90 + 3 da. = $\$.9870\frac{5}{8}$. By Art. 87, face of the note = $\$6,000 \div .9870\frac{5}{8} = \$6,078.51+$. Ans.

(27) Referring to the compound-interest table, the amount of \$1 for 5 yr. at 4% is \$1.216653. The simple interest for 9 mo. on \$1.216653 at 4% is $\$1.216653 \times .03 = \$.03649959$. The total amount is $\$1.216653 + \$.03649959 = \$1.253153-$. To amount to \$8,000 in 5 yr. 9 mo. at 4%, the principal must be $\$8,000 \div 1.253153 = \$6,383.90$. Ans.

(28) See Art. 59.

Principal.....	\$ 2,9 6 0.0 0
Interest from April 1 to July 1 (3 mo.).....	4 4.4 0
Amount.....	\$ 3,0 0 4.4 0
First payment.....	4 0 0.0 0
New principal.....	\$ 2,6 0 4.4 0
Interest from July 1, 1899, to Jan. 21, 1900 (6 mo. 20 da.)....	8 6.8 1
Amount.....	\$ 2,6 9 1.2 1
Sum of 2d and 3d payments.....	7 3 0.0 0
New principal.....	\$ 1,9 6 1.2 1
Interest from Jan. 21 to Mar. 24 (2 mo. 3 da.).....	2 0.5 9
Amount.....	\$ 1,9 8 1.8 0
Fourth payment.....	8 0 0.0 0
New principal.....	\$ 1,1 8 1.8 0
Interest from Mar. 24 to Apr. 25 (1 mo. 1 da.).....	6.1 1
Amount due at time of settlement.	\$ 1,1 8 7.9 1
	Ans.

(29) See Art. 84.

Maturity, Oct. 2, 1900.

Amount of note at maturity.....	\$ 1,2 2 4.8 0
Term of discount, 31 da.	
Discount.....	6.3 3
Proceeds.....	\$ 1,2 1 8.4 7. Ans.

(30) The bank discount on \$1 at 6% for 60 da. + 3 da. = \$.0105.
To yield \$45.80, the face of the note must be $\$45.80 \div .0105 = \$4,361.90$.
Ans.

(31) By Art. 38,

$$\text{Time} = \frac{\overset{5}{100} \times \overset{23.22}{139.32}}{\underset{48}{899} \times 5} = \frac{116.1}{48} = 2.7 \text{ yr.}$$

.7 yr. = .7 × 12 mo. = 8.4 mo.; .4 mo. = .4 of 30 da. = 12 da. Hence,
the time is 2 yr. 8 mo. 12 da. Ans.

(32) By Art. 38,

$$\text{Time} = \frac{\overset{20}{100} \times \overset{.4}{58.40}}{\underset{145}{2,920} \times 5} = .4 \text{ yr.}$$

.4 yr. = .4 of 365 da. = 146 da. 146 da. after April 1, 1900, is
Aug. 25, 1900. Ans.

(33) From Feb. 22, 1900, to July 4, 1900, is 132 da. = $\frac{1}{3}\frac{1}{3}$ yr.
 $\$10,000 \times .045 \times \frac{1}{3}\frac{1}{3} = \162.74 . Ans.

(34)

Principal	\$ 1 2,0 0 0.
Exact interest from May 10, 1894, to Jan. 9, 1895 (244 da.)	4 8 1.3 2
Amount	\$ 1 2,4 8 1.3 2
Sum of first and second payments (\$200 + \$960).....	1,1 6 0.0 0
New principal.	\$ 1 1,3 2 1.3 2
Exact interest from Jan. 9 to Mar. 1 (51 da.).....	9 4.9 0
Amount	\$ 1 1,4 1 6.2 2
Third payment	1 5 0.0 0
New principal.....	\$ 1 1,2 6 6.2 2
Exact interest from Mar. 1 to May 1 (61 da.)	1 1 2.9 7
Amount	\$ 1 1,3 7 9.1 9
Fourth payment	5 0 0.0 0
New principal.....	\$ 1 0,8 7 9.1 9
Interest from May 1 to Sept. 1 (123 da.).....	2 1 9.9 7
Amount due Sept. 1, 1895, by U. S. rule.....	\$ 1 1,0 9 9.1 6

By Art. 62.

Principal	\$ 1 2,0 0 0.
Interest of \$12,000 from May 10, 1894, to Sept. 1, 1895 (1 yr. 114 da.).....	9 4 4.8 7 6
Amount	\$ 1 2,9 4 4.8 7 6

Amount of \$200 from Sept. 1, 1894, to Sept. 1, 1895 (1 yr.)..	\$ 2 1 2.0 0
Amount of \$960 from Jan. 9 to Sept. 1 (235 da.).....	9 9 7.0 8
Amount of \$150 from Mar. 1 to Sept. 1 (184 da.).....	1 5 4.5 4
Amount of \$500 from May 1 to Sept. 1 (123 da.).....	5 1 0.1 1
Sum of payments with interest.....	\$ 1,8 7 3.7 3

Amount due at time of settlement ($12,944.876 - 1,873.735$) = \$11,071.14.
 The amount due by the U. S. rule is \$11,099.16; the difference is
 $\$11,099.16 - \$11,071.14 = \$28.02$. Ans.

(35) By Art. 34,

$$\text{Rate} = \frac{100 \times 148.31}{1,280 \times 2\frac{1}{8}} = \frac{\overset{5}{10} \times 148.31 \times \overset{15}{60}}{\underset{16}{1,280} \times 149} = 4\frac{1}{2}\%. \text{ Ans.}$$

(36) The bank discount for 93 da. at 6% = $.0155 \times \$10,860 = \168.33 .
 The owner should receive the true present worth. By Art. 48,

$$\text{Present worth} = \frac{100 \times 10,860}{100 + \frac{6}{100} \times 93} = \frac{10,860,000}{101.55} = \$10,694.24.$$

The bank discount should be $\$10,860 - \$10,694.24 = \$165.76$. The
 difference is $\$168.33 - \$165.76 = \$2.57$. Ans.

(37) By Art. 31,

$$\text{Principal} = \frac{100 \times 66}{6 \times \frac{3}{80}} = \frac{6,600}{\frac{3}{4}} = 6,600 \times \frac{4}{3} = \$12,000.$$

This is really the amount, or the face of the note and the interest.
 Then, by Art. 40,

$$\text{Principal or face} = \frac{100 \times 12,000}{100 + 6 \times \frac{3}{80}} = \frac{1,200,000}{1\frac{9}{10}} = \$11,912.64. \text{ Ans.}$$

(38) See Art. 29.

Interest for 1 yr. = $\$8,000 \times .06$	\$ 4 8 0
Interest for 5 yr. 7 mo. 24 da. ($\frac{339}{80}$ yr.) = $\$480 \times \frac{339}{80}$.	\$ 2,7 1 2
Interest of \$480 for $4\frac{3}{80} + 3\frac{3}{80} + 2\frac{3}{80} + 1\frac{3}{80} + \frac{3}{80}$ = inter-	
est for $13\frac{1}{4}$ yr. = $\$480 \times .06 \times 13\frac{1}{4}$	3 8 1.6 0
	<u>\$ 3,0 9 3.6 0</u>
	8,0 0 0

$$\text{Amount due} = \$8,000 + \$3,093.60 \dots\dots \$ 11,0 9 3.6 0 \text{ Ans.}$$

(39) The difference in the rate is $5\frac{1}{2}\% - 4\frac{1}{2}\% = 1\%$. 1 yr. 9 mo.
 18 da. = $\frac{9}{10}$ yr.

By Art. 31,

$$\text{Principal} = \frac{100 \times 42}{1\frac{1}{2} \times \frac{9}{10}} = \frac{4,200}{\frac{3}{2}} = \frac{\overset{200}{4,200}}{1} \times \frac{10}{21} = \$2,000. \text{ Ans.}$$

(40) Since $\frac{3}{4}$ of A's money equals $\frac{3}{4}$ of B's, then the interest on $\frac{3}{4}$ of B's = $1\frac{1}{2}\%$ = \$660, and the interest on $\frac{3}{4}$ of A's = $1\frac{1}{2}\%$ = \$660. By Art. 31, principal, or $\frac{3}{4}$ of A's money = $\frac{100 \times 660}{5\frac{1}{2} \times 4} = \frac{66,000}{22} = 3,000$;

$$3,000 = \frac{3}{4}; \frac{4}{4} = \frac{4}{3} \times \frac{1,000}{3,000} = \$4,000, \text{ A's money. Ans.}$$

$$\text{Since } \frac{3}{4} \text{ of B's money} = \$3,000, \frac{3}{4} = \frac{3}{2} \times \frac{1,500}{3,000} = \$4,500, \text{ B's money.}$$

Ans.

ARITHMETIC.

(1) Apply rule, Art. 14,

$$(113\frac{1}{8} + \frac{1}{8}) \times 480 = \$54,600. \quad \text{Ans.}$$

(2) Yearly dividends $= 2 \times 3\frac{1}{2}\% = 7\%$. At \$100 a share, the yearly dividend on 1 share would be $\$100 \times .07 = \7 , and to yield \$5,600 it would take $5,600 \div 7 = 800$ shares. Cost $= (108\frac{1}{2} + \frac{1}{8}) \times 800 = \$86,900.$
Ans.

(3) Apply rule, Art. 14, $38,979 \div (79\frac{3}{4} + \frac{1}{8}) = 488$ shares. Ans.

(4) (a) At $\frac{1}{4}\%$ commission, the broker receives on 1 share $.0025 \times \$100 = \$.25$, and to receive \$50 he bought $50 \div .25 = 200$ shares. Ans.

(b) Cost $= \$110\frac{1}{2} \times 200 = \$22,100.$ Ans.

(5) Apply rule, Art. 14 $(114\frac{1}{8} + \frac{1}{8}) \times 125 = \$14,281.25.$ Ans.

(6) Yearly dividend $= 2\% \times 4 = 8\%$. By Art. 16, number of shares $= 4,800 \div 8 = 600.$ Cost $= (112\frac{3}{8} + \frac{1}{8}) \times 600 = \$67,500.$ Ans.

(7) Yearly dividend $= 4 \times 3\% = 12\%$, or \$12 on 1 share. Since he paid \$120 for 1 share, he receives per annum on his investment $12 \div 120 = 10\%.$ Ans.

(8) Per cent. received on investment in first case is $10 \div 110 = 9\frac{1}{11}\%$; in the second case, $8 \div 90 = 8\frac{2}{9}\%$. Therefore, the former is better by $9\frac{1}{11}\% - 8\frac{2}{9}\% = \frac{2}{99}\%.$ Ans.

(9) Use rule, Art. 27. $\$4,875 \times .01375 = \$67.03.$ $\$4,875 - \$67.03 = \$4,807.97.$ Ans.

(10) Use rule, Art. 27.

$$\$5,600 \times .01125 = \$63. \quad \$5,600 + \$63 = \$5,663. \quad \text{Ans.}$$

(11) Apply rule given in Art. 30. Interest on \$8,000 at 6% for 60 + 3 da. $= \$8,000 \times .0105 = \$84.$ Proceeds of \$8,000 $= \$8,000 - \$84 = \$7,916.$ Premium $= \$8,000 \times .015 = \$120.$ Therefore, the cost is $\$7,916 + \$120 = \$8,036.$ Ans.

(12) Interest of \$6,800 for 93 da. at 5% = $\$6,800 \times .0129\frac{1}{2} = \$87.83+$.
 Proceeds of \$6,800 = $\$6,800 - \$87.83+ = \$6,712.17-$. The discount
 = $\$6,800 \times .0125 = \85 . Cost of the draft = $\$6,712.17 - \85
 = $\$6,627.17-$. Ans.

(13) Use rule, Art. 27. $\$2,800 \times .0075 = \21 ; $\$2,800 + \$21 = \$2,821$.
 Ans.

(14) Use rule, Art. 33. Interest of \$1 for 63 da. at 6% = .0105.
 Proceeds of \$1 = $\$1 - \$.0105 = \$.9895$. Discount on \$1 = \$.015.
 Proceeds - discount = $.9895 - .015 = \$.9745$. Face of draft
 = $\$12,000 \div .9745 = \$12,314.01$. Ans.

(15) Apply rule, Art. 33. Proceeds of \$1 = $\$1 - \$.0125 = \$.9875$;
 $\$10,000 \div .9875 = \$10,126.58$. Ans.

(16) Apply rule, Art. 40. £400 15s. = $\pounds 400\frac{3}{4}$. $\$4.855 \times 400.75$
 = $\$1,945.64$. Ans.

(17) 1 franc = $\frac{\$1}{5.22}$; $\frac{\$1}{5.22} \times 20,000 = \frac{20,000}{5.22} = \$3,831.42$. Ans.

(18) $6,000 \times \$.40\frac{1}{2} = \$2,422.50$. Ans.

(19) Since 1 reichsmark is worth $94\frac{1}{2}$ cents, or \$.94625, \$1,541
 = $\$1,541 \div .94625 = 1,628.53$ reichsmarks. Ans.

(20) $10,000 \div 24.85 = \pounds 402.4145$. $402.4145 \times \$4.84 = \$1,947.69$.
 Ans.

(21) The example may be solved in the following manner:

$10,000 \times 1.00\frac{1}{2}$ reichsmarks	11.85 guilders.
$1 \times 1.00\frac{1}{2}$ guilders	.425 reichmark.
£1	\$.r.
\$4.85	£1.

Reducing the decimals to common fractions,

$$\frac{10,000 \times \frac{801}{800} \times \frac{801}{800} \times 4.85}{11.85 \times .425} = \frac{10,000 \times 801 \times 801 \times 4.85}{11.85 \times .425 \times 800 \times 800} = \$9,654.27+.$$

Ans.

(22)	2	
	6	
	30 bu. wheat	x bu. oats
	5 bu. oats	3 bu. corn
	7 bu. corn	4 bu. rye
	8 bu. rye	5 bu. wheat
	2	

$$x = 2 \times 5 \times 7 \times 2 = 140 \text{ bu. of oats. Ans.}$$

As in example in Art. 44, place the unknown quantity x on the right-hand side of the vertical line of division, and the quantity to

which x is equivalent on the left-hand side. Arrange the other quantities so that a quantity of the same kind appears on each side of the line. Thus, 30 bu. of wheat are equivalent to a certain number of bushels of oats; hence, we place 30 bu. of wheat on the left and x bu. of oats on the right. Now, since 3 bu. of corn are worth 5 bu. of oats, and we already have oats on the right-hand side, we place the 5 bu. of oats on the left-hand side, and the 3 bu. of corn on the right-hand side. We then place 7 bu. of corn on the left and 4 bu. of rye on the right, and also 8 bu. of rye on the left and 5 bu. of wheat on the right. We have then wheat, oats, corn, and rye on each side of the line. Canceling and dividing the product of the numbers on the left by the product of those on the right, $x = 140$ bu. of oats.

(23)	1 cu. in. gold	x cu. in. aluminum
	19.3 cu. in. lead	11.36 cu. in. gold
	11.36 cu. in. silver	10.5 cu. in. lead
	21 cu. in. zinc	14.24 cu. in. silver
	7.12 cu. in. aluminum	2.58 cu. in. zinc

$$x = \frac{1 \times 19.3 \times 11.36 \times 21 \times 7.12}{11.36 \times 19.3 \times 14.24 \times 2.58} = \frac{19.3}{2.58} = 7.48 \text{ cu. in. of aluminum.}$$

Ans.

(24)	A earns in 1 da.	E earns in x da.
	E " " 9 "	D " " 5 "
	D " " 11 "	C " " 6 "
	C " " 7 "	B " " 3 "
	B " " 5 "	A " " 4 "

$$x = \frac{1 \times 9 \times 11 \times 7 \times 5}{5 \times 6 \times 3 \times 4} = \frac{77}{8} = 9.625 \text{ da.}$$

It is first necessary to find how many days of E's work are equivalent to 1 da. of A's work; we find this to be $9\frac{5}{8}$, or 9.625, da. In other words, A earns 9.625 times as much as E. Hence, while E is earning \$100, A earns $9.625 \times \$100 = \962.50 . Ans.

(25)	A goes 50 mi.	D goes x mi.	100	$2x$
	D " $8\frac{1}{2}$ "	C " 9 "	17	18
	C " $9\frac{1}{2}$ "	B " 10 "	19	20
	B " $11\frac{1}{2}$ "	A " 12 "	23	24

or,

We multiply all the numbers by 2 to get rid of the fractions. Therefore,

$$x = \frac{100 \times 17 \times 19 \times 23}{2 \times 18 \times 20 \times 24} = \frac{5 \times 17 \times 19 \times 23}{2 \times 18 \times 24} = 42.99+ \text{ mi. Ans.}$$

(26) Use rule, Art. 51.

$$\$320 \text{ for 5 mo.} = \$320 \times 5 = \$1600 \text{ for 1 mo.}$$

$$\$450 \text{ for 8 mo.} = \$450 \times 8 = \underline{3600} \text{ for 1 mo.}$$

$$\$5200 \text{ for 1 mo.}$$

$\$5,200$ for 1 mo. equals $\$500$ for $\frac{5200}{500}$ mo. = 10.4 mo. = 10 mo. 12 da.

Ans.

(27) $\$800 \text{ for 30 da.} = \24000 for 1 da.

$$\$500 \text{ for 20 da.} = \underline{10000} \text{ for 1 da.}$$

$$\$400 \text{ for 60 da.} = \underline{24000} \text{ for 1 da.}$$

$$\$58000 \text{ for 1 da.}$$

$\$58,000$ at interest for 1 da. equals $\$1,000$ at interest for $\frac{58000}{1000}$, or 58 da. Ans.

(28) $\$1200 \text{ for 30 da.} = \36000 for 1 da.

$$\$3600 \text{ for 60 da.} = \$216000 \text{ for 1 da.}$$

$$\$1200 \text{ for 90 da.} = \underline{\$108000} \text{ for 1 da.}$$

$$\begin{array}{r} \$6000 \\) \$360000 \end{array}$$

60 da.

The whole debt may be equitably paid 60 da. after the obligation is incurred. Ans.

(29) Applying rule, Art. 51,

$$\$360 \times 0 = 0$$

$$\$560 \times 40 = \$22400$$

$$\$800 \times 76 = \$60800$$

$$\$1000 \times 92 = \underline{\$92000}$$

$$\begin{array}{r} \$2720 \\) \$175200 \end{array}$$

64.4+, or 64 da. after July 1, or Sept. 3. Ans.

Taking July 1, the time when the first debt is due, as the date of reference, or the time from which to determine the terms of credit, the term of credit for $\$360$ is 0 da. The term of credit for $\$560$ is from July 1 to Aug. 10, or 40 da.; for $\$800$, the term of credit is from July 1 to Sept. 15, or 76 da.; for $\$1,000$ the term of credit is from July 1 to Oct. 1, or 92 da. We multiply each debt by its term of credit and divide the sum of the products by the sum of the debts. The quotient, 64 da., is the number of days after July 1 when one payment of the whole indebtedness may equitably be made, or Sept. 3.

(30) $\$800 \times 0 = 0$

$$\$900 \times 39 = \$35100$$

$$\$1000 \times 59 = \underline{59000}$$

$$\$1200 \times 91 = \underline{109200}$$

$$\begin{array}{r} \$3900 \\) \$203300 \end{array}$$

52+

52 da. after Sept. 1, or Oct. 23. Ans.

(85) Use rule, Art. 72.

$$\begin{array}{r}
 \$1.20 \times 84 = \$100.80 \\
 \$.90 \times 48 = \quad 43.20 \\
 \$.75 \times 48 = \quad 36.00 \\
 \hline
 180 \quad) \$180.00 \\
 \hline
 \$1.00 \text{ Ans.}
 \end{array}$$

(86) Use rule, Art. 72.

$$\begin{array}{r}
 \$1080 \times 24 = \$25920 \\
 \$2160 \times 36 = \quad 77760 \\
 \$3240 \times 48 = 155520 \\
 \hline
 108 \quad) \$259200 \\
 \hline
 \text{Average cost of a lot} = \$2400 \\
 \text{Average gain on 1 lot} = \quad 100 \\
 \hline
 \text{Selling price, } \$2500 \text{ Ans.}
 \end{array}$$

(87) Use rule, Art. 78.

$$40 \left\{ \begin{array}{l} 29 \\ 37 \\ 41 \\ 47 \end{array} \right. \begin{array}{l} 7 \\ 1 \\ 3 \\ 11 \end{array} \begin{array}{l} 7 \times 29 = 203 \\ 1 \times 37 = \quad 37 \\ 3 \times 41 = 123 \\ 11 \times 47 = 517 \end{array} \\
 \hline
 880 \text{ Ans.}$$

(88) Use rule, Art. 78.

$$87 \left\{ \begin{array}{l} \$5 \\ 6\frac{1}{2} \\ 8 \end{array} \right. \left\{ \begin{array}{l} 1 \\ 1 \\ 2 \end{array} \right. \left\{ \begin{array}{l} 1 \\ 1 \\ 2\frac{1}{2} \end{array} \right. \times 2 = \left\{ \begin{array}{l} 2 \\ 2 \\ 5 \end{array} \right. \text{ Ans.}$$

(89) Use rule, Art. 78.

$$85 \left\{ \begin{array}{l} 22\frac{1}{2} \\ 31 \\ 41 \\ 71\frac{1}{2} \end{array} \right. \left\{ \begin{array}{l} 2 \\ 2 \\ 2 \\ 2 \end{array} \right. \left\{ \begin{array}{l} 2\frac{1}{2} \\ 2\frac{1}{2} \\ 2\frac{1}{2} \\ 4\frac{1}{2} \end{array} \right. \times 2 = \left\{ \begin{array}{l} 5 \\ 5 \\ 5 \\ 9 \end{array} \right. \text{ Ans.}$$

(40) If 1,280 bu cost \$960, 1 bu. cost $\$960 \div 1,280 = \$.75$.

$$75 \left\{ \begin{array}{l} 100 \\ 80 \\ 60 \\ 40 \end{array} \right. \left\{ \begin{array}{l} 35 \\ 15 \\ 5 \\ 25 \end{array} \right. \left\{ \begin{array}{l} 35 \times 16 = 560 \\ 15 \times 16 = 240 \\ 5 \times 16 = \quad 80 \\ 25 \times 16 = 400 \end{array} \right. \text{ Ans.} \\
 \hline
 80 \quad 1280$$

$$1,280 \div 80 = 16.$$

$$\begin{array}{rcl}
 75 \left\{ \begin{array}{l} 100 \\ 80 \\ 60 \\ 40 \end{array} \right. & \begin{array}{l} 15 \\ 35 \\ 25 \\ 5 \end{array} & \begin{array}{l} 15 \times 16 = 240 \\ 35 \times 16 = 560 \\ 25 \times 16 = 400 \\ 5 \times 16 = 80 \end{array} \left. \vphantom{\begin{array}{l} 100 \\ 80 \\ 60 \\ 40 \end{array}} \right\} \text{Ana.} \\
 & 80 & \underline{80} \qquad \underline{1280}
 \end{array}$$

The sum of the proportions 35, 15, 5, and 25 = 80 bu. Also 15, 35, 25, and 5 = 80 bu. The miller bought 1,280 bu., or $1,280 \div 80 = 16$ times as many. Hence, the proportions must be multiplied by 16.

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INDEX.

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